Jñānābha, Vol. 27, 1997

ON A UNIFIED FINITE INTEGRAL

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(Received : December 15, 1996)

In the present paper, we first evaluate a unified finite integral involving the product of a general class of polynomials $S_V^U[x]$, the generalized polynomial set $S_n^{\alpha,\beta,\tau}[x]$ and the multivariable H-function. Being of very general nature, due to the presence of these polynomials and function, our result provides interesting unification and extension of a large number of new and known results obtained by several authors and hitherto lying scattered in the litrature. For the sake of illustration, we have evaluated here three new integrals as special cases of our main integral. Further, a reference to 12 integrals which also follow as simple special cases of our results has also been given.

1. INTRODUCTION

(*i*) Srivastava [3, eq. (1), p. 1] has introduced the following general class of polynomials :

$$S_V^U[x] \sum_{K=0}^{[V/U]} \frac{(-V)_{UK} A(V,K)\, x^K}{K!}, \quad V = 0, 1, 2, \ldots \qquad \ldots (1.1)$$

where U is an arbitrary positive integer and the coefficients $A(V,K)(V, K \geq 0)$ are arbitrary constants real or complex. On suitably specializing the coefficients $A(V,K)$, $S_V^U[x]$ yields a number of known polynomials as its special cases. These include, among others, Laguerre polynomials, Hermite polynomials, Gould-Hopper polynomials, Brafman polynomials and several others [5, pp. 158-161].

(*ii*) The generalized polynomial set is defined by the following Rodrigues type formula [12, eq. (2.1.8), p.64].

$S_n^{\alpha,\beta,\tau}[x : r, s, q, A, B, m, k, l]$

$$= (Ax+B)^{-\alpha}(1-\tau x)^{-\beta/\tau}\, T_{k,l}^{m+n}\, [(Ax+B)^{\alpha+qn}(1-\tau x^r)^{\beta/\tau+sn}] \qquad \text{... (1.2)}$$

with the differential operator $T_{k,l}$ being defined as

$$T_{k,l} \equiv x^l\left(k + x\frac{d}{dx}\right) \qquad \text{... (1.3)}$$

The explicit form of this generalized polynomial set [12, eq. (2.3.4), p. 71] is

$$S_n^{\alpha,\beta,\tau}[x : r, s, q, A, B, m, k, l]$$

$$= B^{qn}\, x^{l(m+n)}\, (1-\tau x^r)^{sn}\, l^{m+n} \sum_{v=0}^{m+n}\sum_{e=0}^{v}\sum_{\delta=0}^{m+n}\sum_{p=0}^{\delta} \frac{(-1)^{\delta}(-\delta)_p}{p!\,\delta!} \qquad \text{... (1.4)}$$

$$\frac{(\alpha)_\delta(-v)_e(-\alpha-qn)_p}{e!\, v!(1+\alpha-\delta)_p}\cdot(-\beta/\tau-\lambda_n)_v \left(\frac{p+k+re}{l}\right)_{m+n}\left(\frac{-\tau x^r}{1-\tau x^r}\right)^{v}\left(\frac{Ax}{B}\right)^{\delta}$$

It may be pointed out here that the polynomial set defined by (1.2) is very general in nature and it unifies and extends a number of classical polynomials introduced and studied by various research workers such as Chatterjea [11], Dhillon [14], Gould and Hopper [7], Krall and Frink [2], Singh [1], Singh and Srivastava [10] etc. Some of the special cases of (1.3) are given by Raizada in a tabular form [12, p. 65].

