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INSTABILITY OF TWO ROTATING VISCOELASTIC SUPERPOSED FLUIDS WITH SUSPENDED PARTICLES IN POROUS MEDIUM

By

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ABSTRACT

The instability of the plane interface between two Oldroydian viscoelastic superposed fluids permeated with suspended particles and uniform rotation in porous medium is considered. The cases of two uniform viscoelastic fluids separated by a horizontal boundary and exponentially varying density, viscosity and suspended particles number density are considered. The system is found to be stable for potentially stable configuration/stable stratification and unstable for potentially unstable configuration/unstable stratification. The viscoelasticity, suspended particles, rotation and porous medium effects do not affect the stability or instability of the system.

1. INTRODUCTION. A detailed account of the instability of the plane interface between two Newtonian viscous fluids under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar[1], Bhatia and Steiner[2] have studied the problem of thermal instability of a Maxiwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destablizing effect in contrast to the stabilizing effect on Newtonian fluid. Bhatia and Steiner[3] have also considered the problem of thermal instability of a Maxwellian fluid in hydromagnetics and have found that the magnetic field has stabilizing effect on viscoelastic fluid just as in the case of Newtonian fluid. Eltayeb[4] has studied the convective instability in a rapidly rotating Oldroydian viscoelastic fluid. An experimental demonstration by Toms and Strawbridge[5] reveals that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of oldroyd fluid.

In geophysical situations, more often than not, the fluid is not pure, but may instead be permeated with suspended (or dust) particles. Scanlon and Segel[6] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The effect of suspended particles was thus found to destablize the layer. The medium has been considered to be non- porous in all the above studies.

The flow through porous medium has been of considerable interest in recent years particularly among petroleum engineers and geophysical fluid dynamicists. A macroscopic equation which describes incompressible flow of Newtonian fluid of viscosity μ through a homogeneous and isotropic porous medium of permeability k_1 is the Darcy's equation. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-(\mu/k_1)\vec{q}$, where $\vec{q}$ is the filter velocity of the fluid. The thermal instability of fluids in a porous medium in the presence of suspended particles has been studied by Sharma and Sharma[7]. The effects of suspended particles and medium permeability were found to destabilize the layer.

The present paper attempts to study the stability of the plane interface separating two incompressible superposed rotating, Oldroydian viscoelastic fluids, permeated with suspended particles, in a porous medium. The constitutive relations of the type of Oldroyd fluid were proposed and studied by Oldryod[8]. The knowledge regarding fluid-particle mixtures is not commensurate with thier scientific and industrial importance. The analysis would be relevant to the stability of some polymer solutions like a dilute solution of methyl methacrylate in n-butyl acetate and to the stability of some Maxwellian viscoelastic fluids and the problem finds its usefulness in several geophysical situations and in chemical technology. These aspects form the motivation for the present study.

2. PERTURBATION EQUATIONS. Let $T_{ij}, z_{ij}, e_{ij}, \delta_{ij}, \mu, \lambda, \lambda_0(<\lambda)$, p, v_i, x_i and d/dt denote respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the Kronecker delta, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the velocity vector, the position vector and the mobile operator. Then the Oldroydian viscoelastic fluid is described by the constitutive relations

$$T_{ij} = -p\delta_{ij} + z_{ij},$$

$$(i + \lambda \frac{d}{dt}) z_{ij} = 2\mu \,(1 + \lambda_0 \frac{d}{dt})\, e_{ij}, \qquad \ldots(2.1)$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

Relations of the type (2.1) were posed and studied by Oldroyd[8]. Oldroyd[8] also showed that many rheology equations of state of general validity, reduce to (2.1) when linearized. When $\lambda_0 = 0$ the fluid is termed as Maxwellian viscoelastic whereas $\lambda = \lambda_0 = 0$ yields the fluid to be Newtonian viscous fluid.

