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EQUILIBRIUM SOLUTIONS OF THE RESTRICTED PROBLEM OF (2+2) BODIES WHEN PRIMARIES ARE TAKEN AS TWO MAGNETIC DIPOLES ASSOCIATED WITH TWO CARRIER STARS

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ABSTRACT

The equilibrium solutions of the restricted problem of (2+2) bodies, in which two charged particles are moving under the action of Lorentz force of two magnetic dipoles located at the two primaries and the Coulomb's forces between the charged particles (minor bodies), are calculated, when magnetic moments of the two dipoles are taken perpendicular to the plane of motion of the primaries.

1. INTRODUCTION

Stormer (1907) has studied the motion of a single charged particle in the field of one magnetic dipole. The motion of a single charged particle in the field of two rotating magnetic dipoles was studied by Mavraganis (1978) in which he has studied the areas of motion in planar magnetic binary case. The same problem has been studied by Goudas et all (1984-85), in which they have taken the direction of the dipole moments as perpendicular to the plane of motion of the primaries, but Mavraganis has not fixed the direction of the dipole's moment in his problem.

The option to consider the two dipoles stationary or moving, is impotant in view of its Astronomical and Engineering applications.

The simplest motion tht the two dipoles should perform is the circular motion about a fixed point 0 resting anywhere along a straight line connecting the dipoles. If these dipoles are associated with two stars S_1 and S_2 then their centre of mass is the centre of rotation of the dipoles. In such a case, the formulation of the problem can follow the lines of the elliptical or circular restricted problem.

The first author has generalized in his paper (1993) the problem of Goudas et al (1984) by considering the motion of $P_j (j = 1, 2, ..., \nu)$ charged particles with charges q_j and masses m_j in place of a single charged particle. In that paper author has determined the equation of motion, the first integral analogous to Jacobian integral and the hyper surfaces of zero velocity.

In this paper, we have studied the equilibrium solutions for the restricted problem of (2+2) bodies. It is found that there exists twenty two equilibrium solutions in the plane of motion of the primaries, out of which six equilibrium solutions are on the x-axis about the three collinear equilibrium points calculated by Goudas et al (1985) and off this axis, there exists sixteen equilibrium solutions in the neighbourhood of the four off-axis equilibrium points calculated by Goudas et al.

2. EQUILIBRIUM SOLUTIONS

In the synodic coordinate system by using dimensionless variables for the restricted problem of (2+2) bodies, the equations of motion of two charged particles may be written as (Prasad, 1993).

$$\ddot{x}_i - \dot{y}_i\left\{2\omega + \frac{q_i}{c\mu_i}\left(\frac{\partial A_{y_i}}{\partial x_i} - \frac{\partial A_{x_i}}{\partial y_i}\right)\right\} + \frac{q_i}{c\mu_i}\dot{z}_i\left(\frac{\partial A_{x_i}}{\partial z_i} - \frac{\partial A_{z_i}}{\partial x_i}\right) = \frac{\partial T}{\partial x_i} \quad \text{... (1)}$$

$$\ddot{y}_i + \dot{x}_i\left\{2\omega + \frac{q_i}{c\mu_i}\left(\frac{\partial A_{y_i}}{\partial x_i} - \frac{\partial A_{x_i}}{\partial y_i}\right)\right\} + \frac{q_i}{c\mu_i}\dot{z}_i\left(\frac{\partial A_{y_i}}{\partial z_i} - \frac{\partial A_{z_i}}{\partial y_i}\right) = \frac{\partial T}{\partial y_i} \quad \text{... (2)}$$

and

$$\ddot{z} - \frac{q_i}{c\mu_i}\left\{\dot{x}_i\left(\frac{\partial Ax_i}{\partial z_i} - \frac{\partial Az_i}{\partial x_i}\right) + \dot{y}_i\left(\frac{\partial A_{y_i}}{\partial z_i} - \frac{\partial A_{z_i}}{\partial y_i}\right)\right\} = \frac{\partial T}{\partial z_i} \quad \text{... (3)}$$

where

$$T = \sum_{i=1}^{2}\left[\frac{\omega^2}{2}\left(\left[x_i^2 + y_i^2\right) + \frac{q_i}{c\mu_i}\left(x_i A_{Y_i} - y_i A_{x_i}\right) - \frac{(-1^{i+1}}{4\pi\varepsilon_0}\frac{q_i q_{3-i}}{\mu_i |\bar{r}|}\right.$$

$$\bar{r}^2 = (x_i - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2,$$

$$\left.\begin{aligned} A_{x_i} &= \frac{M_{1y} z_i - M_{1z} y_i}{|\bar{R}_{1i}|^3} + \frac{M_{2y} z_i - M_{2z} y_i}{|\bar{R}_{2i}|^3} \\ A_{y_i} &= \frac{M_{iz}(x_i - \mu) - M_{1x} z_i}{|\bar{R}_{1i}|^3} + \frac{M_{2z}(x_i + 1 - \mu) - M_{2x} z_i}{|\bar{R}_{2i}|^3} \\ A_{z_i} &= \frac{M_{1x} y_i - M_{1y}(x_i - \mu)}{|\bar{R}_{1i}|^3} + \frac{M_{2x} y_i - M_{2y}(x_i + 1 - \mu)}{|\bar{R}_{2i}|^3} \end{aligned}\right] \quad \text{... 3(a)}$$

c = velocity of light,

$\omega =$ mean angular velocity of the primaries about z-axis,

$\epsilon_0 =$ permitivity of the medium between charged particles,

$\mu = \dfrac{M_2}{M_1 + M_2}$, q_i = charge of the minor particle

$\mu_i = \dfrac{m_i}{M_1 + M_2}$, $i = 1, 2$

M_1 and M_2 are the masses of the primaries and m_1, m_2 are the masses of the minor bodies.

