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A HEAT CONDUCTION PROBLEM AND CERTAIN PRODUCTS OF THE MULTIVARIABLE *H*-FUNCTION WITH TWO GENERAL CLASSES OF POLYNOMIALS

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ABSTRACT

Special functions and their applications are becoming increasingly important to boundary value problems in partial differential equations of engineering and physics. So far as its technical applications is concerned, space research and nuclear reactor also give rise to several problems of the applications of special functions. The aim of this paper is to discuss the applications of certain products of the multivariable H-function with two general classes of polynomials in obtaining a solution of the partial differential equation

$$\frac{\partial \theta}{\partial t} = L\,\frac{\partial^2 \theta}{\partial x^2}$$

related to a (simple but typical) problem of heat conduction in a rod with one end at zero degree and the other end insulated. The results so derived may be found useful for computing different values of $f(x)$ for several special functions (in one and more arguments).

1. INTRODUCTION As an example of the applications of certain products of the multivariable H-function with two general classes of polynomials in applied mathematics, we shall consider the problem of obtaining a solution of heat conduction problem in one dimensional rod with constant thermal properties having no source, with one end at zero degree and the other end insulated, such that no solutions of heat equation develop exponentially in time.

Now, we formulate a simple (but typical) problem of heat flow to find a formula for the temperature $\theta(x, t)$ in one dimensional rod $0 \le x \le F$ must satisfy the heat equation

$$\frac{\partial \theta}{\partial t} = L \frac{\partial^2 \theta}{\partial x^2}, \quad t \geq 0 \qquad \text{...(1)}$$

and the initial conditions

$$\theta(0, t) = 0 \qquad \text{...(2)}$$

$$\frac{\partial \theta}{\partial x}(E, t) = 0 \qquad \text{...(3)}$$

$$\theta(x, t) \text{ is finite when } t \to \infty \qquad \text{...(4)}$$

$$\theta(x, 0) = f(x) \qquad \text{...(5)}$$

Here we shall consider

$$f(x) = \sin\left(\frac{\pi x}{F}\right)^{\gamma-1} S_n^m\left[y\left(\sin\frac{\pi x}{F}\right)^h\right] S_{n'}^{m'}\left[y'\left(\sin\frac{\pi x}{F}\right)^{h'}\right]$$
$$H\left\{z_1\left(\sin\frac{\pi x}{F}\right)^{2h_1}, \ldots, z_r\left(\sin\frac{\pi x}{F}\right)^{2h_r}\right\}, \qquad \text{...(6)}$$

where the multivariable H-function is the H-function of several complex variables introduced by Srivastava and Panda ([8]; see also [6], p.251) and for the general class of polynomials ([5]; see also [7]).

2. THE MAIN INTEGRAL

$$\int_0^F \left(\sin\frac{\pi x}{F}\right)^{\gamma-1} \sin(2v+1)\left(\frac{\pi x}{2F}\right) S_n^m\left[y\left(\sin\frac{\pi x}{F}\right)^h\right]$$
$$S_{n'}^{m'}\left[y'\left(\sin\frac{\pi x}{F}\right)^{h'}\right] H\left(z_1\left(\sin\frac{\pi x}{F}\right)^{2h_1}, \ldots, z_r\left(\sin\frac{\pi x}{F}\right)^{2h_r}\right) dx$$
$$= F\, 2^{1-\gamma} \sin(2v+1)\left(\frac{\pi}{4}\right) \sum_{s=0}^{(n/m)} \frac{(-n)_{ms}}{s!} A_{n,s}\,(2^h y)^s$$
$$\sum_{s'=0}^{(n'/m')} \frac{(-n')_{m's'}}{s'!} A_{n',s'}\,[2^{h'} y']^{s'}\; H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',D'];\ldots;[B^{(R)},D^{(r)}]}$$
$$\left(\begin{matrix}[1-\gamma-hs-h's':2h_1;\ldots;2h_r],..[(a):\theta':\ldots;\theta^{(r)}]: \\ [(c):\psi';\ldots;\psi^{(r)}],[(1-2\gamma-2hs-2h's-2v)/4:h_1;\ldots;h_r],[(3-2\gamma-2hs-2h's'+2v)/4;h_1;\ldots;h_r]:\end{matrix}\right.$$
$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)});\phi^{(\rho)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1 4^{-h_1};\ldots;z_r 4^{-h_r}\right) \qquad \text{...(7)}$$

