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MHD FLUCTUATING HELE-SHAW FLOW OF A VISCOUS FLUID PAST A CIRCULAR CYLINDER

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ABSTRACT

In the present problem an attempt has been made to study the effects of boundary fluctuations and the transverse magnetic field on the Hele-Shaw flow of incompressible viscous fluid past a circular cylinder $x^2+y^2=r^2, -d \le z \le d$.

1. INTRODUCTION. In 1897, Prof. Hele-Shaw established that the Hele-Shaw cell is an arrangement of two parallel plates very close together, the space between the plates being occupied partly by fluid and partly by obstacles in the form of cylinders with generators normal to the plates. Several researchers have paid their attention towards the Hele-Shaw flows. Lamb (1932), Thompson (1968), Lee & Fung (1969) and Buckmaster (1970) have discussed such flows. In these investigations, the pressure gradient was assumed to be a constant and the flow in the cell to be steady. Swaminathan (1975) studied the unsteady Hele-Shaw flow of viscous incompressible fluid under time dependent pressure gradient. Lal & Singh (1981) have extended the work of Swaminathan to discuss the unsteady Hele-Shaw flow through porous media under time varying pressure gradient.

Reigels (1989) has discussed *MHD* Hele-Shaw flow of Rivlin-Ericksen fluid. Sengupta and Roy (1991) have also investigated *MHD* flow of n-immiscible viscoelastic fluid under gravity between two parallel plates. Recently, Ghosh & Sengupta (1995) have studied *MHD* Hele-Shaw flow of Rivlin-Ericksen fluid.

In this paper, we have studied the effects of boundary fluctuations under the influence of transverse magnetic field on the Hele-Shaw flow under constant pressure. The expressions for extact velocity distributions have been found and graphically represented under the influence of magnetic field of different intensities.

2. GOVERNING EQUATIONS OF MOTION. Let the viscous incompressible fluid flows past a vertical cylinder $x^2+y^2=r^2$, $-d \le z \le d$; placed between two parallel plates $z=\pm d$.

The equations of motion for a viscous incompressible fluid in the unsteady Hele-Shaw cell under transverse magnetic field are

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u_2}{\partial z^2}-\frac{\sigma}{\rho}B_0^2 u \quad \text{... (2.1)}$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial z^2}-\frac{\sigma}{\rho}B_0^2 v \quad \text{... (2.2)}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial z} \quad \text{... (2.3)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{... (2.4)}$$

where ρ the density, p the pressure of the field, σ the electrical conductivity, B_0 the strength of magnetic field. The boundary conditions are

$$u=u_0 e^{int},\ v=u_0 e^{int} \text{ at } z=\pm d \quad \text{... (2.5)}$$

Normal velocity at the surface of the cylinder is zero. ... (2.6)

U is the finite and $V=0$ as $|x|$, $|y| \to \infty$ ... (2.7)

3. MATHEMATICAL ANALYSIS. We introduce the following non-dimensional quantities

$$y'=\frac{y}{d},\ x'=\frac{x}{d},\ r'=\frac{r}{d},\ u'=\frac{u}{u_0},\ t'=\frac{\mu t}{\rho d^2},\ v'=\frac{v}{u_0},\ p'=\frac{pd}{\mu u_0},$$

$$d'=\frac{n\rho d^2}{\mu} \text{ and } M=B_0 d\sqrt{\sigma/\mu},$$

where M is the Hartmann number.

The equations (2.1) to (2.4) under the constant pressure (after dropping dashes) reduce to :

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial z^2}-M^2 u \quad \text{... (3.0)}$$

$$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial z^2}-M^2 v \quad \text{... (3.1)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{... (3.2)}$$

The boundary conditions, accordingly, are

$$u = v = e^{int} \text{ at } z = \pm 1 \qquad \dots (3.3)$$

The conditions for ϕ are

$$\frac{\partial \phi}{\partial r} = 0 \text{ at the surface of the cylinder where } x^2 + y^2 = R^2 \qquad \dots (3.4)$$

$$\frac{\partial \phi}{\partial x} = 1 \text{ and } \frac{\partial \phi}{\partial y} = 0 \text{ as } |x| \to \infty,\ |y| \to \infty \qquad \dots (3.5)$$