(*iii*) The multivariable H-function introduced and studied by Srivastava and Panda [6], occurring in this paper will be defined and represented as follows [4, pp. 251-252, eqns. (C.1)-(C.3)].

$$H_{P,Q:P_1,Q_1;\ldots;P_t,Q_t}^{O,N:M_1,N_1;\ldots;M_t,N_t}\left[\begin{matrix} z_1 \\ \cdot \\ \vdots \\ z_t \end{matrix}\,\middle|\, \begin{matrix}(a_j : \alpha_j^{(1)},\ldots,\alpha_j^{(t)})_{1,P} : (c_j^{(1)},\gamma_j^{(1)})_{1,P_1};\ldots;(c_j^{(t)},\gamma_j^{(t)})_{1,P_t} \\ (b_j : \beta_j^{(1)},\ldots,\beta_j^{(t)})_{1,Q} : (d_j^{(1)},\delta_j^{(1)})_{1,Q_1};\ldots;(d_j^{(t)},\delta_j^{(t)})_{1,Q_t}\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^t}\int_{L_1}\cdots\int_{L_t}\phi_1(\xi_1)\ldots\phi_t(\xi_t)\,\psi(\xi_1,\ldots,\xi_t)\, z_1^{\xi_1}\ldots z_t^{\xi_t}\, d\xi_1\ldots d\xi_t \qquad \text{... (1.5)}$$

where $\omega = \sqrt{-1}$

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{M_i} \Gamma(d_j^{(i)} - \delta_j^{(i)}\xi_i) \prod_{j=1}^{N_i} \Gamma(1 - c_j^{(i)} + \gamma_j^{(i)}\xi_i)}{\prod_{j=M_i+1}^{Q_i} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)}\xi_i) \prod_{j=N_i+1}^{P_i} \Gamma(c_j^{(i)} - \gamma_j^{(i)}\xi_i)}$$

$(i = 1, \ldots, t)$ ... (1.6)

and

$$\psi(\xi_1, \ldots, \xi_t) = \frac{\prod_{j=1}^{N} \Gamma\left(1 - a_j + \sum_{i=1}^{t} \alpha_j^{(i)}\xi_i\right)}{\prod_{j=N+1}^{P} \Gamma\left(a_j - \sum_{i=1}^{t} \alpha_j^{(i)}\xi_i\right) \prod_{j=1}^{Q} \Gamma\left(1 - b_j + \sum_{i=1}^{t} \beta_j^{(i)}\xi_i\right)}$$

... (1.7)

All the Greek letters occuring on the left hand side of (1.5) are assumed to be positive real numbers for standardization purposes; the definition of the multivariable H-function will, however, be meaningful even if some of these quantities are zero. For the convergence and existence conditions of the multivariable H-function we refer to the book by Srivastava et al. [4, eqns. (C.4)-(C.8), pp. 252-253]. Throughout the paper it is assumed that this function satisfies the above cited conditions.

2. MAIN INTEGRAL

$$\int_0^a x^{\rho-1} (a-x)^{\sigma-1} (c+bx)^{-\eta} S_n^{\alpha,\beta,\tau} [y x^{\mu} (a-x)^{\nu} (c+bx)^{-\lambda}]$$

$$S_V^U [z x^{\zeta} (a-x)^{\xi} (c+bx)^{-\gamma}] H [z_1 x^{p_1} (a-x)^{q_1} (c+bx)^{-r_1} \ldots$$

$$z_t x^{p_t} (a-x)^{q_t} (c+bx)^{-r_t}] \, dx$$

$$= \frac{a^{\rho+\sigma-1}}{c^{\eta}} \sum_{v=0}^{m+n} \sum_{e=0}^{v} \sum_{\delta=0}^{m+n} \sum_{p=0}^{\delta} \sum_{K=0}^{[V/U]} \theta(k, e, v, \delta, p)$$

$$H^{O, N+3 : M_1, N_1 ; \ldots ; M_t, N_t : 1, 1 ; 1, 0}_{P+3, Q+2 : P_1, Q_1 ; \ldots ; P_t, Q_t : 1, 1 ; 0, 1} \left[z_1 \frac{a^{p_1+q_1}}{c^{r_1}}, \ldots, z_t \frac{a^{p_t+q_t}}{c^{r_t}}, \frac{-\tau y^r a^{(\mu+\nu)r}}{c^{\lambda r}}, \frac{ab}{c} \,\middle|\, \begin{matrix} C : E ; (1-v+sn, 1) : - \\ D : F ; (0, 1) \quad : (0, 1) \end{matrix} \right] \ldots (2.1)$$