Consider a static state in which an incompressible Oldroydian viscoelastic fluid permeated with suspended partiles is arranged in horizontal strata in porous medium and the pressure p and density ρ are functions of the vertical coordinate z only. A uniform rotation

$\vec{\Omega}(0, 0, \Omega)$ pervades the whole system. The character of the equilibrium of this initial static state is determined, as usual by supposing that the system is slightly disturbed and then by following its further evolution.

Let ρ, μ, p, $\boldsymbol{u}$ (u, v, w) and $\vec{\Omega}$ $(0, 0, \Omega)$ denote respectively, the density, the viscosity, the pressure, the velocity of pure fluid and the uniform rotation; $\boldsymbol{v}$ $(\bar{x}, t)$ and N $(\bar{x}, t)$ denote the velocity and number density of the particles respectively. $k = \sigma\pi\rho\upsilon\eta$ where η is the particle radius is the Stokes' drag coefficient, $\boldsymbol{v} = (l, r, s)$, $\bar{x} = (x, y, z)$ and $\vec{\lambda} = (0, 0, 1)$. Let $\in$, k_1 and g stand for medium porosity, medium permeability and acceleration due to gravity respectivley. Then the equations of motion and continuity for the rotating Oldroydian viscoelastic fluid permeated with suspended particles in a porous medium are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial \boldsymbol{u}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{u}.\nabla)\,\boldsymbol{u}\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)[-\nabla(p-\frac{\rho}{2}|\vec{\Omega}\times \boldsymbol{x}|^2)$$

$$-\rho g\vec{\lambda}+\frac{kN}{\varepsilon}(\boldsymbol{v}-\boldsymbol{u})+\frac{2\rho}{\varepsilon}(\boldsymbol{u}\times\vec{\Omega})]-(1+\lambda_0\frac{\partial}{\partial t})\frac{\mu}{k_1}\boldsymbol{u} \qquad \text{...(2.2)}$$

$$\nabla.\boldsymbol{u}=0 \qquad \text{...(2.3)}$$

Since the density of a fluid particle moving with the fluid remains unchanged, we have

$$\varepsilon\frac{\partial\rho}{\partial t}+(\boldsymbol{u}\,.\,\nabla)\rho=0. \qquad \text{...(2.4)}$$

In the equations of motion (2.2), by assuming a uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity differences between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion of the particles. The distances between particles are assumed quite large compared with their diameter so that interparticle reactions are ignored. The effects of pressure, gravity and Darcian force on the suspended particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$mN\left[\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v}\,.\,\nabla)\boldsymbol{v}\right]=kN\,(\boldsymbol{u}-\boldsymbol{v}), \qquad \text{...(2.5)}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\,.\,(N\boldsymbol{v})=0. \qquad \text{...(2.6)}$$

Let $\delta\rho$, δp, $\boldsymbol{u}(u, v, w)$ and $\boldsymbol{v}$ (l, r, s) denote respectively, the perturbations in fluid density ρ, fluid pressure p, fluid velocity (0, 0,

0) and particle velocity (0, 0, 0). Then the linearized perturbation equations of the fluid-particle layer are

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{\partial \boldsymbol{u}}{\partial t}=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p+\boldsymbol{g}\delta\rho+\frac{kN}{\varepsilon}(\boldsymbol{v}-\boldsymbol{u})+\frac{2\rho}{\varepsilon}(\boldsymbol{u}\times\vec{\Omega})\right]-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}\boldsymbol{u} \quad \text{...(2.7)}$$

$$\nabla\cdot\boldsymbol{u}=0, \quad \text{...(2.8)}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho=-w\frac{d\rho}{dz}, \quad \text{...(2.9)}$$

$$\left(\frac{m}{k}\frac{\partial}{\partial t}+1\right)\boldsymbol{v}=\boldsymbol{u}. \quad \text{...(2.10)}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

$$\exp\,(ik_x x+ik_y y+nt), \quad \text{...(2.11)}$$

where k_x, k_y are wave numbers along x and y-directions, $k^2=k_x^2+k_y^2$ and n is in general, a complex constants.