4. SOLUTION. Let

$$u = U(z)\frac{\partial \phi}{\partial x} e^{int}, -1 \le z \le 1 \qquad \dots (4.1)$$

$$v = U(z)\frac{\partial \phi}{\partial y} e^{int}, -1 \le z \le 1 \qquad \dots (4.2)$$

$$u = v = U(z)\, e^{int} \text{ at } z = \pm 1 \qquad \dots (4.3)$$

The conditions (3.3) suggests that

$$U(z) = 1 \text{ at } z = \pm 1 \qquad \dots (4.4)$$

Now substituting from (4.1) to (4.3) in equations (3.0) to (3.2), we get

$$\frac{\partial^2 u}{\partial z^2} - (M^2 + in)\, U = 0 \qquad \dots (4.5)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \dots (4.6)$$

The solution of equation (4.5) under the conditions (4.4) can easily be obtained as,

$$U = \frac{\cosh(\sqrt{M^2 + in})z}{\cosh(\sqrt{M^2 + in})} \qquad \dots (4.7)$$

Also the solution of (4.6) under the conditions (3.4) and (3.5) can be obtained as

$$\phi = \left(R + \frac{r^2}{R} \right) \cos\theta \qquad \dots (4.8)$$

Substituting from equations (4.7) and (4.8) in equations (4.1) and (4.2), the exact velocity distributions are obtained as

$$u = \frac{\cosh(\sqrt{M^2 + in})z}{\cosh(\sqrt{M^2 + in})} \left\{ 1 - \frac{r^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right\} e^{int} \qquad \dots (4.9)$$

and

$$v = \frac{\cosh(\sqrt{(M^2 + in})z}{\cosh(\sqrt{M^2 + in})} \left\{ - \frac{2r^2 xy}{(x^2 + y^2)^2} \right\} e^{int} \qquad \dots (4.10)$$