where

$$\theta(K, e, v, \delta, p) = B^{qn}\, l^{m+n}\, \frac{a^{l(\mu+\nu)(m+n)}\,(-1)^{\delta}\,(-\delta)_p\,(\alpha)_\delta}{c^{\lambda l(m+n)}\, p!\,\delta!\,e!\,v!\,K!}$$

$$\frac{(-v)_e\,(-\alpha-qn)_p\,(-V)_{UK}\,A(V,K)\,z^{K}\,z^{K(\zeta+\xi)}}{(1-\alpha-\delta)_p\,\Gamma(v-sn)\,c^{\gamma K}}\,(-\beta/\tau-sn)_v$$

$$\left(\frac{p+k+re}{l}\right)_{m+n} + \left(\frac{-\tau a^{(\mu+\nu)}}{c^{\lambda r}}\right)^{v}\left(\frac{Aa^{(\mu+\nu)}}{Bc^{\lambda}}\right)^{\delta} \quad \text{... (2.2)}$$

$$C = (1-\rho-\mu R-\zeta K : p_1, \ldots, p_t, \mu r, 1), (1-\sigma-\nu R-\xi K : q_1, \ldots, q_t, \nu r, 0),$$
$$(1-\eta-\lambda R-\gamma K : r_1, \ldots, r_t, \lambda r, 1), (a_j : \alpha_j^{(1)}, \ldots, \alpha_j^{(t)}, 0, 0)_{1,P}$$

$$D = (b_j : \beta_j^{(1)}, \ldots, \beta_j^{(t)}, 0, 0)_{1,Q}, (1-\eta-\lambda R-\gamma K : r_1, \ldots, r_t, \lambda r, 0),$$
$$(1-\rho-\sigma-(\mu+\nu)R-(\zeta+\xi)K : p_1+q_1, \ldots, p_t+q_t, (\mu+\nu)r, 1)$$

$$E = (c_j^{(1)}, \gamma_j^{(1)})_{1,P_1}; \cdots; (c_j^{(t)}, \gamma_j^{(t)})_{1,P_t}$$

$$F = (d_j^{(1)}, \delta_j^{(1)})_{1,Q_1}; \cdots; (d_j^{(t)}, \delta_j^{(t)})_{1,Q_t} \quad \text{... (2.3)}$$

$$R = l(m+n) + rv + \delta \quad \text{... (2.4)}$$

and the following conditions are satisfied :

(*i*) a, b, c are positive numbers such that $\left|\frac{ab}{c}\right| < 1$,

$\text{Re}(\eta) > 0$, $\min\{\mu, \nu, \xi, \lambda, \zeta, \gamma, p_i, q_i, r_i\} \geq 0$, $i = 1, \ldots, t$

(not all zero simultaneously)

(*ii*) $$\text{Re}(\rho) + \sum_{i=1}^{t} p_i \min_{1 \leq j \leq M_i} \left\{ \text{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\} > 0,$$

$$\text{Re}(\sigma) + \sum_{i=1}^{t} q_i \min_{1 \leq j \leq M_i} \left\{ \text{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\} > 0,$$

$(i = 1, \ldots, t)$ ... (2.5)

Proof . To establish the integral (2.1), we first express the polynomials $S_V^U[x]$ and $S_n^{\alpha,\beta,\tau}[x]$ occurring in the left-hand side of (2.1) in their respective series forms with the help of equations (1.1) and (1.4) and then interchange the order of summations and integration (which is permissible under the conditions stated with

