Jñānābha, Vol.26,1996

STRAIGHT CAPILLARIC MODEL FOR MICRO-CIRCULATION

By

D.K. Wagh and S.D. Wagh
Department of Applied Mathematics
Shri G.S. Institute of Technology and Science
Indore, M.P., India

(*Received : November 23, 1994*)

1. INTRODUCTION

The phenomenological description of flow through porous media could be made possible by introducing the concept of permeability. However, the real understanding of the porous flow phenomena requires to link the concept of permeability with the average pore size or with the pore size distribution. This lead to the development of models for porous media. The simplest model developed for this purpose was the straight capillaric model proposed by Scheidegger [1]. In this the porous medium is identified by a bundle of straight and parallel capillaries of uniform average diameter. It follows therefore that there is a close resemblance between the flow through porous media and flow through the bundle of capillaries. Since micro- circulation is a phenomenon in which the flow through the complex geometry of different flow channels similar to porous flow it seems possible to consider microcirculation phenomenon as porous flow and investigate it on the basis of capillaric model. Taking this approach it is found that for a given pressure gradient the seepage velocity of microcirculation is proportional to η and to the fourth power of δ where η is the number of capillaries per unit area of cross-section of the conceptual straight capillaric model representing the actual system of capillaries and δ is the average diameter of the capillaries of the conceptual model.

2. STATEMENT OF THE PROBLEM

We consider the phenomenon of microcirculation through a bundle of capillaries. The bundle of capillaries is assumed to be horizontal and flow is supposed to be uni-directional.

3. STRAIGHT CAPILLARIC MODEL

We now replace the actual system or capillaries through which the microcirculation takes place by a conceptual model which consists of straight and parallel capillaries of uniform average diameter δ. The flow through a single capillary is given by the well known Hagen- Poiseville law

$$Q = -\frac{\pi\delta^4}{128\mu}\frac{dp}{dx} \qquad ...(3.1)$$

where μ is the viscosity and $\frac{dp}{dx}$ is the pressure gradient along the capillary. If there are η such capillaries per unit area of cross-section taken perpendicular to the parallel capillaries, then the flow per unit area will be

$$q = -\frac{\eta\pi\delta^4}{128\mu}\frac{dp}{dx} \qquad ...(3.2)$$

It is well known that the flow through porous media is governed by Darcy's law

$$q = -\frac{k}{\mu}\frac{dp}{dx} \qquad ...(3.3)$$

where k is a constant depending on the properties of the porous medium and μ is called permeability of the medium.

A comprison of equations (3.2) and (3.3) shows that

$$k = \frac{\eta\pi\delta^4}{128} \qquad ...(3.4)$$

We therefore conclude that the actual system of capillaries can be considered as a porous medium whose permeability is given by (3.4).

4. BASIC EQUATIONS

The solution of a flow problem in a porous medium involves determination of three unknowns namely the seepage velocity v, the pressure p and the density ρ. To determine these three unknowns we have three equations - the equation of continuity, the Darcy's equation and the equation of state (the relation between p and ρ). The solution of these equations satisfying prescribed boundary conditions gives the solution of the flow problem.

The equation of continuity in steady state for incompressible fluid may be written as

$$div\ \vec{v} = 0 \qquad ...(4.1)$$

Darcy's law is given by

$$\vec{v} = -\frac{k}{\mu}\ \text{grad}\ p, \qquad ...(4.2)$$

where $\vec{v}$ is the seepage velocity, p is the pressure, μ is viscosity of the fluid and k is permeability of the medium. The equation of state for an incompressible fluid is

$$\rho = \rho_0\ \text{(constant)} \qquad ...(4.3)$$

Substituting (4.2) in (4.1), we get

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0. \qquad ...(4.4)$$

Thus the pressure p satisfies the Laplace equation and the solution of the problem consists in determing the solution of equation (4.4) satisfying boundary conditions of the problem.

5. SOLUTION OF THE PROBLEM

For the uni-directional flow through the bundle of capillaries considered in the present paper the equation (4.4) reduces to

$$\frac{\partial^2 p}{dx^2} = 0 \qquad ...(5.1)$$

which gives

$$p = c_1 x + c_2 \qquad ...(5.2)$$

If the capillary bundle is of length l and p_1 and p_2 are pressures at the entry and exit respectively, then

$$p_1 = c_2,\ p_2 = c_1 l + c_2 \text{ or } c_1 = \frac{p_2 - p_1}{l}.$$

Hence

$$p = \frac{p_2 - p_1}{l} x + p_1 \qquad ...(5.3)$$

Substituting this in equation (4.2), we get the seepage velocity

$$v = -\frac{k}{\mu}\left(\frac{p_2 - p_1}{l}\right) \qquad ...(5.4)$$

Now the permeability for the bundle of capillaries is given by equation (3.4). Substituting this in (5.4), we get

$$v = -\frac{\eta\pi\delta^4}{128\mu}\left(\frac{p_2 - p_1}{p}\right) \qquad ...(5.5)$$

Thus, for a given pressure gradient the seepage velocity is proportional to η, and to the fourth power of δ where η is the number of capillaries per unit area of cross-section of the conceptual straight capillaric model representing the actual system of capillaries and δ is the average diameter of the capillaries of the conceptual model.

REFERENCE

[1] A.E. Scheidegger, *The Physics of Flow Through Porous Media,* University of Toronto Press, (1963), 115-118.