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STABILITY OF STRATIFIED RIVLIN-ERICKSEN FLUID IN POROUS MEDIUM IN PRESENCE OF SUSPENDED PARTICLES AND VARIABLE MAGNETIC FIELD

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ABSTRACT

The stability of stratified Rivlin-Ericksen viscoelastic fluid in stratified porous medium is considered, separately, in presence of suspended particles (fine dust) and variable horizontal magnetic field. A variable horizontal magnetic field stabilizes certain wave-number range whereas the system was unstable for all wave numbers in the absence of magnetic field, for unstable stratifications. The growth rates are found to decrease with the increase in kinematic viscosity, kinematic viscoelasticity and magnetic field whereas the growth rates increase with the increase in medium permeability. The suspended particles (fine dust) do not affect the stability or instability, as was the situation in their absence.

1. INTRODUCTION The stability of superposed fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been treated in detail by Chandrasekhar [1]. The medium has been assumed to be non-porous and the fluid to be Newtonian. The flow through a porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamicists. A macroscopic equation which describes incompressible flow of a Newtonian fluid of viscosity μ through a macrosocopically homogeneous and isotropic porous medium of permeability k_1 is the usual Darcy's equation wherein the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\left(\frac{\mu}{k_1}\right)\vec{q}$, where $\vec{q}$ is the filter velocity of the fluid.

The effect of suspended particles on the stability of superposed fluids importance in geophysics and chemical engineering. Further motivation for the stability of fluids in presence of suspended particles is the fact that knowledge concerning fluid-particle mixtures is not commensurate with thier industrial and scientific importance. Scanlon and Segel [2] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The suspended particles were thus found to destabilize the layer. Palaniswamy and Purushotham [3] have considered the stability of shear flow of stratified fluids with fine dust and have found that the fine dust (suspended particles) increases the region of instability.

Generally the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [4] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable.

Sharma [5] has studied the stability of a layer of electrically conducting Oldroyed fluid (i.e. fluid described by the Oldroyd [6] constitutive relation) heated from below in the presence of a uniform magnetic field and has found that the magnetic field has a stabilizing effect. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relation or Oldroyd's constitutive relation. Rivlin-Ericksen fluid is one such class of viscoelastic fluids. Garg, Srivastava and Singh [7] have studied the drag on a sphere oscillating in conducting dusty Rivlin-Ericksen elastico-viscous liquid. It is this class of elastico-viscous fluid we are interested in.

Owing to the importance of suspended particles and variable magnetic field in chemical engineering and geophysics, the present paper considers the stability of stratified Rivlin- Ericksen fluid in stratified porous medium in the presence of suspended particles and variable horizontal magnetic field separately.

2. EFFECT OF VARIABLE MAGNETIC FIELD

2.1 PERTURBATION EQUATIONS

The initial stationary state whose stability we wish to examine is that of an incompressible, infinitely conducting, Rivlin-Ericksen fluid of variable density, kinematic viscosity and kinematic viscoelasticity arranged in horizontal strata in a porous medium of variable porosity and permeability and acted on by gravity force $\vec{g}(0, 0, -g)$ and variable horizontal magnetic field $\vec{H}(H_0(z), 0, 0)$. Consider an infinite horizontal layer of thickness d bounded by the planes $z = 0$ and $z = d$.

Let $\delta\rho$, δp, $\vec{v}(u, v, w)$ and $\vec{h}(h_x, h_y, h_z)$ denote respectively, the perturbations in density ρ, pressure p, velocity (0,0,0) and magnetic field $\vec{H}(H_0(z), 0, 0)$. Then the linearized perturbation equations governing the motion of Rivlin-Ericksen fluid and Maxwell's equation are

$$\frac{\rho}{\varepsilon}\frac{\partial \vec{v}}{\partial t} = -\nabla\delta p + \vec{g}\delta\rho + \frac{\mu_e}{4\pi}[(\nabla \times \vec{h}) \times \vec{H} + (\nabla \times \vec{H}) \times \vec{h}] - \frac{\rho}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{v}, \quad \text{...(1)}$$