(2.1)). The left-hand side of the integral (2.1) (say I), then takes the following form :

$$I = B^{qn}\, l^{m+n} \sum_{v=0}^{m+n} \sum_{e=0}^{v} \sum_{\delta=0}^{m+n} \sum_{p=0}^{\delta} \sum_{K=0}^{[V/U]} \sum_{u=0}^{\infty} \frac{(-1)^{\delta}(-\delta)_p(-v)_e(\alpha)_\delta}{p!\,\delta!\,e!\,v!\,K!},$$

$$\frac{(-1)^{v}(-\alpha-qn)_p\, y^{R+ur}(-V)_{KU}(v-sn)_u(\tau)^{u+v} A(V,K)\, z^K}{u!\,(1-\alpha-\delta)_p}$$

$$(-\beta/\tau - sn)_v \left(\frac{p+k+re}{l}\right)_{m+n} \left(\frac{A}{B}\right)^{\delta} \int_0^a x^{\rho+\mu(R+ur)+\zeta K-1}$$

$$(a-x)^{\sigma+\nu(R+ur)+\xi K-1}(c+bx)^{-\eta-\lambda(R+ur)-\gamma K}$$

$$H\left[z_1 x^{p_1}(a-x)^{q_1}(c+bx)^{-r_1}, \ldots, z_t x^{p_t}(a-x)^{q_t}(c+bx)^{-r_t}\right] dx \qquad \ldots (2.6)$$

Evaluating the x-integral occurring in the above equation (2.6) with the help of [9, eq. (5.2.6), p. 244] and expressing the result thus obtained in terms of H-function of $(t+2)$ variables, we arrive at the desired result (2.1).

3. SPECIAL CASES

(*i*) If we take $A = 1, B = q = s = m = k = 0, l = -1$ and let $\tau \to 0$ in the main integral (2.1), the polynomial set $S_n^{\alpha,\beta,\tau}[x]$ reduces into Gould and Hopper polymials $H_n^{(r)}[x, \alpha, \beta]$ [7] and we arrive at the following interesting integral after a little simplification :

$$\int_0^a x^{\rho-1}(a-x)^{\sigma-1}(c+bx)^{-\eta} H_n^{(r)}\left[yx^{\mu}(a-x)^{\nu}(c+bx)^{-\lambda}, \alpha, \beta\right]$$

$$S_V^U\left[zx^{\zeta}(a-x)^{\xi}(c+bx)^{-\gamma}\right] H\left[\frac{z_1 x^{P_1}(a-x)^{q_1}}{(c+bx)^{r_1}}, \ldots, \frac{z_t x^{p_t}(a-x)^{q_t}}{(c+bx)^{r_t}}\right] dx$$

$$= \frac{a^{\rho+\sigma-1}}{c^{\eta}} \sum_{v=0}^{n} \sum_{e=0}^{v} \sum_{K=0}^{[V/U]} \theta_1(K, e, v)$$

$$H^{O,N+3:M_1,N_1;\ldots;M_t,N_t;1,0}_{P+3,Q+2:P_1,Q_1;\ldots;P_t,Q_t;0,1}\left[\frac{z_1 a^{p_1+q_1}}{c^{r_1}}, \ldots, \frac{z_t a^{p_t+q_t}}{c^{r_t}}, \frac{ab}{c} \,\middle|\, \begin{matrix} C_1 : E : - \\ D_1 : F : (0,1) \end{matrix}\right] \qquad \ldots (3.1)$$

where

$$\theta_1(K, e, v) = \frac{(-1)^n (-v)_e (-\alpha - re)_n \beta^v a^{(\mu+\nu)R_1 + K(\zeta+\xi)} (-V)_{UK} A(V, K) z^K}{e!\, v!\, K!\, c^{\lambda R_1 + \gamma K}} \quad \dots (3.2)$$

$$C_1 = (1 - \rho - \mu R_1 - \zeta K : p_1, \dots, p_t, 1), (1 - \sigma - \nu R_1 - \xi K : q_1, \dots, q_t, 0),$$