where $\text{Re}\left(\gamma + \sum_{i=1}^{r} h_i d_j^i/\delta_j^i\right) > 0$, $h_i > 0$, $h > 0$, $h' > 0$,

$$|\arg(z_i)| < T_i \frac{\pi}{2}, T_i > 0, 1 \le i \le r,$$

$$1 \le j \le u^{(i)},$$

and m and m' are arbitrary integers and the coeffients $A_{n,s}$ and $A_{n',s'}$ $(n, n', s, s' \ge 0)$ are constants, real or complex.

The integral in (7) can be established by making use of the definition of the general class of polynomials ([8]; see also [6]) and a result recently obtained by Sharma ([3], p.372).

3. SOLUTION OF THE PROBLEM

The solution of the problem will be found to be

$$\theta(x,t) = 2^{2-\gamma-hs-h's'} \sum_{v=0}^{\infty} \sin(2v+1)\left(\frac{\pi}{4}\right) e^{-(2v+1)(\pi/2F)Lt}$$

$$\sin\left(v+\frac{1}{2}\right)\left(\frac{\pi x}{F}\right) \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\, y^s \sum_{s'=0}^{[n'/m']} \frac{(-n')_{m's'}}{s'!} A_{n',s'}\, y^{s'}$$

$$H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left(\begin{matrix}[l-\gamma-hs-h's':2h_1;\ldots;2h_r] \\ [(c):\psi';\ldots;\psi^{(r)}],\end{matrix}\right.$$

$$[(a):\theta';\ldots;\theta^{(r)}]:$$
$$[(1-2\gamma-2hs-2h's'-2v)/4:h_1;\ldots;h_r],[(3+2v-2\gamma-2hs-2h's')/4;h_1;\ldots;h_r];$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix} z_1 4^{-h_1},\ldots,z_r 4^{-h_r}\right) \quad \ldots(8)$$

where $h > 0, h' > 0, h_i > 0, \mathrm{Re}\left(\gamma + \sum_{i=1}^{r} h_i d_j^i/\delta_j^i\right) > 0,$

$$|\arg(z_i)| < T_i \frac{\pi}{2}, T_i > 0, 1 \le i \le r, 1 \le j \le u^{(i)},$$

and m and m' are arbitrary integers and the coefficients $A_{n,s}$ $(n, s \ge 0)$ and $A_{n',s'}$ $(n', s' \ge 0)$ are arbitrary constants, real or complex.

The general solution of the problem expressed in section 1 by setting $\theta = X(x)\ T(t)$ can be easily established as ([4], p.75)

$$\theta(x,t) = \sum_{v=0}^{\infty} B_v \sin\left(\left(v+\frac{1}{2}\right)\frac{\pi x}{F}\right) e^{(-(2v+1)\pi/2F)Lt} \quad \ldots(9)$$

If $t = 0$, then by virtue of (5) we have

$$\left(\sin\frac{\pi x}{F}\right)^{\gamma-1} S_n^m\left[y\left(\sin\frac{\pi x}{F}\right)^h\right] S_{n'}^{m'}\left[y'\left(\sin\frac{\pi x}{F}\right)^{h'}\right]$$

$$H\left(z_1\left(\sin\frac{\pi x}{F}\right)^{2h_1}, \ldots, z_r\left(\sin\frac{\pi x}{F}\right)^{2h_r}\right) = \sum_{v=0}^{\infty} B_v \sin\left(v+\frac{1}{2}\right)\left(\frac{\pi x}{F}\right) \quad \ldots(10)$$

