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UNSTEADY MHD FLOW OVER A MOVING WALL THROUGH POROUS MEDIA

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ABSTRACT

In this paper we study the flows over moving walls which are of interests in a number of technical applications, specially in metallurgy and chemical process, industries such flows belong to separate class of problems of boundary layer theory which is distinct from those over stationary bodies.

1. **INTRODUCTION :** No doubtedly we are living in the world of a high speed computer characterized by vast explosion of knowledge in science and technology. The development of numerical methods approximating differential equation having the non-linear terms has strongly influenced and motivated researches in the area of fluid dynamics. Subsequently several investigations considered the behaviour of boundary layer on moving surfaces under different situation. All these studies pertain to study flows. The unsteady flow over a moving wall with forced flow has been studied by Yang (1958).

Sakadis (1961) was probably first to study the flow over a moving boundary in a fluid at rest. When the free stream velocity varies inversely as a linear function of time. Also the unsteady flow over a moving wall in a fluid at rest has been studied recently by Surma Devi and Nath (1986).

The aim of present analysis is to study the unsteady laminar incompressible forced flow over a moving boundary with an applied magnetic field when the free stream velocity and the wall velocity vary arbitrarily with time. It has also been seen that the self similar solution is possible when the free stream velocity, wall velocity and square root of magnetic field vary inversely as a linear function of time. It may be noted that here the wall is not moving as a rigid boundary as considered by Sakiades (1961), but it is stretched.

The partial differential equations governing the self similar case and the ordinary differential equations governing the self similar case have been solved numerically using a finite difference scheme in combination with quasilinearization process. The result have been compared with those available in the literature.

2. FORMULATION OF THE PROBLEM : The present part denotes the study of unsteady *MHD* flow over a moving wall through porous media of permeability K. We consider a two dimensional an axisymmetric body moving with time dependent velocity U_w in a laminar incompressible fluid with free stream velocity V_e, which is also varies with time. The fluid is assumed to be electrically conducting and magnetic field B fixed relative to the fluid is applied in the direction perpendicular to the body. The magnetic Reynold's number is assumed to be small. Hence the induced magnetic field will be small compared to the applied magnetic field and can be neglected. Since we are interested in the stagnation point region, the viscous dissipation and Joule heating term are neglected as they are small in the neighbourhood of stagnation point. The Hell effect is also neglected.

3. GOVERNING EQUATIONS : We consider *MHD* flow of a viscous incompressible and electrically conducting fluid through porous medium bounded by an infinite horizontal plate. When the wall and free stream temperature are taken to be constant. By assuming that Prandtl's boundary layers assumptions are valid in the present case. The governing equations can be expressed as

$$\frac{\partial}{\partial x}(r' u) + \frac{\partial}{\partial y}(r' v) = 0 \qquad \text{...(A)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= \frac{\partial V_e}{\partial t} + V_e\frac{\partial V_e}{\partial k}\,\nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2 V_e}{\rho}x\left(1 - \frac{u}{V_e}\right) - \frac{\nu}{k}u \qquad \text{...(B)}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = Pr^{-1}\,T_{yy} \qquad \text{...(C)}$$

with suitable initial and boundary conditions.

These equations can also be written in the following from :

$$(r'u)_x + (r'v)_y = 0 \qquad \text{...(1)}$$

$$u_t + uu_x + vu_y = (u_e)_t + u_e(u_e)_x + \nu . u_{yy} + \frac{\sigma B^2 u_e}{\rho}\left(1 - \frac{u}{u_e}\right) - \frac{\nu}{K}u \quad ...(2)$$

$$T_t + uT_x + vT_y = Pr^{-1}\nu \, . \, T_{yy} \quad ...(3)$$

The initial and boundary conditions are given by initial condition :

$$\left.\begin{aligned} u\,(x, y, 0) &= u_t\,(x, y) \quad ...(1a) \\ v(x, y, 0) &= u_t\,(x, y) \quad ...(1b) \\ T(x, y, 0) &= T_t\,(x, y) \quad ...(1c) \end{aligned}\right| \quad ...(4)$$

Boundary condition :

$$\left.\begin{aligned} u(x, 0, t) &= u_w(x, t) \\ v(x, 0, t) &= 0 \\ T(x, 0, t) &= T_w \\ u(x, \infty, t) &= u_e\,(x, t) \\ T(x, \infty, t) &= T_\infty \end{aligned}\right| \quad ...(5)$$