$m_1, m_2 << M_1 < M_2$.

Here the distance betwen the primaries is taken as the unit of distance, the sum of their masses as the unit of mass.

Now if we adopt the unit of time such that $\omega = 1$, the unit of charge such that $\dfrac{1}{4\pi\varepsilon_0} = 1$ and take $\dfrac{q_i}{c\mu_i} = p < 1$. Then T can be written as

$$T = \sum_{i=1}^{2} \frac{1}{2}(x_i^2 + y_i^2) + p(x_i A_{y_i} - y_i A_{x_i}) - (-1)^{i+1} cp \frac{q_{3-i}}{|\bar{r}|} \quad \text{... (4)}$$

$$T = \sum_{i=1}^{2} (\Omega(x_i, y_i, z_i)) - (-1)^{i+1} cp \frac{q_{3-i}}{|\bar{r}|}.$$

$$\Omega(x_i, y_i, z_i) = \frac{1}{2}\left(x_i^2 + y_i^2\right) + p\left(x_i A_{y_i} - y_i A_{x_i}\right).$$

The equilibrium solutions of this system are those points of the phase space which satisfy

$$\dot{x}_i = \dot{y}_i = \dot{z}_i = \frac{\partial T}{\partial x_i} = \frac{\partial T}{\partial y_i} = \frac{\partial T}{\partial z_i} = 0 \quad \text{... (5)}$$

By evaluation of the partial derivatives appearing in equation (5) and considering $\overline{M}_1 \equiv (0, 0, 1), \overline{M}_2 \equiv (0, 0, \lambda)$, we get from equation (3(a)).

$$A_{x_i} = -y_i\left(\frac{1}{|\overline{R}_{1i}|^3} + \frac{\lambda}{|\overline{R}_{2i}|^3}\right)$$

$$A_{y_i} = \frac{x_i - \mu}{|\overline{R}_{1i}|^3} + \frac{\lambda(x_i + 1 - \mu)}{|\overline{R}_{2i}|^3}$$

$$A_{z_i} = 0, i = 1, 2$$

From equation (5) we get

$$x_i + p\left[-3x_i\left\{\frac{(x_i-\mu)^2}{|\bar{R}_{1i}|^5} + \frac{\lambda(x_i+1-\mu)^2}{|\bar{R}_{2i}|^5}\right\} - 3y_i^2\left\{\frac{x_i-\mu}{|\bar{R}_{1i}|^5} + \frac{\lambda(x_i+1-\mu)}{|\bar{R}_{2i}|^5}\right\}\right.$$

$$\left. + \frac{2x_i-\mu}{|\bar{R}_{1i}|^3} + \frac{\lambda(2x_i+1-\mu)}{|\bar{R}_{2i}|^3}\right] + (-1)^{i+1} cpq_{3-i}\frac{x_1-x_2}{|\bar{r}|^3} = 0 \qquad \text{... (6)}$$

$$y_i + p\left[-3x_iy_i\left\{\frac{x_i-\mu}{|\bar{R}_{1i}|^5} + \frac{\lambda(x_i+1-\mu)}{|\bar{R}_{2i}|^5}\right\} - 3y_i^2\left\{\frac{1}{|\bar{R}_{1i}|^5} + \frac{\lambda}{|\bar{R}_{2i}|^5}\right\}\right.$$

$$\left. + 2y_i\left\{\frac{1}{|\bar{R}_{1i}|^3} + \frac{\lambda}{|\bar{R}_{2i}|^3}\right\}\right] + (-1)^{i+1} cpq_{3-i}\frac{y_1-y_2}{|\bar{r}|^3} = 0 \qquad \text{... (7)}$$

and

$$p\left[z_i\left\{-3x_i\left(\frac{x_i-\mu}{|\bar{R}_{1i}|^5} + \frac{\lambda(x_i+1-\mu)}{|\bar{R}_{2i}|^5}\right) - 3y_i^2\left(\frac{1}{|\bar{R}_{1i}|^5} + \frac{\lambda}{|\bar{R}_{2i}|^5}\right)\right\}\right]$$

$$+ (-1)^{i+1} cpq_{3-i}\frac{z_1-z_2}{|\bar{r}|^3} = 0. \qquad \text{... (8)}$$

(a) Computation of the Collinear Equilibrium Solutions :

Equations (7) and (8) are directly satisfied for $y_i = 0$, $z_i = 0$ $(i = 1, 2)$ and in that case equation (6) becomes

$$x_i + p\left[-3x_i\left\{\frac{1}{|x_i-\mu|^3} + \frac{\lambda}{|x_i+1-\mu|^3}\right\} + \frac{2x_i-\mu}{|x_i-\mu|^3} + \frac{\lambda(2x_i+1-\mu)}{|x_i+1-\mu|^3}\right]$$

$$+ cpq_{3-i}\frac{x_i-x_{3-i}}{|\bar{r}|^3} = 0 \qquad \text{... (9)}$$

where $|\bar{r}|^2 = (x_1 - x_2)^2$.