$$(1 - \eta - \lambda R_1 - \gamma K : r_1, \dots, r_t, 1), (a_j : \alpha_j^{(1)}, \dots, \alpha_j^{(t)}, 0)_{1, P}$$

$$D_1 = (b_j : \beta_j^{(1)}, \dots, \beta_j^{(t)}, 0)_{1, Q}, (1 - \eta - \lambda R_1 - \gamma K : r_1, \dots, r_t, 0),$$

$$(1 - \rho - \sigma - (\mu + \nu)R_1 - (\zeta + \xi)K : p_1 + q_1, \dots, p_t + q_t, 1) \quad \dots (3.3)$$

$$R_1 = rv - \eta$$

provided that the conditions easily obtainable from those mentioned with the main integral (2.1) are satisfied.

(*ii*) Taking $\eta = \rho + \sigma$, $\lambda = \mu + \nu$, $\gamma = \zeta + \xi$, $r_1 = p_i + q_i$ $(i = 1, \dots, t)$ and replacing x by $ax/(1+x)$ in (2.1), after making a little simplifications, we arrive at the following infinite form of the integral which itself is quite general in nature.

$$\int_0^\infty x^{\rho-1} (1 + dx)^{-\rho-\sigma} S_n^{\alpha, \beta, \tau} [y_1 x^\mu (1 + dx)^{-\mu-\nu}]$$

$$S_V^U [z' x^\zeta (1 + dx)^{-\zeta-\xi}] H [z_1' x^{p_1} (1 + dx)^{-p_1-q_1}, \dots, z_t' x^{p_t} (1 + dx)^{-p_t-q_t}]\, dx$$

$$= d^{-\rho} \sum_{v=0}^{m+n} \sum_{e=0}^{v} \sum_{\delta=0}^{m+n} \sum_{p=0}^{\delta} \sum_{K=0}^{[V/U]} \theta_2 (K, e, v, \delta, p)$$

$$H^{O, N+2 : M_1, N_1; \dots; M_t, N_t; 1, 1}_{P+2, Q+1 : P_1, Q_1; \dots; P_t, Q_t; 1, 1} \left[z_1' d^{-p_1}, \dots, z_t' d^{-p_t}, -\tau y_1 d^{-\mu r} \left| \begin{matrix} C_2 : E : (1 - v + sn, 1) \\ D_2 : F : (0, 1) \end{matrix} \right. \right] \quad \dots (3.5)$$

where

$$d = \frac{c + ab}{c}, y_1' = y \left(\frac{a}{c} \right)^{\mu+\nu}, z' = z \left(\frac{a}{c} \right)^{\zeta+\xi},$$

$$z_i' = z_i \left(\frac{a}{c} \right)^{p_i + q_i} \text{ for } i = 1, \dots, t \quad \dots (3.6)$$

$$\theta_2 (K, e, v, \delta, p) = B^{qn} l^{m+n} \frac{(-1)^\delta (-\delta)_p (-v)_e (\alpha)_\delta (-\alpha - qn)_p}{p!\, \delta!\, e!\, v!\, K!\, \Gamma(v - sn)}$$

$$\frac{(-V)_{UK}\,A(V,K)\,(z')^K\,(-\beta/\tau - sn)_v\,(-\tau)^V}{(1-\alpha-\delta)_p}\left(\frac{a}{c}\right)^{(\mu+\nu)R}\left(\frac{A}{B}\right)^{\delta}\cdot$$

$$\left(\frac{p+k+re}{l}\right)_{m+n} \quad \ldots (3.7)$$

$$C_2 = (1-\rho-\mu R-\zeta K : p_1, \ldots, p_t, \mu r), (1-\sigma-\nu R-\xi K : q_1, \ldots, q_t, \nu r)$$