Semi Similar Equations : In order to reduce the number of independent variable in eqn. (1) to (3), we apply the following transformations :

$$\left.\begin{aligned} \eta &= (1 + j)^{1/2}\,(a/\nu)^{1/2} y \\ u &= ax\phi(t^*)\,f'\,(\eta, t^*) \\ t^* &= at \\ v &= -\,(1 - j)^{1/2}\,(a - v)^{1/2}\,\phi(t^*)\,f'(\eta, t^*) \\ \nu_w &= a_1 x\phi(t^*) \end{aligned}\right| \quad ...(6)$$

$$\left.\begin{aligned} & u_e = a_1\,x\,\phi(t^*), \qquad \frac{T - T_\infty}{T_w - T_\infty} = g(\eta, t^*) \\ & r = x, \qquad M = \frac{Ha^2}{ReL} \\ & Ha^2 = \frac{\sigma B^2 L^2}{\mu}, \quad ReL = \frac{aL^2}{\nu}, \quad b = \frac{u_w}{u_e} \end{aligned}\right| \quad ...(7)$$

to eqn. (1) to (3) to find that equation (1) is satisfied identically and equation (2) to (3) reduce. Equation (1) and (2) can be written in the type of the form of eqn. (A) and (B), which have been shown in the beginning. Also from eqn. (6) and eqn. (7), we have

$$\frac{\partial u}{\partial t} = a^2\,x\phi' f' + a^2 x f''\phi$$

$$\frac{\partial u}{\partial x} = a\phi f'$$

$$\frac{\partial u}{\partial y} = ax\phi f''(1+j)^{1/2}\,(a/\nu)^{1/2}$$

$$\frac{\partial u_e}{\partial x} = a\phi$$

$$\frac{\partial u_e}{\partial t} = a^2\,x\,\phi'.$$

Also we have $Ha^2 = \frac{\sigma B^2 L^2}{\mu}$, $\sigma B^2 = \frac{Ha^2 \mu}{L^2}$ but $M = \frac{Ha^2}{ReL}$, $\sigma B^2 = \frac{MReL\mu}{L^2}$, $ReL = \frac{aL^2}{\nu}$.

Hence $\sigma B^2 = M\,\frac{aL^2}{\nu L^2} \times \mu = \frac{M\mu a}{\nu}$. But $\frac{\mu}{\nu} = \rho$. Therefore $\sigma B^2 = Ma\rho$.

Substituting these values in equation (2), we get the resulting equation as follows :

$$f''' + \phi f'' + (1+j)^{-1}[\phi(1-f'^2) + \phi^{-1}\phi_{\tau*}\,(1-f') - f'_{t*} + M(1-f') - \frac{\nu}{Ka} f'\,] = 0 \quad \text{...(7a)}$$

Therefore

$$f''' + \phi ff'' + (1+j)^{-1}\,[\phi(1-f'^2) + \phi^{-1}\phi_{\tau*}\,(1-f') - f'_{t*} + M(1-f') + \frac{\rho\nu Mf'}{K\sigma B^2}] = 0 \quad \text{...(7b)}$$

Thus

$$f''' + \phi f'' + (1+j)^{-1}\,[\phi(1-f'^2) + \phi^{-1}\,\phi_{\tau*}\,(1-f') - f'_{t*} + M\,(1-f' + \frac{\mu f' M}{K\sigma B^2})] = 0 \quad \text{...(8)}$$

and $Pr^{-1}g'' + \phi fg' - (1+j)^{-1}\,g_1^* = 0$ ...(9)

The boundary conditions are given by

$$\left.\begin{array}{l} f = 0,\ f' = b,\ g = 1 \text{ at } \eta = 0 \\ f' \to 1,\ g \to 0 \text{ as } \eta \to \infty \end{array}\right|\ \text{for } t^* \geq 0 \quad \text{...(10)}$$

The flow is initially assumed to be steady ($t^* = 0$) and then change to unsteady state ($t^* > 0$). Therefore, the initial conditions are given by the steady state equations obtained by putting

$$t^* = 0,\ \phi = 1,\ f'_{t*} = g_{t*} = \phi_{t*} = 0 \qquad ...(11)$$

In equation (8) and (9) and thus steady state equations can be written in the following type of the form :

$$f''' + ff'' + (1+j)^{-1}\,[(1-f'^2) + M(1 - f' - \frac{\mu M f'}{K\sigma B^2})] = 0 \qquad ...(12)$$

and

$$Pr^{-1} g'' + fg' = 0. \qquad ...(13)$$

It is to be noted that (8) and (9) containing two independent variables are known as semi-similar equations.