By taking two charged particles $q_i (i = 1, 2)$ instead of one, as has been taken by Goudas et al (1985), there is an additional term $cpq_{3-i}\frac{x_i - x_{3-i}}{|\bar{r}|^3}$ occurring in equation (9). Assuming this quantity to be small, we may take the solution of equation (9) as a power series in terms of small parameter defined by

$$\epsilon_i = \frac{q_i}{(q_1+q_2)^{2/3}} << 1, \quad i = 1, 2 \qquad \text{... (10)}$$

in the form

$$x_i = L_k + \sum_{j=1}^{\infty} a_{ij}\, \epsilon_{3-i}^j, \quad i = 1, 2, k = 1, 2, 3. \qquad \text{... (11)}$$

where L_k denotes the collinear equilibrium solutions calculated by Goudas et al and will be the roots of the equation (9) when last term is neglected.

Thus for $0(\varepsilon) = \max(\varepsilon_1, \varepsilon_2)$, equation (9) implies that

$$a_{ij}\,\varepsilon_{3-i}\,\Omega^0_{xx} + cpq_{3-i}\,\frac{x_i - x_{3-i}}{|x_1 - x_2|^3} = 0 \qquad \dots (12)$$

where super script "o" denotes the evaluation at one of the collinear equilibrium points calculated by Goudas.

Here

$$\Omega^o_{xx} = 1 + 2p\left[\frac{1}{|L_k - \mu|^3} + \frac{\lambda}{|L_k + 1 - \mu|^3} + \frac{3\mu}{|L_k - \mu|^4} - \frac{3\lambda(1-\mu)}{|L_k + 1 - \mu|^4}\right]$$

Writing down equation (12) for $i = 1, 2$ and after a little simplification, we get

$$\frac{q_1 q_2}{(q_1 + q_2)^{2/3}}(a_{11} + a_{12}) = 0$$

$$\rightarrow a_{12} = -\,a_{11} \text{ as } \frac{q_1 q_2}{(q_1 + q_2)^{2/3}} \neq 0.$$

Thus we conclude that the centre of mass m_1 with charge q_1 and mass m_2 with charge q_2 lies at the equilibrium point $(L_1, 0. 0)$.

For $i = 1$, equation (12) gives

$$a_{11} = \pm\left(-\frac{cp}{\Omega^0_{xx}}\right)^{1/3} = -\,a_{21} \qquad (13)$$

Thus we find that equation (13) has generally two solutions From equation (11) has generally two solutions. From equation (11) we find

$$x_1 = L_k \pm \left\{\frac{cp}{\Omega^o_{xx}(q_1 + q_2)^2}\right\}^{1/3} q_2$$

and

$$x_2 = L_k \pm \left\{\frac{cp}{\Omega^o_{xx}(q_1 + q_2)^2}\right\}^{1/3} (-\,q_1)$$

or in terms of μ_1 and μ_2, we have,

$$x_1 = L_k \pm \left\{\frac{c^2p^2}{\Omega^0_{xx}(\mu_1 + \mu_2)^2}\right\}^{1/3} \mu_2 \qquad (14)$$

and $$x_2 = L_k \pm \left\{\frac{c^2p^2}{\Omega^{o}_{xx}(\mu_1+\mu_2)^2}\right\}^{2/2}(-\mu_1) \qquad \text{... (15)}$$

Thus equations (14) and (15) give approximate locations of the two collinear equilibrium points for each charged particle. Further we may observe by (x, μ) graph fig. (1) that the abscissa of that collinear equilibrium points increases with μ and the rate of increase is almost the same. For each equilibrium points L_1, L_2, L_3 determined by Goudas we get six equilibrium points on the x-axis. The abscissa of these equilibrium points on the x-axis. The abscissa of these equilibrium points x_{ijk} are shown in tables (1-3) for some values of μ and λ where $i = 1, 2, 3$ (collinear equilibrium solution corresponding to Goudas et al 1985), $j = 1, 2$, (No. of solution), $k = 1, 2$ (No. of the charge).

(b) Computation of the Off-Axis Equilibrium Points :

Equation (8) is directly satisfied by $z_i = 0$ $(i = 1, 2)$ and therefore off-axis solutions are given by equations (6) and (7) when $z_i = 0$. Thus off- axis equlibrium points are given by

$$\Omega_{x_i}(x_i, y_i) + cpq_{3-i}\frac{x_i - x_{3-i}}{|\bar{r}|^3} = 0 = \frac{\partial T}{\partial x_i} \qquad \text{... (16)}$$

$$\Omega_{y_i}(x_i, y_i) + cpq_{3-i}\frac{y_i - y_{3-i}}{|\bar{r}|^3} = 0 = \frac{\partial T}{\partial x_i} \qquad \text{... (17)}$$

where

$$\Omega(x_i, y_i) = \tfrac{1}{2}(x_i^2 + y_i^2) + p(x_iA_{y_i} - y_iA_{x_i})$$

$$\bar{r}^2 = (x_1 - x_2)^2 + (y_1 + y_2)^2.$$

The solutions of equations (16) and (17) may be expressed as a power series in terms of the small parameter defined in equation (10) i.e. by