$$\nabla \cdot \vec{v} = 0, \quad \text{...(2)}$$

$$\varepsilon\frac{\partial}{\partial t}(\delta\rho) = -\omega\frac{\partial\rho}{dz}, \quad \text{...(3)}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = (\vec{H} \cdot \nabla)\vec{v} - (\vec{v} \cdot \nabla)\vec{H} \quad \text{...(4)}$$

$$\nabla \cdot \vec{h} = 0. \quad \text{...(5)}$$

Equations (1) and (2) are linearized perturbed equations of motion and continuity whereas equation (3) ensures that the density of every particle remains unchanged as we follow it with its motion. Equations (4) and (5) are linearlized perturbed, Maxwell's equations.

Analysing the disturbance into normal modes, we seek solutions whose dependence on x, y and t is given by

$$\exp(ik_x x + ik_y y + nt), \quad \text{...(6)}$$

where k_x, k_y are horizontal wave numbers, $k = (k_x^2 + k_y^2)^{1/2}$ and n is in general, a complex constant.

Equations (1)-(5), using expression (6), give

$$\frac{\rho}{\varepsilon}\left[\left(1 + \frac{\varepsilon\nu'}{k_1}\right)n + \frac{\varepsilon\nu}{k_1}\right]u = -ik_x\delta p + \frac{\mu_e}{4\pi}h_z\, dH_0 \quad \text{...(7)}$$

$$\frac{\rho}{\varepsilon}\left[\left(1 + \frac{\varepsilon\nu'}{k_1}\right)n + \frac{\varepsilon\nu}{k_1}\right]v = -ik_y\delta p + \frac{\mu_e}{4\pi}(ik_x h_y - ik_y h_x) \quad \text{...(8)}$$

$$\frac{\rho}{\varepsilon}\left[\left(1+\frac{\varepsilon\nu'}{k_1}\right)n+\frac{\varepsilon\nu}{k_1}\right]w=-D\delta p-\frac{g(D\rho)}{\varepsilon n}w+\frac{\mu_e}{4\pi}\left(ik_xh_z-Dh_xh_x\frac{DH_0}{H_0}\right) \quad ...(9)$$

$$ik_xu+ik_yv+Dw=0 \quad ...(10)$$

$$ik_xh_x+ik_yh_y+Dh_z=0 \quad ...(11)$$

$$\varepsilon nh_x=ik_xH_0u-wdH_0 \quad ...(12)$$

$$\varepsilon nh_y=ik_xH_0v \quad ...(13)$$

$$\varepsilon nh_z=ik_xH_0w. \quad ...(14)$$

Eliminating u, v, h_x, h_y, h_z and δp from equations (7)-(9) and using (10)- (14), after a little algebra, we get

$$D\left[\frac{\rho}{\varepsilon}\left\{\left(1+\frac{\varepsilon\nu'}{k_1}\right)n+\frac{\varepsilon\nu}{k_1}\right\}Dw\right]-k^2\frac{\rho}{\varepsilon}\left[\left(1+\frac{\varepsilon\nu'}{k_1}\right)n+\frac{\varepsilon\nu}{k_1}\right]w$$
$$+\frac{gk^2}{\varepsilon n}(D\rho)w+\frac{\mu_ek_x^2}{4\pi\varepsilon n}\left\{H_0^2(D^2-k^2)w+D(H_0^2)Dw-\frac{H_0^2\,(Dw)(D\varepsilon)}{\varepsilon}\right\}=0 \quad ...(15)$$