For perturbations of the form (2.11), writing component equations and eliminating u, v, $\delta\rho$ and δp Eqs. (2.7)-(2.10) yield

$$\left[(1+\lambda n)(zn+1)+(1+\lambda_0 n)(zn+1)\frac{\varepsilon v}{k_1}\right][D(\rho Dw)-k^2\rho w]+n(1+\lambda n)\,[D(mNDw)-mNk^2w)+\left[gk^2\frac{(1+\lambda n)\,(zn+1)}{n}\right](D\rho)w+4(1+\lambda n)^2\,(zn+1)^2\,\Omega^2\left[D\left\{\frac{\rho Dw}{(1+\lambda n)(zn+1)n+(1+\lambda n)\frac{mnN}{\rho}+(1+\lambda_0 n)(zn+1)\frac{v\varepsilon}{k_1}}\right\}\right]=0 \quad \text{...(2.12)}$$

where v $(=\mu/\rho)$ stands for kinematic viscosity.

3. TWO UNIFORM VISCOELASTIC FLUIDS SEPARATED BY A HORIZONTAL BOUNDARY

Here we consider the case when two superposed uniform Oldroydian viscoelastic fluids of densities ρ_1 and ρ_2 uniform viscosities μ_1 and μ_2 uniform suspended particles densities N_1 and N_2 are separated by a horizontal boundary at $z=0$. The subscripts 1 and 2 distinguish the lower and the upper fluids respectively. Then, in each region of constant ρ, contant μ and constant mN, Eq. (2.12) reduces to

$$(D^2 - \chi^2)w = 0, \qquad ...(3.1)$$

where

$$\chi = \frac{k}{\left[1 + \dfrac{4\Omega^2 (1+\lambda n)^2 (zn+1)^2}{\left[n(1+\lambda n)(zn+1)n(1+\lambda n)\dfrac{mN}{\rho} + (1+\lambda_0 n)(zn+1)\dfrac{v\varepsilon}{k_1}\right]^2}\right]^{1/2}}$$

and in case of highly viscous fluid

$$\chi = \frac{k}{\left[1 + \dfrac{1}{2}\dfrac{4\Omega^2 (1+\lambda n)^2 (zn+1)^2}{\left[n(1+\lambda n)(zn+1) + n(1+\lambda n)\dfrac{mN}{\rho} + (1+\lambda_0 n)(zn+1)\dfrac{v\varepsilon}{k_1}\right]^2}\right]} \qquad ...(3.2)$$

The general solution of Eq. (3.1) is

$$w = Ae^{+\chi z} + be^{-\chi z} \qquad ...(3.3)$$

where A and B are arbitrary constants. The boundary conditions to be statisfied in the present problem are

(*i*) The velocity w should vanish when $z \to +\infty$ (for the upper fluid) and $z \to -\infty$ (for the lower fluid).

(*ii*) $w(z)$ is continuous at $z = 0$.

(*iii*) The pressure should be continuous across the interface.

Applying the boundary conditions (*i*) and (*ii*), we have

$$w_1 = Ae^{+\chi z} \ (z < 0), \qquad ...(3.4)$$

$$w_2 = Ae^{-\chi z} \ (z > 0), \qquad ...(3.5)$$

where the same constant A has been chosen to ensure the continuity of w at $z = 0$.