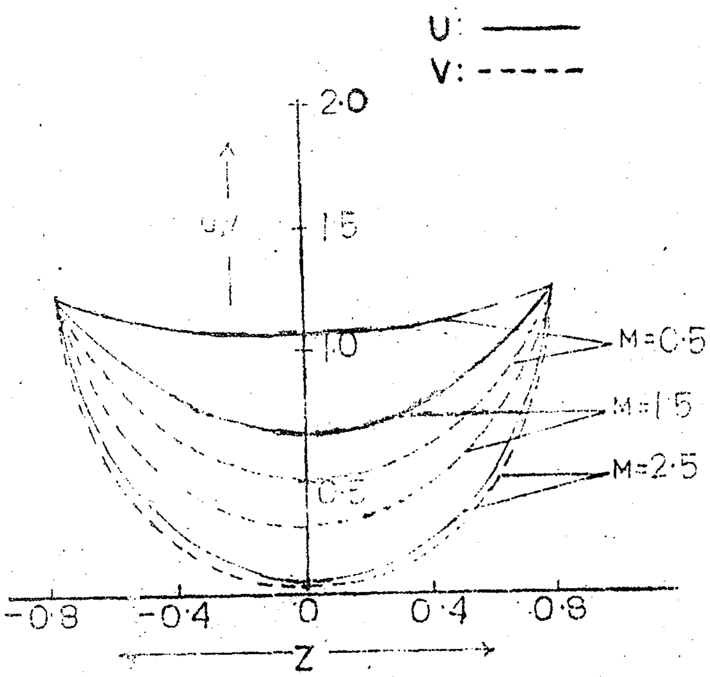

FIG.(I) VELOCITIES PLOTTED AGAINST Z
AT n=1, m=0, X=1 AND Y=α1

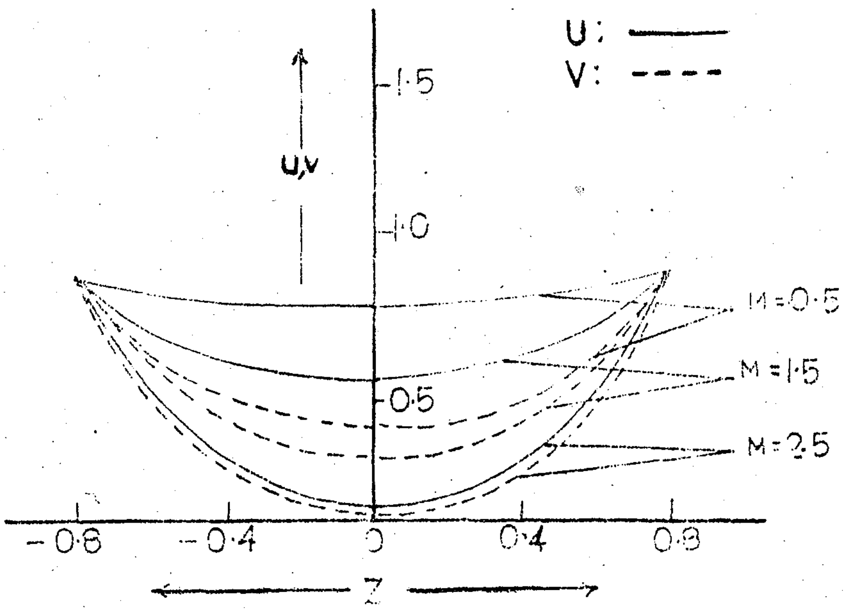

FIG.II VELOCITIES PLOTTED AGAINST Z
AT $nt = \pi/4$, $n=1$, $X=1$, AND $Y=-1$

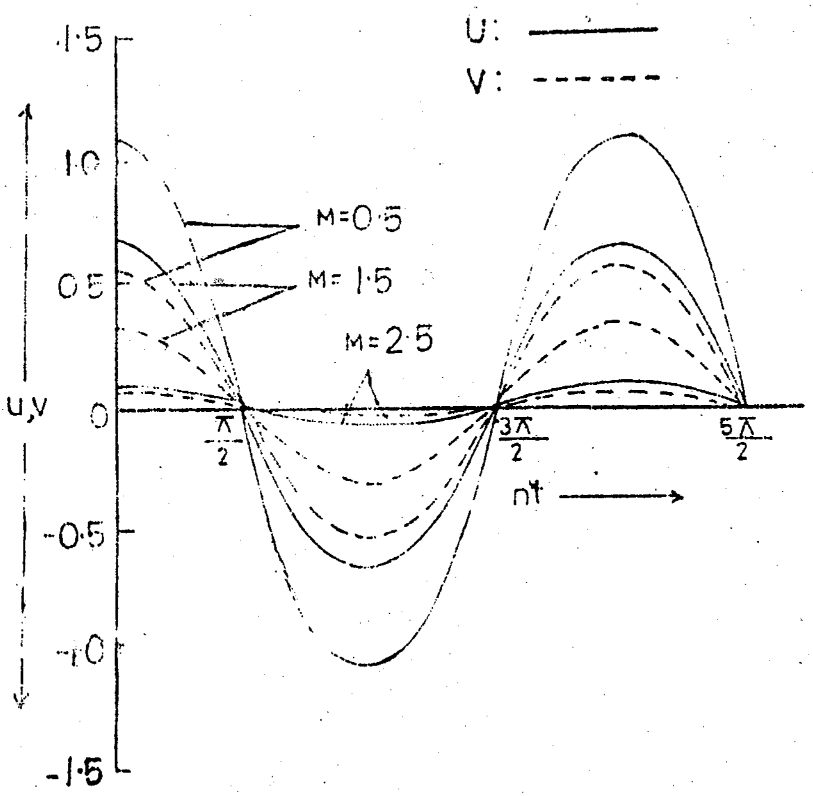

FIG.III VELOCITIES PLOTTED AGAINST nt AT Z=0, n=1, X=1 AND Y=-1

Equating the real parts, we have

$$u = (M_r \cos nt - M_i \sin at)\left\{1 - \frac{r^2(x^2-y^2)}{(x^2+y^2)^2}\right\} \qquad \text{... (4.11)}$$

and

$$v = (M_r \cos nt - M_i \sin nt)\left\{-\frac{2r^2(x^2-y^2)}{(x^2+y^2)^2}\right\} \qquad \text{... (4.12)}$$

where

$$M_r = \frac{\cos A \cos Az \cosh B \cosh Bz + \sin A \sin Az \sinh B \sinh Bz}{\cos^2 A + \sinh^2 B}$$

$$M_i = \frac{\sin A \sinh B \cos Az \cosh Bz - \sin Az \sinh Bz \cos A \cosh B}{\cos^2 A + \sinh^2 B}$$

$$A = -\frac{M^2 + \sqrt{(M^4+n^2)}}{2} \quad \text{and} \quad B = \frac{M^2 + \sqrt{(M^4+n^2)}}{2}.$$

NUMERICAL DISCUSSIONS AND CONCLUSIONS :

In order to study the effects of the boundary fluctuations and the effects of Hartmann number (M) on the distribution of velocity profiles on the 'Hele - Shaw flow' of viscous incompressible fluid, we have plotted graphs of velocities against z, we observe that the velocities decreases with the increase in M or the increase in *nt*. The following results have been established.

[i] An Increase in the Hartmann number decreases the velocity field, i.e. the magnetic field decelerates the flow.

[ii] Boundary fluctuations are held responsible for the flow in the absence of the pressure gradient.

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