Multiplying both the sides of (10) by $\sin\,(w+\frac{1}{2})\frac{\pi x}{F})$, integrating with respect to x from 0 to F and using (7) along with the orthogonality property of sines, we get

$$B_w = 2^{2-\gamma-hs-h's'} \sin\,(2w+1)\,\frac{\pi}{4}) \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\, y^s$$

$$\sum_{s'=0}^{[n'/m']} \frac{(-n')_{m's'}}{s'!} A_{n',s'}\, y'^{s'}\, H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix}[1-\gamma-hs-h's':2h_1;\ldots;2h_r],\ldots\ [(a):\theta';\ldots;\theta^{(r)}]: \\ [(c):\psi';\ldots;\psi^{(r)}],[(1-2\gamma-2hs-2h's')/4:h_1;\ldots;h_r];\ [(3-2\gamma-2hs-2h's'+2w)/4:h_1;\ldots;h_r)]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1\,4^{-h_1},\ldots,z_r\,4^{-h_r}\right) \quad \ldots(11)$$

Putting the values of B_w from (11) in (9), we get the related solution.

4. APPLICATIONS AND PARTICULAR CASES

(A) By applying our results given in (7) and (8) to the case of Hermite polynomials ([9], p.106, eq. (5.54) and [7], p.158) by setting

$$S_n^2\,(x) \rightarrow x^{n/2}\, H_n\left(\frac{1}{2\sqrt{x}}\right)$$

in which case $m = 2$, $A_{n,s} = (-1)^s$ and also letting $m' = 2$. $A_{n',s'} = (-1)^{s'}$. We have the following interesting consequences of the main results :

(A.1)

$$\int_0^F \sin\left(\frac{\pi x}{F}\right)^{\gamma-1} (\sin\,(2v+1)\,\frac{\pi x}{2F})\left[y\left(\sin\frac{\pi x}{F}\right)^h\right]^{n/2} H_n\left[\frac{1}{2\sqrt{y\,(\sin\frac{\pi x}{F})^h}}\right]$$

$$\left[y' (\sin \frac{\pi x}{F})^{h'} \right]^{n'/2} H_{n'}\left[\frac{1}{2\sqrt{y' (\sin \frac{\pi x}{F})^{h'}}} \right]$$

$$H\left(z_1(\sin \frac{\pi x}{F})^{h_1}, \ldots, z_r (\sin \frac{\pi x}{F})^{h_r} \right) dx$$

$$= F.2^{1-\gamma-n-n'-hs-h's'} \sin ((2v+1)\frac{\pi}{4}) \sum_{s=0}^{n/2} \frac{(-n)_{2s}}{s!} (-\frac{y}{4})^s$$

$$\sum_{s'=0}^{(n'/2)} \frac{(-n')_{2s'}}{s'!} (\frac{-y'}{4})^{s'} H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix} (1-\gamma-hs-h's');2h_1;\ldots;2h_r],[(a):\theta';\ldots;\theta^{(r)}]: \\ [(c):\psi';\ldots;\psi^{(r)}],[(1-2\gamma-2hs-2h's'-2v)/4;h_1;\ldots;h_r][(3-2\gamma-2hs-2h's'+2v)/4:h_1;\ldots;h_r]: \end{matrix} \right.$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(\rho)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; \end{matrix} z_1 4^{-h_1},\ldots, z_r 4^{-h_r} \right) \ldots(12)$$

which holds true under the same conditions needed for (7).