Here we assume the kinematic viscosities of both fluids to be equal i.e. $v_1 = v_2 = \nu$ (Chandrasekhar[1], p.443) and $\frac{mN}{\rho} = \frac{mN_1}{\rho_1} = \frac{mN_2}{\rho_2} (= M)$ as these simplifying assumptions do not obscure any of the essential features of the problem. Integrating Eq. (2.12) across the interface, $z = 0$ we obtain the boundary condition

$$\left[(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\frac{v\varepsilon}{k_1}\right]\Delta_0 (\rho Dw)$$

$$+ n(1+\lambda n)\,\Delta_0(mNDw) + (1+n\lambda)(zn+1)\frac{gk^2}{n}\,\Delta_0(\rho)w_0$$

$$+ \frac{4(1+\lambda n)^2(zn+1)^2\,\Omega^2}{\left[(1+\lambda n)(zn+1)n + (1+\lambda n)\dfrac{mnN}{\rho} + (1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v}{k_1}\right]}\,\Delta_0(\rho Dw)$$

$$= 0 \qquad \ldots(3.6)$$

where w_0 is the common value of w at $z = 0$.

Applying the boundary condition (3.6) to the solutions (3.4) and (3.5), we obtain

$$1 + \frac{mn\,(1+\lambda n)\,(N_1+N_2)}{(\rho_1+\rho_2)\left[(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v}{k_1}\right]}$$

$$- \frac{(1+\lambda_n)(zn+1)gk^2(\alpha_2-\alpha_1)}{n\chi\left[(1+\lambda)(zn+1)n + (1+\lambda_0)(zn+1)\dfrac{\varepsilon v}{k_1}\right]}$$

$$+ \frac{4(1+\lambda n)^2(zn+1)^2\Omega^2(\alpha_2+\alpha_1)}{\left[(1+\lambda n)(zn+1)n + (1+\lambda n)Mn + (1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v}{k_1}\right]\left[(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v}{k_1}\right]}$$

$$= 0, \qquad \ldots(3.7)$$

where
$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1+\rho_2}.$$

Equation (3.7), after substituting the value of χ from (3.2) and simplification, yields

$$A_{13}n^{13} + A_{12}n^{12} + A_{11}n^{11} + \ldots + A_2 n^2 + A_1 n + A_0 = 0, \qquad \ldots(3.8)$$

where
$$A_{13} = \lambda^4 z^4,$$

$$A_0 = -gk(\alpha_2-\alpha_1)\frac{\varepsilon v}{k_1}\left[\frac{\varepsilon^2 v^2}{k_1^2} + 2\Omega^2\right] \qquad \ldots(3.9)$$

and the coefficients $A_1 - A_{12}$ being quite lengthy and not needed in the discussion of stability, have not been written here.

For the potentially stable arrangement $(\alpha_2 < \alpha_1)$ all the coefficients of Eq. (3.8) are positive. So, all the roots of Eq. (3.8) are either real and negative or there are complex roots (which occur in pairs) with negative real parts and the rest negative real roots.

The system is therefore stable in each case.

For the potentially unstable arrangement ($\alpha_2 > \alpha_1$), the constant term is negative and so there is atleast one change of sign in Eq. (3.8). Therefore, Eq. (3.8) allows atleast one positive root of n meaning thereby instability of the system.

4. SPECIAL CASE. In the absence of rotation, Eq. (3.7) after some algebraic simplifications, gives

$$A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \qquad \text{...(4.1)}$$

where $A_4 = (\rho_2 + \rho_1)\lambda z,$

$$A_3 = \left[(\rho_2 + \rho_1)(z + \lambda + \lambda_0 z \frac{\varepsilon v}{k_1}) + m\lambda(N_1 + N_2) \right]$$

$$A_2 = \left[(\rho_2 + \rho_1)(1 + \frac{zv\varepsilon}{k_1} + \lambda_0 \frac{\varepsilon v}{k_1}) + m(N_1 + N_2) - (\rho_1 + \rho_2)\lambda zgk(\alpha_2 - \alpha_1) \right]$$

$$A_1 = \left[(\rho_2 + \rho_1) \left\{ \frac{\varepsilon v}{k_1} - (\alpha_2 - \alpha_1)gk(z + \lambda) \right\} \right]$$

and $A_0 = -(\rho_2 + \rho_1)gk(\alpha_2 - \alpha_1).$ ...(4.2)

For the potentially stable arrangement ($\alpha_2 < \alpha_1$) all the coefficients of Eq. (4.1) are positive. So, all the roots of Eq. (4.1) are either real and negative or there are complex roots (which occur in pairs) with negative real parts and the rest negative real roots. The system is, therefore, stable in each case.