$$x_i = L_{x_k} + a_{i1}\,\varepsilon_{3-i} \qquad \text{... (17(a))}$$

$$y_i = L_{y_k} + b_{i1}\,\varepsilon_{3-i}\,,\ i = 1, 2,\ k = 4, 5, 6, 7 \qquad \text{... (17(b))}$$

where L_{x_k}, L_{y_k} represent the off-axis equilibrium points determined by Goudas et al (1985). It may be noted that L_{x_k} and L_{y_k} satisfy

$$\Omega_{x_i}(x_i, y_i) = 0 \text{ and } \Omega_{y_i}(x_i, y_i) = 0.$$

Expanding $\Omega_{x_i}(x_i, y_i)$ and $\Omega_{y_i}(x_i, y_i)$ in the neighbourhood of (L_{x_k}, L_{y_k}) we get

$$\Omega_{x_i} = (\Omega_{x_i})^o + a_{i1}\,\varepsilon_{3-i}(\Omega_{x_i x_i})^o + b_{i1}\varepsilon_{3-i}\,(\Omega_{x_i y_i})^o \quad \text{... (18)}$$

$$\Omega_{y_i} = (\Omega_{y_i})^o + a_{i1}\,\varepsilon_{3-i}\,(\Omega_{y_i x_i})^o + b_{i1}\,\varepsilon_{3-i}\,(\Omega_{y_i y_i})^o \quad \text{... (19)}$$

where the super script "o" denotes the evaluation of the partial derivatives at (L_{x_k}, L_{y_k}).

As $(\Omega_{x_i})^o = 0 = (\Omega_{y_i})^o$, equations (16) and (17) with the use of (18) and (19) can be written as

$$a_{i1}\varepsilon_{3-i}\,(\Omega_{x_i x_i})^o + b_{i1}\,\varepsilon_{3-i}\,(\Omega_{x_i y_i})^o + cpq_{3-i}\,\frac{x_i - x_{3-i}}{|\bar{r}|^3} = 0 \quad \text{... (20)}$$

$$a_{i1}\,\varepsilon_{3-i}\,(\Omega_{y_i x_i})^o + b_{i1}\,\varepsilon_{3-i}\,(\Omega_{y_i y_i})^o + cpq_{3-i}\,\frac{y_i - y_{3-i}}{|\bar{r}|^3} = 0 \quad \text{... (21)}$$

For $i = 1, 2$, equation (20) may be written as

$$\left.\begin{aligned} a_{11}\,\varepsilon_2\,\Omega^o_{xx} + b_{11}\,\varepsilon_2\,\Omega^o_{xy} + cpq_2\,\frac{x_1 - x_2}{|\bar{r}|^3} &= 0 \\ a_{21}\,\varepsilon_1\,\Omega^o_{xx} + b_{21}\,\varepsilon_1\,\Omega^o_{xy} + cpq_1\,\frac{x_2 - x_1}{|\bar{r}|^3} &= 0 \end{aligned}\right\} \quad \text{... (22)}$$

and equation (21) may be written as

$$\left.\begin{aligned} a_{11}\,\varepsilon_2\,\Omega^o_{xy} + b_{11}\,\varepsilon_2\,\Omega^o_{yy} + cpq_2\,\frac{y_1 - y_2}{|\bar{r}|^3} &= 0 \\ a_{21}\,\varepsilon_1\,\Omega^o_{xy} + b_{21}\,\varepsilon_1\,\Omega^o_{yy} + cpq_1\,\frac{y_2 - y_1}{|\bar{r}|^3} &= 0 \end{aligned}\right\} \quad \text{... (23)}$$

where

$$\Omega^o_{x_1 x_1} = \Omega^o_{x_2 x_2} = \Omega^o_{xx}$$

$$\Omega^o_{x_1 x_1} = \Omega^o_{x_2 y_2} = \Omega^o_{xy}$$

$$\Omega^o_{y_1 y_1} = \Omega^o_{y_2 y_2} = \Omega^o yy.$$

From equation (22) and (23) we get

$$a_{11}\,\Omega^o_{xx} + b_{11}\Omega^o_{xy} + a_{21}\,\Omega^o_{xx} + b_{21}\,\Omega^o_{xy} = 0 \quad \text{... (24)}$$

$$a_{11}\,\Omega^o_{xy} + b_{11}\,\Omega^o_{yy} + a_{21}\Omega^o_{xy} + b_{21}\Omega^o_{yy} = 0 \quad \text{... (25)}$$

From equations (24) and (25) we get

$a_{21} = -a_{11}$ and $b_{21} = -b_{11}$

Since $$\Omega^o_{xx} \cdot \Omega^o_{yy} - \left(\Omega^o_{xy}\right)^2 \neq 0.$$

Therefore to $0(\varepsilon) \equiv \max(\varepsilon_1, \varepsilon_2)$, we conclude that the centre of mass of m_1 with charge q_1 and of mass m_2 with charge q_2 lies at the off-axis equilibrium points calculated by Goudas et al (1985).