(A.2)

$$\theta(x,t) = 2^{2-\gamma+n+n'-hs-h's'} \sum_{v=0}^{\infty} \sin (2v+1)\frac{\pi}{4}) e^{-[(2v+1)\pi/2F] Lt}$$

$$\sin (v+\frac{1}{2})\frac{\pi x}{F}) \sum_{s=0}^{[n/2]} \frac{(-n)_{2s}}{s!} (-\frac{y}{4})^s \sum_{s'=0}^{(-n'/2)} \frac{(-n')_{2s'}}{s'!} (-\frac{y'}{4})^{s'}$$

$$H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)});}_{A+1,C+2:[B',D'];\ldots;[B^{(r)}D^{(r)}];} \left(\begin{matrix} [1-\gamma-hs-h's':2h_1;\ldots;2h_r] \\ [(c):\psi;\ldots;\psi^{(r)}] \end{matrix} \right.$$

$$\begin{matrix} [(a):\theta';\ldots;\theta^{(r)}]: \\ [(1-2\gamma-2hs-2h's'-2v)/4:h_1;\ldots;h_r],[(3+2v-2\gamma-2hs-2h's')/4;h_1;\ldots;h_r]: \end{matrix}$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; \end{matrix} z_1 4^{-h_1},\ldots, z_r 4^{-h} \right) \ldots(13)$$

valid under the same conditions as obtainable from (8).

(B) For the Laguerre polynomials ([9], p.101, Eq. 15, 1.6) and [7], p.159), setting $S_n^1(x) \to L_n^{(\alpha')}(x)$ in which case

$m = 1, A_{n,s} \binom{n+\alpha'}{n} (\frac{1}{(\alpha'+1)_k})$ and also taking $m' = 1$.

$A_{n',s'} \binom{n'+\alpha''}{n'} (\frac{1}{(\alpha''+1)_{k'}})$ the results in (7) and (8) reduce to the following formulae :

(B.1)

$$\int_0^F (\sin \frac{\pi x}{F})^{\gamma-1} \sin ((2v+1)\frac{\pi x}{2F}) L_n^{\alpha'}\left[y\left(\sin \frac{\pi x}{F}\right)^h \right] L_{n'}^{\alpha''}\left[y'\left(\sin \frac{\pi x}{F}\right)^{h'} \right]$$

$$H\left(z_1\left(\sin \frac{\pi x}{F}\right)^{2h_1}, \ldots, z_r\left(\sin \frac{\pi x}{F}\right)^{2h_r} \right) dx$$

$$= F.2^{1-\gamma-hs-h's'} \sin ((2v+1)\frac{\pi}{4}) \sum_{s=0}^{(n)} \binom{n+\alpha'}{n-s} \frac{(-y)^s}{s!} \sum_{s'=0}^{(n')} \binom{n'+\alpha''}{n'-s'} \frac{(-y')^{s'}}{s'!}$$

$$H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',d'];\ldots;[B^{(r)},D^{(r)}]}\left(\begin{matrix} 1-\gamma-hs-h's':2h_1;\ldots;2h_r], \\ [(c):\psi';\ldots;\psi^{(r)}] \end{matrix} \right.$$

$$[(a):\theta';\ldots;\theta^{(r)}]:$$
$$[(1-2\gamma-2hs-2h's'-2v)/4):h_1;\ldots;h_r], [(3-2\gamma-2hs-2h's'+2v)/4:h_1,\ldots,h_r]:$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(d'):\delta'];\ldots;[(d^{(r)});\delta^{(r)}]; \end{matrix} z_1 4^{-h_1};\ldots;z_r 4^{-h_r} \right) \quad \ldots(14)$$

valid under the same conditions as obtainable from (7).