$$(a_j : \alpha_j^{(1)}, \ldots, \alpha_j^{(t)}, 0)_{1,p}$$

$$D_2 = (b_j : \beta_j^{(1)}, \ldots, \beta_j^{(t)}, 0)_{1,Q}, (1-\rho-\sigma-(\mu+\nu)R-(\zeta+\xi)K :$$

$$p_1+q_1, \ldots, p_t+q_t, (\mu+\nu)r \quad \ldots (3.8)$$

$$R = l(m+n)+rv+\delta \quad \ldots (3.9)$$

provided that the conditions easily obtainable from those mentioned with the main integral (2.1) are satisfied.

Further, if we take $A=1, B=0=q,\ m=-n$, let $\tau \to 0$ and $V=0$, both the polynomials $S_n^{\alpha,\beta,\tau}[x]$ and $S_V^U[x]$ reduce into unity. Again, we put $t=3$ and reduce the H-function of 3-variables thus obtained into the product of H-function of one variable and H-function of two variables by taking $\alpha_j^{(1)} = \beta_j^{(1)} = 0$. Finally taking $\rho+\sigma=\eta$, $p_i+q_i=r_i\ (i=1,2,3), N_i=0, p_1=1, r_1=0, r_2=p_2, r_3=p_3$. We arrive at a known result [4, eq. (8.2.4), p. 136] after making some suitable adjustments and simplifications on the right-hand side.

(*iii*) If we take $\eta=\rho+\sigma$, $\lambda=\mu+\nu$, $\gamma=\zeta+\xi$, $r_i=p_i+q_i$ $(i=1,\ldots,t)$, $c=a$, $b=(f-g)/(h-f)$ and replace x by $a\,(f-x)/(f-g)$ in the main integral (2.1), we arrive at the following form of the integral after a little simplification

$$\int_g^f (f-x)^{\rho-1}(x-g)^{\sigma-1}(x-h)^{-\rho-\sigma} S_n^{\alpha,\beta,\tau}\left[\frac{y(f-x)^{\mu}(x-g)^{\nu}}{(x-h)^{\mu+\nu}}\right]$$

$$S_V^U\left[z\frac{(f-x)^{\zeta}(x-g)^{\xi}}{(x-h)^{\zeta+\xi}}\right] H\left[z_1\frac{(f-x)^{p_1}(x-g)^{q_1}}{(x-h)^{p_1+q_1}}, \ldots, z_t\frac{(f-x)^{p_t}x-g)^{q_t}}{(x-h)^{p_t+q_t}}\right]dx$$

$$= \sum_{v=0}^{m+n}\sum_{e=0}^{v}\sum_{\delta=0}^{m+n}\sum_{p=0}^{\delta}\sum_{K=0}^{[V/U]} \theta_3(K,e,v,\delta,p)$$

$$H^{O,N+2:M_1,N_1;\ldots;M_t,N_t;1,1}_{P+2,Q+1:P_1,Q_1;\ldots;P_t,Q_t;1,1}$$

$$\left[z_1\left(\frac{f-g}{f-h}\right)^{p_1}\left(\frac{f-g}{g-h}\right)^{q_1},\ldots,z_t\left(\frac{f-g}{f-h}\right)^{p_t}\left(\frac{f-g}{g-h}\right)^{q_t},-\tau y^r\left(\frac{f-g}{f-h}\right)^{\mu r}\left(\frac{f-g}{g-h}\right)^{r}\right.$$

$$\left.\begin{array}{|l} C_3 : E : (1-v+sn,\ 1) \\ D_3 : F : (0,\ 1) \end{array}\right] \quad \ldots (3.10)$$

where

$$Y = y\left(\frac{f-h}{f-g}\right)^{\mu+\nu},\ Z = z\left(\frac{f-h}{f-h}\right)^{\zeta+\xi},\ Z_i = z_i\left(\frac{f-h}{f-g}\right)^{p_i+q_i}$$