which, on simplification, can be written as

$$\left[\frac{\rho}{\varepsilon}\left\{\left(1+\frac{\varepsilon\nu'}{k_1}\right)n+\frac{\varepsilon\nu}{k_1}\right\}\right](D^2-k^2)w+\left\{\frac{n}{\varepsilon}(D\rho)\left(1+\frac{\varepsilon\nu'}{k_1}\right)+\frac{n\rho}{\varepsilon^2}(D\varepsilon)\left(1+\frac{\varepsilon\nu'}{k_1}\right)\right.$$
$$\left.+\frac{n\rho}{k_1}D\nu'-\frac{n\rho\nu'}{k_1^2}dk_1+\frac{\nu}{k_1}D\rho+\frac{\rho}{k_1}D\nu-\frac{\rho\nu}{k_1^2}Dk_1\right\}Dw+\frac{gk^2(D\rho)w}{\varepsilon n}$$
$$+\frac{\mu_ek_x^2}{4\pi\varepsilon n}\left\{H_0^2(D^2-k^2)w+D(H_0^2)Dw-\frac{H_0^2}{\varepsilon}(Dw)(D\varepsilon)\right\}=0 \quad ...(16)$$

Equation (16) is the general equation governing the stability of a stratified Rivlin-Ericksen fluid in a stratified porous medium in the presence of a variable horizontal magnetic field.

2.2 THE CASE OF EXPONENTIALLY VARYING STRATIFICATIONS

Assume the stratifications in density, kinematic viscosity, kinematic viscoelasticity, medium porosity, medium permeability and magnetic field of the forms

$$\rho=\rho_0e^{\beta z},\ \mu=\mu_0e^{\beta z},\ \mu'=\mu_0'e^{\beta z},\ \varepsilon=\varepsilon_0e^{\beta z},\ k_1=k_{10}e^{\beta z},\ H_0^2=H_1^2e^{\beta z} \quad ...(17)$$

where $\rho_0, \mu_0, \mu_0', \varepsilon_0, k_{10}, H_1$ and β are constants. Equations (17) imply that the kinematic viscosity $\nu\left(=\frac{\mu}{\rho}=\frac{\mu_0}{\rho_0}\right)$, the kinematic viscoelasticity $\nu'\left(=\frac{\mu'}{\rho}=\frac{\mu_0'}{\rho_0}\right)$, and the Alfvén velocity $V_A\left(=\sqrt{\frac{\mu_e H_0^2}{4\pi\rho}}=\sqrt{\frac{\mu_e H_1^2}{4\pi\rho_0}}\right)$ are constant everywhere.

Using stratifications of the form (17), equation (16) transforms to

$$\left[n+\frac{\varepsilon_0}{\kappa_{10}}(\nu_0+\nu_0' n)+\frac{k_x^2 V_A^2}{n}\right](D^2-k^2)w+\frac{gk^2\beta}{n}w=0 \qquad \text{...(18)}$$

The general solution of equation (18) is

$$w=Ae^{m_1 z}+Be^{m_2 z} \qquad \text{...(19)}$$

where A, B are arbitrary constants; m_1, m_2 are given by

$$m_{1,2}=\pm k\left(1-\frac{g\beta}{nL}\right)^{1/2}$$

and

$$L=\left[n+\frac{\varepsilon_0}{\kappa_{10}}(v_0+nv_0')+\frac{k_x^2 V_A^2}{n}\right] \qquad \text{...(20)}$$

Here we consider the fluid to be confined between two rigid planes $z=0$ and $z=d$. The boundary conditions for the case of two rigid surfaces are

$$w=0 \text{ at } z=0 \text{ and } z=d. \qquad \text{...(21)}$$

The vanishing of w at $z=0$ is satisified by the choice

$$w=A(e^{m_1 z}-e^{m_2 z})$$

while the vanishing of w at $z=d$ requires

$$e^{(m_1-m_2)d}=1$$

i.e.

$$(m_1-m_2)d=2is\pi$$

where s is an integer.