Now by the use of $a_{21} = -a_{11}$ and $b_{21} = -b_{11}$ in equations (22) and (23) we get

$$a_{11}\,\Omega^o_{xx} + b_{11}\,\Omega^o_{xy} + cp\,\frac{a_{11}}{|a_{11}^2 + b_{11}^2|^{3/2}} = 0 \qquad \dots (26)$$

$$a_{11}\,\Omega^o_{xy} + b_{11}\,\Omega^o_{yy} + cp\,\frac{b_{11}}{|a_{11}^2 + b_{11}^2|^{3/2}} = 0 \qquad \dots (27)$$

From equations (26) and (27) we get

$$a_{11}^2 + a_{11}b_{11}\left(\frac{\Omega^o_{yy} - \Omega^o_{xx}}{\Omega^o_{xy}}\right) - b_{11}^2 = 0$$

i.e. $$(a_{11} + a_1 b_{11})\,(a_{11} + a_2 b_{11}) = 0 \qquad \dots (28)$$

where

$$\alpha_1 = \beta + \gamma, \qquad \alpha_2 = \beta - \gamma$$

$$\beta = \frac{\Omega^o_{yy} - \Omega^0_{xx}}{2\Omega^o_{xy}} \text{ and } \gamma = \frac{\{(\Omega^o_{yy} - \Omega^o_{xx})^2 + 4\,(\Omega^o_{xy})^2\}^{1/2}}{2\Omega^o_{xy}}.$$

Thus from equation (28) we get two cases as

case (*i*) $\quad a_{11} = -\alpha_1 b_{11}$

case (*ii*) $\quad a_{11} = -\alpha_2 b_{11}$.

By using case (i) in equation (27) we get

$$\alpha_1\,\Omega^o_{xy} - \Omega^o_{yy} = \frac{cp}{|b_{11}|^3\,(1 + \alpha_1^2)^{3/2}} \qquad \dots (29)$$

and by case (ii) from equation (26) we get

$$\frac{\Omega^o_{xy}}{\alpha_1} - \Omega^o_{xx} = \frac{cp}{|a_{11}|^3(1 + 1/\alpha_1^2)^{3/2}} \qquad \dots (30)$$

From equations (29) and (30) we get

$$b_{11} = \pm \left\{ \frac{cp}{\alpha_i \Omega^o_{xy} - \Omega^o_{yy}} \right\}^{1/3} \left(\frac{1}{1 + \alpha_i^2} \right)^{1/2}$$

and

$$a_{11} = \pm \left\{ \frac{cp}{\dfrac{\Omega_{xy}}{\alpha_i} - \Omega^o_{xx}} \right\}^{1/3} \left(\frac{1}{1 + \dfrac{1}{\alpha_i^2}} \right)^{1/2}$$

where $\alpha_i = \beta + (-1)^{i+1} \gamma$, $i = 1, 2$.

Therefore equations 17(a) and 17(b) may be written as

$$x_i = L_{x_k} + (-1)^i \alpha_j \mu_{3-i} \left\{ \frac{c^2 p^2}{(\mu_1 + \mu_2)^2 (\alpha_j \Omega^o_{xy} - \Omega^o_{yy})} \right\}^{1/3} \left(\frac{1}{1 + \alpha_j^2} \right)^{1/2} \quad \ldots (31)$$

$$y_i = L_{y_k} \pm \frac{(-1)^{i+1} \mu_{3-i}}{(1 + \alpha_j^2)^{1/2}} \left\{ \frac{c^2 p^2}{(\mu_1 + \mu_2)^2 (\alpha_j \Omega^o_{xy} - \Omega^o_{yy})} \right\}^{1/3} \quad \ldots (32)$$

For $i = 1, 2$, $k = 4, 5, 6, 7$ and $j = 1, 2$

or

$$x_i = L_{x_k} \pm \frac{(-1)^{i+1} \mu_{3-i}}{(1 + \alpha_j^2)^{1/2}} \left\{ \frac{c^2 p^2}{(\mu_1 + \mu_2)^2 \left(\dfrac{\Omega^o_{xy}}{\alpha_j} - \Omega^o_{xx} \right)} \right\}^{1/3} \quad \ldots (33)$$

and

$$y_i = L_{y_k} \pm \frac{(-1)^{i+1} \mu_{3-i}}{(1 + \alpha_j^2)^{1/2}} \left\{ \frac{c^2 p^2}{(\mu_1 + \mu_2)^2 \left(\dfrac{\Omega^o_{xy}}{\alpha_j} - \Omega^o_{xx} \right)} \right\}^{1/3} \quad \ldots (34)$$

where $i = 1, 2$, $j = 1, 2$, $k = 4, 5, 6, 7$.

Thus equations (31) to (34) give approximate positions of the off-axis equilibrium points. We find that there exists sixteen off-axis equilibrium points in the neighbourhood of four off-axis equilibrium points determined by Goudas et al (1985). We observe that four off-axis equilibrium points exists in the neighbourhood of each off-axis equilibrium point determined by Goudas et al (1985). Further it may be seen that two classes of off-axis equilibrium solutions exist where one class of solutions lie approximately along the line passing through the off-axis equilibrium points calculated

TABLE - 1

S.No.	μ	λ	L_1 (Goudas)	x_{111}	x_{121}	x_{112}	x_{122}
	About L_1, collinear equilibrium points of charge (q_1)			About L_1, collinear equilibrium pts. of charge (q_2)			
1.	0.05	1.0	1.069987	1.498087	0.641887	0.641887	1.498087
2.	0.10	1.0	1.137093	1.559300	0.714886	0.714886	1.559300
3.	0.15	1.0	1.202000	1.618962	0.785038	0.785038	1.618962
4.	0.20	1.0	1.265171	1.677381	0.852961	0.852961	1.677381
5.	0.25	1.0	1.326935	1.734784	0.919086	0.919086	1.734784