(B.2)

$$\theta(x,t) = 2^{2-\gamma+n+n'-hs-h's'} \sum_{v=0}^{\infty} \sin (2v+1)\frac{\pi}{4}) e^{-[(2v+1)\pi/2F]Lt}$$

$$\sin (v+\frac{1}{2})\frac{\pi x}{F}) \sum_{s=0}^{n} \frac{(-y)^s}{s!} \binom{n+\alpha'}{n-s} \sum_{s'=0}^{n'} \frac{(-y')^{s'}}{s'!} \binom{n'+\alpha''}{n'-s'}$$

$$H^{0,\lambda+1:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+1,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}:\left(\begin{matrix} [1-\gamma-hs-h's':2h_1;\ldots;2h_r], \\ [(c):\psi';\ldots;\psi^{(r)}], \end{matrix} \right.$$

$$[(a):\theta';\ldots;\theta^{(r)}]:$$
$$[(1-2\gamma-2hs-2h's'-2v)/4:h_1;\ldots;h_r], [(3+2v-2\gamma-2hs-2h's')/4:h_1;\ldots;h_r]:$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]: \\ [(d').\delta'];\ldots;[(d^{(r)}).\delta^{(r)}]; \end{matrix} z_1 4^{-h_1},\ldots,z_1 4^{-h_r} \right) \quad \ldots(15)$$

holds true under the same conditions as those required for (8).

(C) for the Jacobi polynomials ([9], p.68, Eq. (4.3.2) and [7], p.159), setting $S_n^1(x) \to P_n^{(\alpha',\beta')}(1-2x)$ and $S_{n'}^1(x) \to p_{n'}^{(\alpha'',\beta'')}(1-2x)$ in which case $m=1, A_{n,s} = \binom{n+\alpha'}{n} \frac{(\alpha'+\beta'+n+1)_s}{(\alpha'+1)_s}$ and also let $m'=1$.

$$A_{n',s'} = \binom{n'+\alpha''}{n'} \frac{(\alpha''+\beta''+n'+1)_{s'}}{(\alpha''+1)_{s'}},$$

From equations (7) and (8), we obtain the following results.

(C.1)

$$\int_0^F (\sin \frac{\pi x}{F})^{\gamma - 1} \sin (2v + 1) \frac{\pi x}{2F}) p_n^{(\alpha', \beta')} \left[1 - 2y \left(\sin \frac{\pi x}{F} \right)^h \right]$$

$$P_{n'}^{(\alpha'', \beta'')} \left[1 - 2y' \left(\sin \frac{\pi x}{F} \right)^{h'} \right] H (z_1 (\sin \frac{\pi x}{F})^{2h_1}, ..., z_r (\sin \frac{\pi x}{F})^{2h_r}) \, dx$$

$$= F.2^{1 - \gamma - hs - h's'} \sin (2v + 1) \frac{\pi}{4}) \sum_{s=0}^{n} (-y)^s \binom{n + \alpha'}{n - s} \binom{\alpha' + \beta' + n + s}{s}$$

$$\sum_{s'=0}^{(n')} (-y')^{s'} \binom{n' + \alpha''}{n' - s'} \binom{\alpha'' + \beta'' + n' + s'}{s'}$$

$$H^{0, \lambda + 1 : (u', v');...; (u^{(r)}, v^{(r)})}_{A + 1, C + 2 : [B', D'];...; [B^{(r)}, D^{(r)}]} \left(\begin{matrix} [1 - \gamma - hs - h's' : 2h_1 ;...; 2h_r], \\ [(c) : \psi' ;...; \psi^{(r)}], \end{matrix} \right.$$

$$\begin{matrix} [(a) : \theta' ;...; \theta^{(r)}]: \\ [(1 - 2\gamma - 2hs - 2h's' - 2v)/4v : h_1;...;h_r] \; [(3 - 2\gamma - 2hs - 2h's' + 2v)/4 : h_1 ;...; h_r]: \end{matrix}$$

$$\left. \begin{matrix} [(b') ; \phi'] ;...; [(b^{(r)}) : \phi^{(r)}]; \\ (d') : \delta'];...; [(d^{(r)}); \delta^{(r)}]; \end{matrix} \; z_1 4^{-h_1} ;...; z_r 4^{-h_4} \right), \qquad ...(16)$$

valid under the same conditions needed for (7).