Here x and y are distances along and perpendicular to the surface respectively. u and v are the components of velocity along x and y directions, respectively. t, t^* are the dimensional and non-dimensional times respectively. T is temperature, μ and ν are coefficients of viscosity, kinematic coefficient of viscosity, respecitvely. η be the similarity variable, f be the dimensionless stream functions. f' and g are the dimensionless velocity, temperature respectively. B is the magnetic field, a is the velocity gradient at $t^* = 0$. r is the radius of axisymmetric body. $y = 0$ and 1 for two dimensional and axisymmetric flows, respectively, M, Ha and ReL are the magnetic parameter, Hartmann number and Reynold's number, respectively. a_1 is the graident of wall velocity at $t^* = 0$, b is the ratio of velocity of the wall and the free stream velocity (b >=< 0 according to whether the velocities of the wall and free stream are in the same direction or in opposite direction), σ, Pr and L respectively, electrical conductivity, Prandtl number and characteristic length. ϕ is an arbitrary function of t^* having continuous first derivative for $t^* \geq 0$. The subcripts i denotes the initial condition, the subscripts e, w and ∞ denote the conditions at the edge of the boundary layer, on the wall and in the free stream respectively. The subscripts t, t^* x and y denote partial derivatives with respect to t, t^* x and y respectively and primes denote derivatives with respect to η.

The skin friction and heat transfer coefficients can be expressed as

$$\left.\begin{aligned} Cf &= \frac{2T_w}{\rho\,(ue)^2_{t*}} = 0 \\ Cf &= 2(Rex)^{-1/2}\,\phi(t^*)f''_w \\ Nu &= \frac{x\left(\frac{\partial T}{\partial y}\right)_w}{T_w - T_\infty} = -\,(Rex)^{1/2}\,g'_w \end{aligned}\right| \qquad ...(14)$$

where

$$\left.\begin{aligned} T_w &= \mu\left(\frac{\partial u}{\partial y}\right)_w \\ Rex &= \frac{ax^2}{v} \\ (u_e)_{att^*} &= 0 = ax \end{aligned}\right| \qquad ...(15)$$

Here Cf and Nu are surface skin friction and Nusselt number (heat transfer coefficient) respectively, Tw is the shear stress at the wall, f''_w and $-g''_w$ are the skin friction and heat transfer parameters at the wall, Rex is the local Reynold's number and ρ is the density.

4. SELF SIMILAR EQUATIONS :

The equations (1) to (3) are partial differential equations with three independent variables. It can be shown that if the free stream velocity and the wall velocity vary inversely as a linear function of time and directly as a linear function of x i.e.

$$u_e = ax(1 - \lambda t^*)^{-1}$$

$$u_w = a_1 x\,(1 - \lambda t^*)^{-1}$$

and the magnetic field as a square root of linear function of time then equations (1) to (3) admit self similar solutions that is they are reduced to a set of ordinary differential equations.

We apply the following transformations as follows :

$$\left.\begin{aligned}
&\eta = (1+j)^{1/2}(a/v)^{1/2}(1-\lambda t^*)^{-1/2}y \\
&t^* = at,\ \partial t^* < 1 \\
&u = ax(1-\lambda t^*)^{-1} f'(\eta) \\
&v = -(1+j)^{1/2}(1-\lambda t^*)^{-1/2} f(\eta) \\
&\frac{T-T_\infty}{T_w - T_\infty} = g(\eta) \\
&B = B_0 (1-\lambda t^*)^{-1/2} \\
&M = \frac{Ha^2}{ReL} \\
&Ha^2 = \frac{\partial B_0^2 L^2}{\mu} \\
&u_e = ax(1-\lambda t^*)^{-1} \\
&u_w = a_1 x(1-\lambda t^*)^{-1}
\end{aligned}\right| \qquad \ldots(16)$$

