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CONCIRCULAR CURVATURE TENSOR AND RELATIVISTIC GRAVITATION

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ABSTRACT

In this paper the influence of the vanishing of the concircular curvature tensor and its divergence on a relativistic gravitational field is explored.

1. INTRODUCTION A C_2 concircular curvature tensor in an n-dimensional space V_n denoted by C_2 and defined by

$$C_2(X, Y, Z, T) \underline{\underline{\text{def}}} K(X, Y, Z, t) - \frac{r}{n(n-1)} g(Y, Z)s(X, t) - g(X, Z)s(Y, t) \quad ...(1.1)$$

where S is Ricci tensor of (0,2) type and studied its Retatinistic influence. It is shown that the vanishing of concircular curvature tensor on a relativistic gravitational field implies an Einstein space- time. In this last section it is found that the vanishing of the divergence of C_2 curvature tensor of a relatinstic gravitational field gives the equation of State for disorder radiation (See also Mishra [2]).

1. If C_2 curvature tensor is flat then

$$C_2(X, Y, Z, t) = 0 \quad ...(1.2)$$

from (1.1) and (1.2), we have

$$K(X, Y, Z, t) = \frac{r}{n(n-1)} g(Y, Z)S(X, t) - g(X, Z)S(X, t) \quad ...(1.3)$$

contracting (1.3), we have

$$S(Y, Z) = \frac{r}{n} g(Y, Z) \quad ...(1.4)$$

where r is the scalar curvature.

Thus, if C_2 curvature tensor is flat, then the manifold is an Einstein.

From (1.1), it is clear that if the Ricci tensor S and curvature tensor C_2 vanish together than $K(X, Y, Z, t) = 0$.

Hence from (1.1) and (1.4), we can say that if a part of gravitational space which is free from matter has a constant curvature (Narlikar [1]) then the latter is zero. Again, if $S(X, Y) \neq 0$, but

$$S(X, Y) - \frac{1}{2} g(X, Y)r = KT(X, Y) \qquad ...(1.5)$$

where T is the stress energy tensor defined by (Narlikar [1]).

$$T(X, Y) = (+p)A(X)A(Y) - pg(X, Y) \qquad ...(1.6)$$

where g is the energy density function, p the pressure function and A is 1-form defined by $A(X) = g(X, P)$, P is the unit vector field and K is the coupling constant and $C_2(X, Y, Z, t) = 0$. We have from (1.4) for a Riemannian manifold

$$S(X, Y) = (r/4)\, g(X, Y)$$

gives the Einstein space. Hence in a Riemannian space-time of constant curvature is an Einstein space which has application in the inflationary scenario in early universe.

2. Now let

$$\text{div } C_2 = 0 \qquad ...(2.1)$$

then using Bianchi identiels from (1.1) and (2.1), we get

$$dr(X) = 0 \qquad ...(2.2)$$

It is known (Narlikar [1]), that

$$T^* = g^{ij} T_{ij} = \rho - 3p \qquad ...(2.3)$$

and

$$r = KT^* \qquad ...(2.4)$$

Differentiating covariantly the equation (2.4) and using (2.2), we have

$$T^* = \text{const. i.e. } \rho = 3p + \text{constant} \qquad ...(2.5)$$

If the constant vanishes then $\rho = 3p$ is the equation of state for disorder radiation.

REFERENCES

[1] J.V. Narlikar, *Lectures on General Relativity and Cosmology*, Macmillian India ltd., New Delhi, 1978.

[2] R.S. Mishra, *Structure on a Differentiable Manifold and Their Application*, Chandrama Prakashan, Allahabad, 1984.