(C.2)

$$\theta (x, t) = 2^{2 - \gamma - hs - h's'} \sum_{v=0}^{\infty} \sin (2v + 1) \frac{\pi}{4}) e^{[-(2v + 1) \pi/2F] Lt}$$

$$\sin (v + \frac{1}{2}) \frac{\pi x}{F}) \sum_{s=0}^{n} (-y)^s \binom{n + \alpha'}{n - s} \binom{\alpha' + \beta' + n + s}{s}$$

$$\sum_{s'=0}^{n'} (-y')^{s'} \binom{n' + \alpha''}{n' - s'} \binom{\alpha'' + \beta'' + n' + s'}{s'}$$

$$H^{0, \lambda + 1 : (u', v');...;(u^{(r)}, v^{(r)}]}_{A + 1, C + 2 : [B', D'];...;[B^{(r)} : D^{(r)}]} \left(\begin{matrix} [1 - \gamma - hs - h's' : 2h_1;...;2h_r], \\ [(c) : \psi' :...; \psi^{(r)}], \end{matrix} \right.$$

$$\begin{matrix} [(a) : \theta' ;...; \theta^{(r)}]: \\ [(1 - 2\gamma - 2hs - 2h's' - 2v)/4 : h_1;...;h_r], [(3 - 2\gamma - 2hs - 2h's' + 2v)/4 : h_1 ;...; h_r]: \end{matrix}$$

$$\left. \begin{matrix} [(b') : \phi'] ;...; [(b^{(r)}) : \phi^{(r)}]: \\ [(d') : \delta'] ;...; [(d^{(r)}) : \delta^{(r)}]; \end{matrix} \; z_1 4^{-h_1};...; z_r 4^{-h} \right) \qquad ...(17)$$

valid under the same conditions as obtainable from (8).

(D) Taking $\lambda = A$, $u^{(i)} = 1$, $v^{(i)} = B^{(i)}$ and replacing $D^{(i)}$ by $D^{(i)} + 1$, $\forall\ i = 1, ..., v$ in (7) and (8), we get

(D.1)

$$\int_0^F (\sin \frac{\pi x}{F})^{\gamma - 1} \sin (2v + 1) \frac{\pi x}{2F})\, S_n^m \left[y (\sin \frac{\pi x}{F})^h \right] S_{n'}^{m'} \left[y' \left(\sin \frac{\pi x}{F} \right)^{h'} \right].$$

$$F^{A : B' ;...; B^{(r)}}_{C : D' ;...; D^{(r)}} \left(\begin{matrix} [1 - (a) : \theta' ;...; \theta^{(r)}] : [1 - (b') : \phi'] ;...; [1 - (b^{(r)}) : \phi^{(r)}] ; \\ [1 - (c) : \psi' ;...; \psi^{(r)}] : [1 - (d') : \delta'] ;...; [1 - (d^{(r)}) : \delta^{(r)}] ; \end{matrix} \right.$$

$$\left. - z_1 (\sin \frac{\pi x}{F})^{2h_1}, ..., - z_r (\sin \frac{\pi x}{F})^{2h_r} \right) dx$$

$$= F.2^{1 - \gamma - hs - h's'} \sin ((2v + 1) \frac{\pi}{4}) \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n, s}\, y^s \sum_{s'=0}^{[n'/m']} \frac{(-n')_{m's'}}{s'!} A_{n', s'}\, y'^{s'}$$

$$\frac{\Gamma (\gamma + hs + h's')}{\Gamma [(3 + 2\gamma + 2hs + 2h's' + 2v)/4]\, \Gamma [(1 + 2\gamma + 2hs + 2h's' - 2v)/4]}$$

$$F^{A + 1 : B' ;...; B^{(r)}}_{C + 2 : D' ;...; D^{(r)}} \left(\begin{matrix} [\gamma + hs + h's' : 2h_1 ;...; 2h_r] ; \\ [1 - (c) : \psi' ;...; \psi^{(r)}] \end{matrix} \right.$$