Inserting the values of m_1, m_2 from (20) in above equation, we obtain

$$1-\frac{g\beta}{nL}=-\frac{s^2\pi^2}{k^2d^2},$$

which on simplification gives

$$\left(1+\frac{\varepsilon_0\nu_0'}{k_{10}}\right)n^2+\frac{\varepsilon_0\nu_0}{k_{10}}n-\frac{g\beta k^2d^2}{k^2d^2+s^2\pi^2}=0. \qquad ...(22)$$

If $\beta<0$ (stable stratification), equation (22) does not admit any positive root of n and so the system is always stable for disturbance of all wave numbers.

If $\beta>0$ (unstable stratification), the system is stable or unstable according as

$$k_x^2\,V_A^2><gk^2\beta/L \qquad ...(23)$$

The system is clearly unstable in the absence of a magnetic field. However, the system can be completely stabilized by large enough magnetic field as can be seen from equation (23), if

$$V_A^2>gk^2\beta/Lk_x^2.$$

Thus if $\beta>0$ and $k_x^2V_A^2<gk^2B/L$ equation (22) has atleast one positive root. Let n_0 denotes the positive root of equation (22). Then

$$\left(1+\frac{\varepsilon\nu'}{k_1}\right)n_0^2+\frac{\nu\varepsilon}{k_1}n_0+\left(k_x^2\,V_A^2-\frac{gk^2\beta}{L}\right)=0 \qquad ...(24)$$

as $$\nu_0=\frac{\mu_0}{\rho_0}=\frac{\mu}{\rho}=\nu,\ \frac{\varepsilon_0}{\kappa_{10}}=\frac{\varepsilon}{\kappa_1},\ \nu_0'=\frac{\mu_0'}{\rho_0}=\frac{\mu'}{\rho}=\nu'$$

and $$V_A^2=\frac{\mu_e H_1^2}{4\pi\rho_0}=\frac{\mu_e H_0^2}{4\pi\rho}.$$

To find the role of kinematic viscosity, kinematic viscoelasticity, medium permeablity and magnetic field on the growth rate of unstable modes, we examine the natures of

$$\frac{dn_0}{dv},\frac{dn_0}{dv'},\frac{dn_0}{dk_1}\text{ and }\frac{dn_0}{dV_A}.$$

Equation (24) yields

$$\frac{dn_0}{d\nu} = -\frac{\varepsilon n_0}{2n_0 (k_1 + \varepsilon v') + \varepsilon \nu}, \quad ...(25)$$

$$\frac{dn_0}{d\nu'} = -\frac{\varepsilon n_0^2}{2n_0 (k_1 + \varepsilon \nu') \varepsilon \nu}, \quad ...(26)$$

$$\frac{dn_0}{dk_1} = \frac{\varepsilon n_0 (\nu + n\nu')}{k_1 [2n_0 (k_1 + \varepsilon \nu') + \varepsilon \nu]}, \quad ...(27)$$

$$\frac{dn_0}{dV_A} = -\frac{2k_x^2 V_A}{2n_0 (k_1 + \varepsilon \nu') + \varepsilon \nu}. \quad ...(28)$$

It is clear from equation (25)-(28) that the growth rates decrease with the increase in kinematic viscosity, kinematic viscoelasticity and magnetic field whereas the growth rates increase with the increase in medium permeability.

3. EFFECT OF SUSPENDED PARTICLES

3.1 PERTURBATION EQUATIONS

The initial stationary state whose stability we wish to examine is that of an incompressible Rivlin-Ericksen fluid of variable density, kinematic viscosity and kinematic viscoelasticity arranged in horizontal strata embedded by suspended particles (fine dust) in porous medium of variable porosity and permeability. Consider an infinite horizontal fluid-particle layer of thickness d bounded by the planes $z = 0$ and $z = d$. The relevant equations of motion for the fluid are given by

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{V}}{\partial t} + \frac{1}{\varepsilon}(\vec{V}\cdot\nabla)\vec{V}\right] = -\nabla p + \rho\vec{g} - \frac{\rho}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{V} + \frac{kN}{\varepsilon}(\vec{u} - \vec{v}) \quad ...(29)$$

$$\nabla \cdot \vec{v} = 0, \quad ...(30)$$

$$\varepsilon\frac{\partial \rho}{\partial t} + (\vec{v}\cdot\nabla)\rho = 0. \quad ...(31)$$