For the potentially unstable arrangement ($\alpha_2 > \alpha_1$) the constant term is negative and so there is atleast one change of sign in Eq. (4.1). Equation (4.1) allows atleast one positive root of n thereby meaning instability of the system.

5. THE CASE OF EXPONENTIALLY VARYING DENSITY, VISCOSITY AND SUSPENDED PARTICLES NUMBER DENSITY

Let us assume the stratifications in density, viscosity and suspended particles number density of the form

$$\rho = \rho_0 e^{\beta z}, \ \mu = \mu_0 e^{\beta z}, \ n = N_0 e^{\beta z} \qquad \text{...(5.1)}$$

where ρ_0, μ_0, N_0 and β are constants. Equations (5.1) imply that the coefficient of kinematic viscosity ν is constant everywhere.

Using the stratification of the form (5.1), Eq. (2.12) transforms to

$$\left[(1 + \lambda n)(zn + 1)n + (1 + \lambda_0 n)(zn + 1) \frac{\varepsilon v_0}{k_1} + M_0 n(1 + \lambda_n) \right.$$

$$+\frac{4\Omega^2(1+\lambda n)^2\,(zn+1)^2}{(1+\lambda n)(zn+1)n+(1+\lambda n)M_0 n+(1+\lambda_0 n)(zn+1)\,\dfrac{\varepsilon v_0}{k_1}}\Bigg]D^2 w$$

$$+\Bigg[\,(1+\lambda n)(zn+1)n+(1+\lambda_0 n)(zn+1)\,\frac{\varepsilon v_0}{k_1}+M_0 n(1+\lambda n)$$

$$+\frac{4\Omega^2(1+\lambda n)^2(zn+1)^2}{(1+\lambda n)(zn+1)n+(1+\lambda n)M_0 n+(1+\lambda_0 n)(zn+1)\,\dfrac{\varepsilon v_0}{k_1}}\Bigg]\beta D w$$

$$-\Bigg[\,(1+\lambda n)(zn+1)n+(1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}+M_0 n(1+\lambda n)$$

$$-\frac{g\beta(1+\lambda n)(zn+1)}{n}\Bigg]k^2 w=0 \qquad \text{...(5.2)}$$

The general solution of Eq. (5.2) is

$$w=A_1 e^{q_1 z}+A_2 e^{q_2 z}, \qquad \text{...(5.3)}$$

where A_1, A_2 are two arbitrary constant and q_1, q_2 are the roots of the equation

$$\Bigg[\,(1+\lambda n)(zn+1)n+(1+\lambda_0 n)(zn+1)\frac{\varepsilon \nu_0}{k_1}+M_0 n(1+\lambda n)$$

$$+\frac{4\Omega^2\,(1+\lambda n)^2(zn+1)^2}{(1+\lambda n)(zn+1)n+(1+\lambda n)M_0 n+(1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v_0}{k_1}}\Bigg]q^2$$

$$+\Bigg[\,(1+\lambda n)(zn+1)n+(1+\lambda_0 n)(zn+1)\frac{\varepsilon n_0}{k_1}+M_0 n(1+\lambda n)$$

$$+\frac{4\Omega^2\,(1+\lambda n)^2(zn+1)^2}{(1+\lambda n)(zn+1)n+(1+\lambda n)M_0 n+(1+\lambda_0 n)(zn+1)\dfrac{\varepsilon v_0}{k_1}}\Bigg]\beta q$$