TABLE - 2

S.No.	μ	λ	L_2 (Goudas)	x_{211}	x_{221}	x_{212}	x_{222}
	About L_2, collinear equilibrium points of charge (q_1)			About L_2, collinear equilibrium pts. of charge (q_2)			
1.	0.05	1.0	− 0.048734	− 0.009384	− 0.088084	− 0.088084	− 0.009384
2.	0.10	1.0	− 0.088261	− 0.013759	− 0.162763	− 0.162763	− 0.013759
3.	0.15	1.0	− 0.110178	− 0.008128	− 0.212228	[illegible]	− 0.008128
4.	0.20	1.0	− 0.114831	0.007835	− 0.237497	− 0.237497	0.007835
5.	0.25	1.0	− 0.107397	0.031517	− 0.246311	− 0.246311	0.031517

TABLE - 3

About - L_3, collinear equilibrium points of charges

	About L_3, collinear equilibrium positions of charge (q_1)				About L_3, collinear equilibrium positions of charge (q_2)		
S.No.	μ	λ	L_3 (Goudas)	x_{311}	x_{321}	x_{312}	x_{322}
1.	0.05	1.0	− 2.119889	− 1.204441	− 3.035337	− 3.035337	− 1.204441
2.	0.10	1.0	− 2.065715	− 1.238192	− 2.893238	− 2.893238	− 1.238192
3.	0.15	1.0	− 2.011305	− 1.246454	− 2.776156	− 2.2776156	− 1.246454
4.	0.20	1.0	− 1.956637	− 1.239852	− 2.673422	− 2.673422	− 1.239852
5.	0.25	1.0	− 1.901685	− 1.223586	− 2.579784	− 2.579784	− 1.223586

TABLE - 4

About L_4 equilibrium position of charge q_1

S.No.	μ	λ	L_{x4}	L_{y4}	x_{411}	y_{411}	x_{421}	y_{421}
1.	0.0	1.0	− 0.2949	0.7487	− 0.02800872	0.475584218	− 0.56179128	1.021815782
2.	0.01	1.5	0.4177	0.7384	0.23463015	0.314040634	− 0.39923015	1.162759366
3.	0.04	1.0	− 0.225846	0.787876	0.054655069	0.516592263	− 0.506347069	1.059159737
4.	0.05	1.5	− 0.3	0.8276	− 0.019745836	0.512529575	− 0.580254164	1.142671125
5.	0.05	1.0	− 0.2053	0.8009	0.062957356	0.501649104	− 0.473557356	1.10015[illegible]96

TABLE - 5

About - L_4, collinear equilibrium points of charges q_1

S.No.	μ	λ	L_{x4}	L_{y4}	x_{431}	y_{431}	x_{441}	y_{441}
1.	0.0	1.0	– 0.2949	0.7487	0.249329621	1.280526243	– 0.839129621	0.216873757
2.	0.01	0.5	0.4177	0.7384	0.236009636	0.388629636	0.599390356	1.088170364
3.	0.04	1.0	– 0.225846	0.787876	– 1.018254844	– 0.0314563	0.566562844	1.6072083
4.	0.05	0.5	– 0.3	0.8276	– 0.861892806	0.327798094	0.261892806	1.327401906
5.	0.05	1.0	– 0.2053	0.8009	– 1.33975942	– 0.216062986	0.92915942	1.817862986

TABLE - 6

About L_4 equilibrium position of charge 2

S.No.	μ	λ	L_{x4}	L_{y4}	x_{412}	y_{412}	x_{422}	y_{422}
1.	0.0	1.0	– 0.2949	0.7487	– 0.56179128	1.021815782	– 0.2800872	0.475584218
2.	0.01	1.5	0.4177	0.7384	– 0.39923015	1.162759366	1.23453015	0.314040634
3.	0.04	1.0	– 0.225846	0.787876	– 0.506347069	1.059159737	0.054655069	0.516592263
4.	0.05	1.5	– 0.3	0.8276	– 0.580254164	1.142670425	– 0.019745836	0.512529575
5.	0.05	1.0	– 0.2053	0.8009	– 0.473557356	1.100150896	0.062957356	0.501649104

TABLE - 7

About - L_4 equilibrium position of charge q_2

S.No.	μ	λ	L_{x4}	L_{y4}	x_{432}	y_{432}	x_{442}	y_{442}
1.	0.0	1.0	– 0.2949	0.7487	– 0.839129621	0.216873757	0.249329621	1.280526243
2.	0.01	0.5	0.4177	0.7384	0.599390356	1.088170364	0.236009644	0.388629636
3.	0.04	1.0	– 0.225846	0.787876	0.566562844	1.6072083	– 1.018254844	– 0.0314563
4.	0.05	0.5	– 0.3	0.8276	0.261892806	1.327401906	– 0.861892806	0.327798094
5.	0.05	1.0	– 0.2053	0.8009	0.92915942	1.817862986	– 1.33975942	– 0.216062986