Here $\vec{u}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the particles respectively, $k = 6\pi\rho\nu\eta$, where η is particle radius, is the Stokes drag coefficient, $\vec{u} = (l, r, s)$ and $\bar{x} = (x, y, z)$.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t} + \frac{1}{\varepsilon}(\vec{u}\cdot\nabla)\vec{u}\right] = kN(\vec{v} - \vec{u}) \quad ...(32)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla \cdot (N\vec{u}) = 0. \qquad \text{...(33)}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (30). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (32). The buoyancy force on the particles is neglected. Inerparticle reactions are not considered either since we assume that the distances between particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion for the particles (32).

Let $\delta\rho$, δp and $\vec{v}(u, v, w)$ denote, respectively the perturbations in fluid density ρ, pressure p and velocity (0,0,0). Then the linearized perturbation equations of the fluid-particle layer become

$$\frac{\rho}{\varepsilon}\frac{\partial \vec{v}}{\partial t} = -\nabla\, \delta p + \vec{g}(\delta\rho) - \frac{\rho}{k_1}\left(\nu + \nu' \frac{\partial}{\partial t}\right)\vec{v} + \frac{kN}{\varepsilon}(\vec{u} - \vec{v}) \qquad \text{...(34)}$$

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v} \qquad \text{...(35)}$$

together with equations (2) and (3).

Eliminating $\vec{u}$ from equation (34) by making use of (35) and using expression (6), equations (34), (2) and (3) yield

$$\frac{\rho}{\varepsilon}\left[n + \frac{\varepsilon}{\kappa_1}(\nu + n\nu') + \frac{mNn/\rho}{\frac{mn}{K} + 1}\right]u = -ik_x\delta p \qquad \text{...(36)}$$

$$\frac{\rho}{\varepsilon}\left[n + \frac{\varepsilon}{\kappa_1}(\nu + n\nu') + \frac{mNn/\rho}{\frac{mn}{k} + 1}\right]v = -ik_y\delta p \qquad \text{...(37)}$$

$$\frac{\rho}{\varepsilon}\left[n + \frac{\varepsilon}{\kappa_1}(\nu + n\nu') + \frac{mNn/\rho}{\frac{mn}{k} + 1}\right]w = -D(\delta p) - g(\delta\rho) \qquad \text{...(38)}$$

$$ik_x u + ik_y v + Dw = 0 \qquad \text{...(39)}$$

$$\varepsilon n \delta\rho = -wD\rho. \qquad \text{...(40)}$$

Eliminating u, v and δp from equations (36)-(38) and using (39) and (40), we obtain

$$\left[n + \frac{\varepsilon}{\kappa_1}(\nu + n\nu') + \frac{mNn/\rho}{\frac{mn}{k}+1} \right](D^2 - k^2)w + \left[\frac{1}{k_1} D\{\varepsilon(\nu + n\nu')\} - \frac{\varepsilon\,(\nu + n\nu')Dk_1}{k_1^2} + \frac{mn}{(mn/k+1)}\left\{ N\left(-\frac{1}{\rho^2} \right)D\rho + \frac{1}{\rho}DN \right\}\right]Dw + \frac{gk^2(d\rho)w}{\rho n} = 0 \quad ...(41)$$

3.2 THE CASE OF EXPONENTIALLY VARYING STRATIFICATIONS

Assume the stratifications in fluid density, fluid viscosity, fluid viscoelasticity, suspended particles number density, medium porosity and medium permeability of the forms

$$\rho = \rho_0 e^{\beta z},\ \mu = \mu_0 e^{\beta z},\ \mu_0' = \mu_0' e^{\beta z},\ N = N_0 e^{\beta z},\ \varepsilon = \varepsilon_0 e^{\beta z},\ k_1 = k_{10} e^{\beta z} \quad ...(42)$$

where ρ_0, μ_0, μ_0', N_0, ε_0, k_{10} and β are constants and so the kinematic viscosity $\nu = \frac{\mu}{\rho} = \frac{\mu_0}{\rho_0} (= \nu_0)$ and the kinematic viscoelasticity $\nu' = \frac{\mu'}{\rho} = \frac{\mu_0'}{\rho_0} (= \nu_0')$ are contrants everywhere.