$$-\Bigg[\,(1+\lambda n)(zn+1)n+(1+\lambda n)(zn+1)\frac{\varepsilon v_0}{k_1}+M_0 n(1+\lambda n)$$

$$-\frac{g\beta(1+\lambda n)(zn+1)}{n}\Bigg]k^2=0. \qquad \text{...(5.4)}$$

If the fluid is suspoesd to be confined between two rigid planes at $z = 0$ and $z = d$, then the vanishing of w at $z = 0$ is statisfied by the choice

$$w = A(e^{q_1 z} - e^{q_2 z}), \qquad \text{...(5.5)}$$

while the vanishing of w at $z = d$ requires

$$\exp{(q_1 - q_2)d} = 1, \qquad \text{...(5.6)}$$

which implies that

$$(q_1 - q_2)d = 2im\pi \qquad \text{...(5.7)}$$

where m is an integer.

Equation (5.4) gives

$$q_{1,2} = \Bigg[-\frac{\beta}{2}\Bigg\{(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1} + M_0 n(1+\lambda n)$$

$$+ \frac{4\Omega^2 (1+\lambda n)^2 (zn+1)^2}{(1+\lambda n)(zn+1)n + (1+\lambda n)M_0 n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}}\Bigg\}$$

$$\pm \frac{1}{2}\Bigg[\beta^2\Bigg\{(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1} + M_0 n(1+\lambda n)$$

$$+ \frac{4\Omega^2 (1+\lambda n)^2 (zn+1)^2}{(1+\lambda n)(zn+1)n + (1+\lambda n)M_0 n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}}\Bigg\}^2$$

$$+ 4k^2 \Bigg\{(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1} + M_0 n(1+\lambda n)$$

$$+ \frac{4\Omega^2 (1+\lambda n)^2 (zn+1)^2}{(1+\lambda n)(zn+1)n + (1+\lambda n)M_0 n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}}\Bigg\}$$

$$\Bigg\{(1+\lambda n)(zn+1)n + (1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1} + M_0 n\,(1+\lambda n)$$

$$- \frac{g\beta(1+\lambda n)(zn+1)}{n}\Bigg\}\Bigg]^{1/2}$$

$$\left[(1+\lambda n)(zn+1)n+(1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}+M_0 n(1+\lambda n)\right.$$

$$\left.+\frac{4\Omega^2(1+\lambda n)^2(zn+1)^2}{(1+\lambda n)(zn+1)n+(1+\lambda n)M_0 n+(1+\lambda_0 n)(zn+1)\frac{\varepsilon v_0}{k_1}}\right]^{-1} \quad \text{...(5.8)}$$

Inserting the values of q_1, q_2 from (5.8) in (5.7) and simplifying, we obtain

$$A_7 n^7 + A_6 n^6 + A_5 n^5 + \ldots + A_2 n^2 + A_1 n^1 + A_0 = 0, \quad \text{...(5.9)}$$

where

$$A_7 = (\beta^2 d^2 + 4m^2\pi^2 + 4k^2 d^2)\lambda^2 z^2,$$

$$A_6 = (\beta^2 d^2 + 4m^2\pi^2 + 4k^2 d^2)\{2\lambda z(\lambda + z) + 2\lambda\lambda_0 z^2\frac{\varepsilon v_0}{k_1} + 2M_0\lambda^2 z\},$$

$$A_0 = 4k^2 d^2\,(-g\beta\,\varepsilon v_0/k_1), \quad \text{...(5.10)}$$

and the coefficients $A_1 - A_5$ being quite lengthy and not needed in the discussion of stability have not been written here.

If $\beta < 0$ (stable stratification), Eq. (5.9) does not admit of any positive root of n and the system is always stable for disturbances of all wave-numbers.

If $\beta > 0$ (unstable stratification), the constant term in eq. (5.9) is negative. Equation (5.9), therefore, has atleast one positive real root and so the system is unstable for all wave numbers.

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