From what has been done it follows that

$$u_x = \frac{\partial u}{\partial x} = a(1-\lambda t^*)^{-1} f'(\eta)$$

$$u_y = \frac{\partial u}{\partial y} = \frac{a^{3/2}x}{v^{1/2}}(1-\lambda t^*)^{-3/2}(1+j)^{1/2} f'(\eta)$$

$$u_{yy} = \frac{\partial^2 u}{\partial y^2} = \frac{a^2 x}{\nu} = (1-\lambda t^*)^{-2}(1+j) f''(\eta)$$

$$u_t = \frac{\partial u}{\partial t} = \frac{a^2 x}{v^2}(1-\lambda t^*)^{-1/2} f''(\eta) + a^2 x f'(\eta)(1-\lambda t^*)^{-2}$$

$$\left.\begin{aligned}
&ue_x = \frac{\partial u_e}{\partial x} = a(1-\lambda t^*)^{-1} \\
&ue_t = \frac{\partial u_e}{\partial t} = a^2 x(1-\lambda t^*)^{-2}
\end{aligned}\right| \qquad \ldots(16a)$$

Also we have $\quad B = B_0 (1-\lambda t^*)^{-1/2}$

$\therefore \quad B^2 = B_0^2 (1-\lambda t^*)^{-1}$

$\therefore \quad \sigma B^2 = \sigma B_0^2 (1-\lambda t^*)^{-1}$

but we have $\quad Ha^2 = \dfrac{\partial B_0^2 L^2}{\mu}$

$$\therefore \qquad \sigma B_0^2 = \frac{Ha^2\mu}{L^2}$$

Hence $$\sigma B^2 = \frac{Ha^2\mu}{L^2}(1-\lambda t^*)^{-1}$$

but $$M = \frac{Ha^2}{ReL}$$

$$\therefore \qquad Ha^2 = M.ReL$$

Therefore, $$\sigma B^2 = \frac{M.ReL\mu}{L^2}(1-\lambda t^*)^{-1}.$$

Since $$ReL = \frac{aL^2}{\nu}$$

Thus, we get

$$\sigma B^2 = M\left(\frac{aL^2}{\nu}\right)\frac{\mu}{L^2}(1-\lambda t^*)^{-1}$$

$$\sigma B^2 = Ma(\mu/\nu)\,(1-\lambda t^*)^{-1}$$

$$\therefore \qquad \sigma B^2 = Ma\rho\,.\,(1-\lambda t^*)^{-1}. \qquad \text{...(16b)}$$

Putting the values of equations (16a) and (16b) in equation (1) to equation (3). We get that equation (1) is satisfied identically. Equations (2) and (3) reduce to

$$f''' + ff'' + (1+j)^{-1}\,[(1-f'^2) + (1-f'-\eta\frac{f''}{2}) + M\,(1-f'-\frac{\upsilon\,.\,\rho}{Kb^2\sigma})] = 0 \qquad \text{...(17)}$$

and $$pr^{-1}g'' + fg' - \frac{(1+j)^{-1}\lambda\eta g'}{2} = 0. \qquad \text{...(18)}$$

The boundary conditions are

$$f = 0, f' = b, g = 1 \text{ at } -\eta = 0;\ f' = -1, g = 0 \text{ as } \eta \to \infty \qquad \text{...(19)}$$

Here is the parameter characterizing the unsteadiness in the flow field and B_0 is value of magnetic field at $t^* = 0$, $\lambda >< 0$ accoding as the flow in accelerating or decelerating. Also the magnetic field B is assumed to vary as the square root of a linear function of time as given in equation (16) in order to certain self similar solution.

In actual practice, it may be possible to create a maintian such a magnetic field. Inspite of this weakness the reśult may be used to gain some insight into the characteristic of flow based on more realistic distribution of the magnetic field.

The skin friction and heat transfer coefficinets are given by :

$$\left.\begin{aligned} Cf &= \frac{2T_w}{(\rho u_0^2)} \\ Cf &= 2(1+j)^{1/2}\,(Rex)^{-1/2}\,f_w'' \\ Rex &= \frac{u_e x}{\nu} \\ Nu &= \frac{xT_y}{(T_w - T_\infty)} \\ Nu &= (1+j)^{1/2}\,(Rex)^{1/2}\,g_w' \end{aligned}\right| \qquad ...(20)$$

SOLUTION OF THE GOVERNING EQUATIONS :

ASYMPTOTIC SOLUTION

In this section, we consider the asymptotic behaviour of the governing equations (17) and (18).