TABLE - 8

About L_5 equilibrium position of charge q_1

S.No.	μ	λ	L_{x5}	L_{y5}	x_{511}	y_{511}	x_{521}	y_{521}
1.	0.0	1.0	– 0.2949	– 0.7487	– 0.56179128	– 1.021815782	– 0.02800872	– 0.475584218
2.	0.01	0.5	0.4177	– 0.7384	– 0.39923015	– 1.162759366	1.23463015	– 0.314040634
3.	0.04	1.0	– 0.225846	– 0.787876	– 0.506347069	– 1.059159737	0.054655069	– 0.0516592263
4.	0.05	0.5	– 0.3	– 0.8276	– 0.580254164	– 1.142670425	– 0.019745836	– 0.512529575
5.	0.05	1.0	– 0.2053	– 0.8009	– 0.473557356	– 1.100150896	0.062957355	– 0.501649104

TABLE - 9

About - L_5 equilibrium position of charge 1

S.No.	μ	λ	L_{x5}	L_{y5}	x_{531}	y_{531}	x_{541}	y_{541}
1.	0.0	1.0	– 0.2949	– 0.7487	– 0.839129621	– 0.216873757	0.249329621	– 1.280526243
2.	0.01	0.5	0.4177	– 0.7384	0.599390356	– 1.088100364	0.236009644	– 0.388629636
3.	0.04	1.0	– 0.225846	– 0.787876	0.566562844	– 1.6072083	– 1.018254844	– 0.0314563
4.	0.05	0.5	– 0.3	– 0.8276	0.261892806	– 1.327401906	– 0.861892806	– 0.327798094
5.	0.05	1.5	– 0.2053	– 0.8009	0.92915942	– 1.817862986	– 1.33975942	0.216062986

TABLE - 10

About L_5 equilibrium position of charge (2)

S.No.	μ	λ	L_{x5}	L_{y5}	x_{512}	y_{512}	x_{522}	y_{522}
1.	0.0	1.0	– 0.2949	– 0.7487	– 0.02800872	– 0.475584218	– 0.56179128	– 1.021815782
2.	0.01	1.5	0.4177	– 0.7384	1.23463015	– 0.314040634	– 0.39923015	– 1.162759366
3.	0.04	1.0	– 0.225846	– 0.787876	0.054655069	– 0.516592263	– 0.506347069	– 1.059159737
4.	0.05	1.5	– 0.3	– 0.8276	– 0.019745836	– 0.512529575	– 0.580254164	– 1.142670425
5.	0.05	1.0	– 0.2053	– 0.8009	0.019745836	– 0.501649104	– 0.473557356	– 1.100150896

TABLE - 11

About - L_5 equilibrium position of charge (2)

S.No.	μ	λ	L_{x5}	L_{y5}	x_{532}	y_{532}	x_{542}	y_{542}
1.	0.0	1.0	– 0.2949	– 0.7487	0.249329621	– 1.280526243	– 0.829129621	– 0.216873757
2.	0.01	0.5	0.4177	– 0.7384	0.236009644	– 0.388629636	0.599390356	– 1.088170364
3.	0.04	1.0	– 0.225846	– 0.787876	– 1.018254844	0.0314563	0.566562844	– 1.6072083
4.	0.05	0.5	– 0.3	– 0.8276	– 0.861892806	– 0.327798094	0.261892806	– 1.327401906
5.	0.05	1.0	– 0.2053	– 0.8009	– 1.33975942	0.216062986	0.92915942	– 1.817862086

TABLE - 12

About L_6 equilibrium position of charge 1

S.No.	μ	λ	L_{x6}	L_{y6}	x_{611}	y_{611}	x_{621}	y_{621}
1.	0.04	1.0	0.801166	0.660356	1.258396117	0.16102025	– 0.343935883	1.15969175
2.	0.05	0.5	– 0.0876	0.9459	0.040941151	0.531626678	– 0.216141151	1.360173322
3	0.05	1.0	0.5564	0.8603	1.150467044	0.553094369	– 0.037667044	1.167505631
4.	0.12	2.5	0.0555	0.8065	0.283528573	0.492492686	– 0.172528573	1.120507314
5.	0.15	3.0	0.855	0.7799	1.375743769	0.37809241	– 0.334256231	1.18170759

TABLE - 13

About - L_6 equilibrium position of charge (1)

S.No.	μ	λ	L_{x6}	L_{y6}	x_{631}	y_{631}	x_{641}	y_{64}
1.	0.04	1.0	0.801166	0.660356	0.483092287	0.369103308	1.119239713	0.951608692
2.	0.05	0.5	−0.0876	0.9459	−0.718977	0.749995698	0.543777	1.141804302
3.	0.05	1.0	0.5564	0.8603	0.357655435	0.475972422	0.755144565	1.244627578
4.	0.12	2.5	0.0555	0.8065	0.775019732	1.329007121	−0.664019732	0.283992879
5.	0.15	3.0	0.855	0.7799	0.589442551	0.435736798	1.120557449	1.124063202

TABLE - 14

About L_6 equilibrium position of charge (2)

S.No.	μ	λ	L_{x6}	L_{y6}	x_{612}	y_{612}	x_{622}	y_{5}[illegible]
1.	0.04	1.0	0.801166	0.660356	0.343935883	1.1569175	1.258396117	0.16102025
2.	0.05	0.5	−0.0876	0.9459	−0.216141151	1.360173322	0.040941151	0.531626678
3.	0.05	1.0	0.5564	0.8603	−0.037667044	1.167505631	1.150467044	0.553094369
4.	0.12	2.5	0.05564	0.8065	−0.172528573	1.120507314	0.283528573	0.492492686
5.	0.15	3.0	0.855	0.7799	0.334256231	1.18170759	1.375743769	0.37809241