$$(i = 1, \ldots, t) \quad \ldots (3.11)$$

$$\theta_3\,(K, e, v, \delta, p) = B^{qn}\, l^{m+n}\, \frac{(-1)^{\delta}\,(-\delta)_p\,(\alpha)_\delta\,(-v)_e\,(-\alpha-qn)_p}{p!\,\delta!\,e!\,v!\,K!\,(1-\alpha-\delta)_p}$$

$$\frac{(-\beta/\tau-sn)_v\,(-\tau)^v\,(-V)_{UK}\,A(V, K)\,Z^k}{\Gamma\,(v-sn)}\left(\frac{A}{B}\right)^{\delta}\left(\frac{f-g}{f-h}\right)^{\zeta}\left(\frac{f-g}{g-h}\right)^{\xi}$$

$$\left(\frac{p+k+re}{l}\right)_{m+n} \quad \ldots (3.12)$$

C_3, D_3, R can be obtained from the equations (2.3) and (2.4) respectively. Provided that the conditions easily obtainable from those mentioned with the main integral (2.1) are satisfied.

Now, if we take $A = 1, B = 0 = q, m = -n$ and let $\tau \to 0$ in (3.10) the polynomial set $S_n^{\alpha,\beta,\tau}[x]$ reduces into unity. Further, change t to $(t+1)$ and reduce the H-function of $(t+1)$ variables into the product of H-function of one variable H-function of t-variables by taking $\alpha_j^{(t+1)} = 0 = \beta_j^{(t+1)}$ therein, we arrive at an integral obtained by Chaurasia and Sharma [15, eq. (2.1), p. 42] after a little simplification.

(*iv*) If we take $A = 1, B = 0 = q, m = -n$ and let $\tau \to 0$ in the main integral (2.1), the polynomial set $S_n^{\alpha,\beta,\tau}[x]$ reduces into unity and we arrive at an integral essentially similar to the one obtained by Agrawal [13, eq. (4.5.1), p. 122].

Further, if we put $\eta = 0,\ \gamma = 0,\ r_i = 0\ (i = 1, \ldots, t)$ change t to $(t+1)$ and reduce the H-function of $(t+1)$ variables into the product of H-function of one variable and the H-function of t-variables by taking $\alpha_j^{(t+1)} = 0 = \beta_j^{(t+1)}$, we arrive at another known integral given by Chaurasia and Sharma [15, eq. (2.2), p. 43] after a little simplification. Further, if we take $t = 2$ and $V = 0$ reduce the polynomial $S_0^U[x]$ into unity, we easily arrive at another known integral [4, eq. (8.2.7), p. 138].

The results obtained earlier by Gupta et al. [8, eq. (3.1), p. 69], Srivastava and Singh [5, eq. (2.2), p. 166], Prasad and Singh [16, p.126] and Chaurasia and Sharma [15, eq. (2.3), p. 44 and eq. (2.4), p. 45] can also be obtained as simple special cases of the result referred in the above paragraphs. However, we omit the details here.

(v) If in the main result (2.1) we reduce both the polynomials $S_V^U[x]$ and $S_n^{\alpha,\beta,\tau}[x]$ into unity by the substitutions indicated in special case (ii), we arrive at an integral same in essence obtained by Garg [9, eq. (5.2.6), p. 244]. Further, if we take $t = 2$, $\eta = \rho + \sigma$, $r_i = p_i + q_i$ $(i = 1, 2)$, c=a, b = $\frac{c-d}{l+d}$ and replace x by ax, we get a known integral [4, eq. (8.4.4), p.144] after a little simplification.

Several other interesting results involving a large variety of polynomials, obtained as special cases of $S_V^U[x]$ and $S_n^{\alpha,\beta,\tau}[x]$, and various simpler functions, which are particular cases of the multivariable H-function, can also be obtained as special cases of our main result (2.1) but we do not record them here for lack of space.

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