This will enable us to find the range of value for which similarity solution is valid for large η.

$$f(\eta) = \eta + f_1(\eta),\ g(\eta) = g_1(\eta) \qquad ...(21)$$

From boundary condition (19), it is evident that

$$f_1 \to 0,\ f_1' \to 0,\ g_1 \to 0 \text{ as } \eta \to \infty \qquad ...(22)$$

where f_1 and g_1 are small. Now linearizing equations (17) and (18), using relation given in equation (21) and integrating resulting equation corresponding to eqn. (17) once and using the appropriate boundary conditions, we obtain :

$$f_1'' + \alpha\eta f_1' - (1+j)^{-1}\left(2 + \lambda + M - \frac{\upsilon M}{KB^2}\frac{\rho}{\sigma} + \alpha\right) f_1 = 0 \qquad ...(23)$$

$$Fr^{-1} g'' + \alpha\eta g_1' = 0,\ \alpha = 1 - 2^{-1}(1+j)^{-1}\lambda \qquad ...(24)$$

The solution of equation (24) satisfying the relevant boundary condition given in equation (22) is given by

$$g = g_1 = -A \int_0^{\infty} \exp\left(\frac{pr\alpha\eta^2}{2}\right) d\eta \qquad \text{...(24a)}$$

where A is constant, we apply the following transformation to equation (23)

$$f_1 = \exp(-\alpha\eta^3/4)H \qquad \text{...(25)}$$

Consequently equation (23) would be reduced to

$$H'' - [3/2 + (1+j)^{-1}(2 + \lambda/4 + M - \frac{Mv}{KB^2}\frac{\rho}{\sigma}) + \alpha^2\eta^2/4]H = 0 \qquad \text{...(26)}$$

where $H \to 0$ as $\eta \to \infty$. Equation (26) is Weber's type of equation whose solution for η large can be written in terms of parabolic cylinder functions as

$$H = A_1 \exp\left(-\frac{\alpha^2\eta^2}{4}\right)(\alpha\eta)^n P_1(n) + B_1 \exp\left(\frac{\alpha^2\eta^2}{4}\right)(\alpha\eta)^n P_2(\eta) \qquad \text{..(27)}$$

where

$$\left.\begin{aligned} P_1(\eta) &= 1 - 2^{-1} n(n-1)(\alpha\eta)^{-2} + O(\alpha\eta)^{-4} \\ P_2(\eta) &= 1 + 2^{-1} n(n+1)(\alpha\eta)^{-2} + O(\alpha\eta)^{-4} \\ n &= -2 - (1+j)^{-1}\left(2 + \frac{\lambda}{4} + M - \frac{vM}{KB^2}\frac{\rho}{\sigma}\right) \end{aligned}\right\} \qquad \text{...(28)}$$

Since $H \to 0$ as $\eta \to \infty$, the divergent part of solution H will be omitted. Hence

$$f_1 = A_1 \exp[-\alpha(\alpha+1)\frac{\eta^2}{4}](\alpha\eta)^n p_1(\eta) \qquad \text{...(29)}$$

It is clear from (24a) and (29) that g or g_1 and f_1 decay to zero exponentially if $\alpha > 0$ i.e. $\lambda < 2(1+j)$. This fixes the upper limit of λ. The lower limit of λ is given by that value of $\lambda(\lambda < 0)$ for which skin friction parameter f''_w vanishes.

NUMERICAL SOLUTION

The partial differential equations (8) and (9) under with the boundary condition (10) and initial conditions (12) and (13) and ordinary differential equations (17) and (18) under with the boundary condition (19) have been solved numerically using an implicit finite difference scheme in combination with quasilinearization technique.

The effect of step size $\Delta\eta$ and Δt^* and the edge of the boundary layer η_∞ on the solutions have been studied and optimum value of $\Delta\eta$, Δt^* and η_∞ have been obtained. consequently we have taken $\Delta\eta = 0.05$ and $\Delta t^* = 0.1$ for computation. Also we have taken the value of edge of the boundary layer (η_∞) between 4 and 8 depending on the value of parameters.

For computation the free stream velocity distribution have been taken in the form

$$\phi(t^*) = 1 \pm \in t^*$$

and

$$\phi(t) = \frac{1 + \in_1 \cos w^* t^*}{1 + \in_1}$$

where $\in$ and $\in_1$ are constants and w^* is the frequency parameter.