$$[1 - (a) : \theta' ;...; \theta^{(r)}] :$$
$$[(3 + 2\gamma + 2hs + 2h's' + 2v)/4 : h_1 ;...; h_r],\ [(1 + 2\gamma + 2hs + 2h's' - 2v)/4 : h_1 ;...; h_r] :$$

$$\left. \begin{matrix} [1 - (b') : \phi'] ;...; [1 - (b^{(r)}) : \phi^{(r)}] ; \\ [1 - (d') : \delta'] ;...; [1 - (d^{(r)}); \delta^{(r)}] ; \end{matrix} - z_1\, 4^{-h_1}, ..., - z_r\, 4^{-h_r} \right), \qquad ...(18)$$

valid under the same conditions as required in (7).

(D.2)

$$\theta (x, t) = 2^{2 - \gamma - hs - h's'} \sum_{v = 0}^{\infty} \sin (2v + 1) \frac{\pi}{4})\, e^{- [(2v + 1) \pi / 2F] Lt}$$

$$\sin ((v + \frac{1}{2}) \frac{\pi x}{F}) \sum_{s = 0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n, s}\, y^s \sum_{s' = 0}^{[n'/m']} \frac{(-n')_{m's'}}{s'!} A_{n', s'}\, y'^{s'}$$

$$\frac{\Gamma (\gamma + hs + h's')}{\Gamma [(3 + 2\gamma + 2hs + 2h's' + 2v)/4]\, \Gamma [(1 + 2\gamma + 2hs + 2h's' - 2v)/4]}$$

$$F^{A + 1 : B' ;...; B^{(r)}}_{C + 2; D' ;...; D^{(r)}} \left(\begin{matrix} [\gamma + hs + h's' : 2h_1 ;...; 2h_r] \\ [1 - (c) : \psi' ;....; \psi^{(r)}] ; \end{matrix} \right.$$

$$[1 - (a) : \theta' ;...; \theta^{(r)}] :$$
$$[(3 + 2\gamma + 2hs + 2h's' + 2v)/4 : h_1 ;...; h_r],\ [(1 + 2\gamma + 2hs + 2h's' - 2v)/4 : h_1 ;...; h_r] :$$

$$\left. \begin{matrix} [1 - (b') : \phi'] ;...; [1 - (b^{(r)}) : \phi^{(r)}] ; \\ [1 - (d') : \delta'] ;...; [1 - (d^{(r)}) : \delta^{(r)}] ; \end{matrix} - z_1\, 4^{h_1}, ..., - z_r\, 4^{-h_r} \right) \qquad ...(19)$$

valid under the same conditions as obtainable from (8).

(E) Letting $n' \to 0$ the results in (12) through (19) follow as particular case of the results recently obtained by Sharma [3].

(F) Taking $n \to 0$ and $n' \to 0$, the results in (7) and (8) reduce to the results recently obtained by Chaurasia ([1], p.172, eq.(2.1) and p.173, eq.(3.1)).

The importance of our results lies in its manifold generality. In view of the generality of the H-function of several complex variables, on specializing the various parameters and variables in the H-function of several complex variables, we obtain, from our results, several integrals and solutions involving a remarkably wide variety of useful functions (or product of several such functions), which are expressible in terms of E, F, G and H functions of one and several variables.

Secondly, by suitably specializing the coefficients $A_{n,s}$ and $A_{n',s'}$ and making a free use of the special cases of $S_n^m[x]$ listed by Srivastava and Singh [7], our results can be reduced to a large number of integerals and solutions of the problem (and also its particular cases) involving a product of generalized Hermite polynomials, Jacobi polynomials and its various special cases, Laguerre polynomials, Bessel polynomials. Gould-Hopper polynomials, Brafman polynomials and their various combinations. Thus the results presented in this paper would at once yield a very large number of results involving a large variety of polynomials and various special functions occuring in the problems of mathematical analysis, applied mathematics and mathematical physics.

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