Using stratifications of the form (42), equation (41) transfoms to

$$\left[n + \frac{\varepsilon_0}{\kappa_{10}}(\nu_0 + \nu_0' n) + \frac{mN_0 n/\rho_0}{\left(\frac{mn}{k}+1\right)} \right](D^2 - k^2)w + \frac{gk^2\beta}{n}w = 0 \quad ...(43)$$

The general solution of equation (43) is

$$w = Ae^{q_1 z} + Be^{q_2 z} \quad ...(44)$$

where A, B are constants; q_1, q_2 are given by

$$q_{1,2} = \pm k\left[1 - \frac{q\beta}{nL_2} \right]^{1/2} \quad ...(45)$$

and

$$L_2 = \left[n + \frac{\varepsilon_0}{\kappa_{10}}(\nu_0 + \nu_0' n) + \frac{mN_0 n/\rho_0}{\left(\frac{mn}{k}+1\right)} \right] \quad ...(46)$$

Here also we consider the fluid to be confined between two rigid planes at $z = 0$ and $z = d$. The boundary conditions are

$$w = 0 \text{ at } z = 0 \text{ and } z = d \qquad \text{...(47)}$$

The vanishing of w at $z = 0$ is satisfied by the choice

$$\omega = A\,(e^{q_1 z} - e^{q_2 z}),$$

while the vanishing of w at $z = d$ requires

$$e^{(q_1 - q_2)d} = 1$$

i.e.

$$(q_1 - q_2)d = 2is\pi$$

where s is an integer.

Inserting the values of q_1, q_2 form (45) in above equation, we obtain

$$1 - \frac{g\beta}{nL_2} = -\frac{s^2\pi^2}{d^2},$$

which on simplification yields

$$\left(1 + \frac{\varepsilon_0 \nu_0'}{k_{10}}\right)n^3 + \left[\frac{k}{m}\left(1 + \frac{\varepsilon_0}{k_{10}}\nu_0'\right) + \frac{\varepsilon_0 \nu_0}{k_{10}} + \frac{kN_0}{\rho_0}\right]n^2$$
$$+\left[\frac{\nu_0\varepsilon_0}{k_{10}}\frac{k}{m} - \frac{g\beta k^2 d^2}{k^2d^2 + s^2\pi^2}\right]n - \frac{q\beta k^2 d^2}{k^2d^2 + s^2\pi^2}\frac{k}{m} = 0. \qquad \text{...(48)}$$

If $\beta < 0$ (stable stratification), equation (48) does not allow any positive root of n and so the system is always stable for disturbances of all wave numbers.

If $\beta > 0$ (unstable stratification), equation (48) has one positive root and so the system is unstable for disturbances of all wave numbers. Let n_0 denote the positive root of equation (48). Then

$$\left[1 + \frac{\varepsilon v'}{k_1}\right]n_0^3 + \left[\frac{k}{m} + \frac{\varepsilon v}{k_1} + \frac{k}{m} + \frac{\varepsilon v'}{k_1} + \frac{kN}{\rho}\right]n_0^2$$
$$+\left[\frac{v\varepsilon}{k_1}\frac{k}{m} - \frac{g\beta k^2 d^2}{k^2d^2 + s^2\pi^2}\right]n_0 - \frac{g\beta k^2 d^2}{k^2d^2 + s^2\pi^2}\frac{k}{m} = 0 \qquad \text{...(49)}$$