TABLE - 15

About - L_6 equilibrium position of charge (2)

S.No.	μ	λ	L_{x6}	L_{y6}	x_{632}	y_{632}	x_{642}	y_{642}
1.	0.04	1.0	0.801166	0.660356	1.119239713	0.951608692	0.48309287	0.369103308
2.	0.05	0.5	– 0.0876	0.9459	0.543777	1.141804302	– 0.718977	0.749995698
3.	0.05	1.0	0.5564	0.8603	0.755144565	1.244627578	0.357655435	0.475972422
4.	0.12	2.5	0.0555	0.8065	– 0.664019732	0.283992879	0.775019732	1.329007121
5.	0.15	3.0	0.855	0.7799	1.120557449	1.124063202	0.5894422551	0.435736798

TABLE - 16

About L_7 equilibrium position of charge 1

S.No.	μ	λ	L_{x7}	L_{y7}	x_{711}	y_{711}	x_{721}	y_{721}
1.	0.04	1.0	0.801166	– 0.660356	0.3443935883	– 1.15969175	1.258396117	– 0.16102025
2.	0.05	0.5	– 0.0876	– 0.9459	– 0.216141151	– 1.360173322	0.040941151	– 0.531626678
3.	0.05	1.0	0.5564	– 0.8603	– 0.037667044	– 1.167505631	1.150467044	– 0.553094369
4.	0.12	2.5	0.0555	– 0.8065	– 0.172528573	– 1.120507314	0.283528573	– 0.492492686
5.	0.15	3.0	0.855	– 0.7799	0.334225621	– 1.18170759	1.375743769	– 0.37809241

TABLE - 17

About L_7 equilibrium position of charge (1)

S.No.	μ	λ	L_{x7}	L_{y7}	x_{731}	y_{731}	x_{741}	y_{741}
1.	0.04	1.0	0.801166	– 0.660356	1.119239713	– 0.951608692	0.483092287	– 0.369103308
2.	0.05	0.5	– 0.0876	– 0.9459	0.543777	– 1.141804302	– 0.718977	– 0.749995698
3.	0.05	1.0	0.5564	– 0.8603	0.755144565	– 1.244627578	0.357655435	– 0.475972422
4.	0.12	2.5	0.0555	– 0.8065	– 0.664019732	– 0.283992879	0.775019732	– 1.329007121
5.	0.15	3.0	0.855	– 0.7799	1.120557449	– 1.124063202	0.589442551	– 0.435736798

TABLE - 18

About L_7 equilibrium position of charge (2)

S.No.	μ	λ	L_{x7}	L_{y7}	x_{712}	y_{712}	x_{722}	y_{722}
1.	0.04	1.0	0.801166	– 0.660356	1.258396117	– 0.16102025	0.343935883	– 1.15969175
2.	0.05	0.5	– 0.0876	– 0.9459	0.040941151	– 0.531626678	– 0.216141151	– 1.360173322
3.	0.05	1.0	0.5564	– 0.8603	1.150467044	– 0.553094369	– 0.037667044	– 1.167505631
4.	0.12	2.5	0.0555	– 0.8065	0.283528573	– 0.492492686	– 0.172528573	– 1.120507314
5.	0.15	3.0	0.855	– 0.7799	1.375743769	– 0.37809241	0.334256231	– 1.18170759

TABLE - 19

About L_7, equilibrium position of charge (2)

S.No.	μ	λ	L_{x7}	L_{y7}	x_{732}	y_{732}	x_{742}	y_{742}
1.	0.04	1.0	0.801166	−0.660356	0.483092287	−0.369103308	1.119239713	−0.951608692
2.	0.05	0.5	−0.0876	−0.9459	−0.718977	−0.749995698	0.543777	−1.141804302
3.	0.05	1.0	0.5564	−0.8603	0.357655435	−0.475972422	0.755144565	−1.244627578
4.	0.12	2.5	0.0555	−0.8065	0.775019732	−1.329007121	−0.664019732	−0.283992879
5.	0.15	3.0	0.855	−0.7799	0.589442551	−0.435736798	1.120557449	−1.124063202

by Goudas et al and the other class of solutions lie along perpendicular line to the first class. The abscissas and the ordinates of these equilibrium points (x_{ijk}, y_{ijk}) are shown in tables 4 to 19 for some values of μ and λ where

$i = 4, 5, 6, 7$ (off-axis equilibrium solution corresponding to Goudas et al (1985))

$j = 1, 2, 3, 4$ (Number of the solution)

$k = 1, 2$ (Number of tne charge)

Tables (1 to 19) are given in the last.

REFERENCES

[1] A.G. Mavraganis, *Ast. Phy. and Sp. Sci.* **32** (1975), 115.

[2] A.G. Mavraganis, *Ast. Phy. and Sp. Sci.* **54** (1978), 305-313.

[3] A.G. Mavraganis, *Ast. Phy. and Sp. Sci.* **62** (1979), 251-253.

[4] A.G. Mavraganis, *Cel. Mech.* **30** (1983), 219-224.

[5] C.L. Goudas et al, *D. Raidel pub. Comp.*, (1985), 344-364.

[6] C.L. Goudas and E.G. Petsagouraki's, *Cel. Mech.*, **37** No. 2 (1985), 127-148.