as $\nu_0 = \frac{\mu_0}{\rho_0} = \frac{\mu}{\rho} = \nu,\ \nu_0' = \frac{\mu_0'}{\rho_0} = \frac{\mu'}{\rho} = \nu',\ \frac{N_0}{\rho_0} = \frac{N}{\rho}$

and $\frac{\varepsilon_0}{k_{10}} = \frac{\varepsilon}{k_1}.$

To find the roles of fluid viscosity, fluid viscoelasticity, suspended particles number density and medium permeability on the growth rates of unstable modes, we examine the natures of $\frac{dn_0}{d\nu}$, $\frac{dn_0}{d\nu'}$, $\frac{dn_0}{dN}$ and $\frac{dn_0}{dk_1}$ analytically. Equation (49) yields

$$\frac{dn_0}{d\nu} = -\frac{n_0\varepsilon\left(n_0 + \frac{k}{m}\right)}{k_1\left[3n_0^2\left(1 + \frac{\varepsilon\nu'}{k}\right) + 2n_0\left(\frac{k}{m} + \frac{\varepsilon\nu}{k_1} + \frac{k\varepsilon}{mk_1}\nu' + \frac{kN}{\rho}\right) + \frac{\nu\varepsilon}{k_1}\frac{k}{m} - \frac{g\beta k^2 d^2}{k^2 d^2 + s^2\pi^2}\right]} \quad \text{...(50)}$$

$$\frac{dn_0}{d\nu'} = -\frac{n_0^2\varepsilon\left(n_0 + \frac{k}{m}\right)}{k_1\left[3n_0^2\left(1 + \frac{\varepsilon\nu'}{k_1}\right) + 2n_0\left(\frac{k}{m} + \frac{\varepsilon\nu}{k_1} + \frac{k\varepsilon}{mk_1}\nu' + \frac{kN}{\rho}\right) + \frac{\nu\varepsilon}{k_1}\frac{k}{m} - \frac{g\beta k^2 d^2}{k^2 d^2 + s^2\pi^2}\right]} \quad \text{...(51)}$$

$$\frac{dn_0}{dN} = \frac{n_0\, k/\rho}{3n_0^2\left(1 + \frac{\varepsilon\nu'}{k_1}\right) + 2\left(\frac{k}{m} + \frac{\varepsilon\nu}{k_1} + \frac{k\varepsilon}{mk_1}\nu' + \frac{kN}{\rho}\right)} \quad \text{...(52)}$$

and

$$\frac{dn_0}{dk_1} = \frac{n_0\varepsilon\left[n_0^2\nu' + n_0\left(\nu + \frac{k}{m}\nu'\right) + \nu\frac{k}{m}\right]}{k_1^2\left[3n_0^2\left(1 + \frac{\varepsilon\nu'}{k_1}\right) + 2n_0\left(\frac{k}{m} + \frac{\varepsilon\nu}{k_1} + \frac{k}{m}\frac{\varepsilon\nu'}{k_1} + \frac{kN}{\rho}\right) + \frac{\nu\varepsilon}{k_1}\frac{k}{m} - \frac{g\beta k^2 d^2}{k^2 d^2 + s^2\pi^2}\right]} \quad \text{...(53)}$$

It is clear from equations (50) and (51) that the growth rates decrease or increase with the increase in kinematic viscosity and kinematic viscoelasticity according to

$$3n_0^2\left(1 + \frac{\varepsilon\nu'}{k_1}\right) + 2n_0\left(\frac{k}{m} + \frac{\varepsilon\nu}{k_1} + \frac{k\varepsilon\nu'}{mk_1} + \frac{kN}{\rho}\right) + \frac{\nu\varepsilon}{k_1}\frac{k}{m} >< \frac{g\beta^2 d^2}{k^2 d^2 + s^2\pi^2} \quad \text{...(54)}$$

Equation (52) implies that the growth rates increase with the increase in the suspended particles number density. However, the growth rates increase or decrease with the increase in medium permeability according to (54) as is evident from equation (53).

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