CONCLUSION AND DISCUSSION

The skin friction and heat transfer results are found to be significantly affected by the stream velocity, magnetic field and wall velocity However, their effects on the heat transfer is comparatively

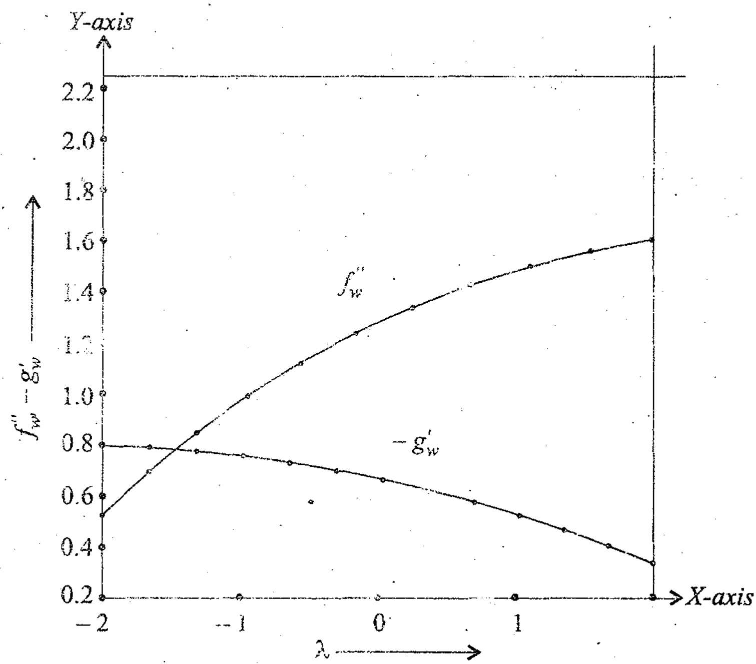

Fig. 1

Comparision of skin-friction and heat transfer results (f''_p, g''_w) for $\phi(t^*) = (1 - \lambda t^*)^{-1}$ [Self similar case], $M = b = j = 0$, $f_r = 0.7$, present results.

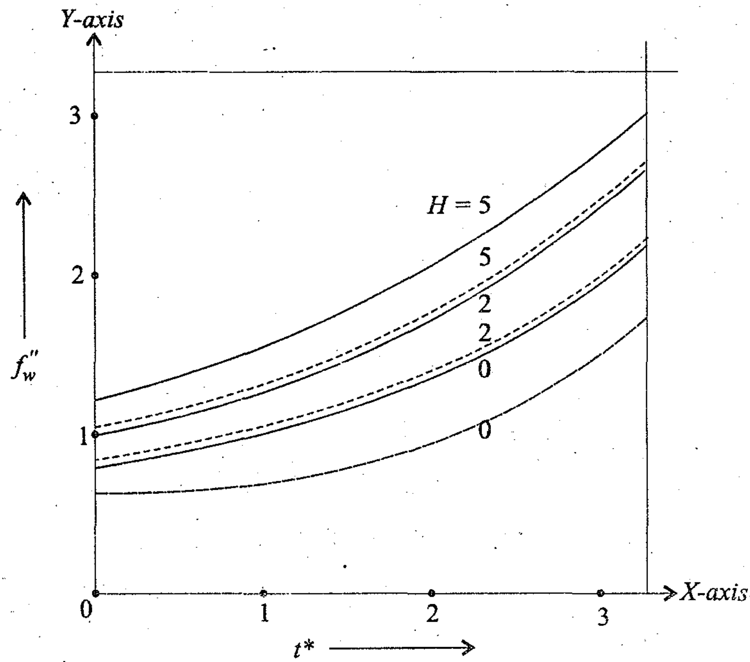

Fig. 2

Skin friction parameter (f''_w) for $\phi(t^*) = 1 + \in t^*$, $\in = 0.25$, $b = 0.5$, $j = 0, \ldots, 1$.

less as compared to that on the skin friction. The self similar solution exists when the free stream velocity, wall velocity and the square of magnetic field vary inversely as a linear function of time. The skin friction and heat transfer increases as the magnetic parameter increases. However, the skin friction decreases as the wall velocity increases, but the heat transfer increases. The skin friction and heat transfer for the axisymmetric flow are found to be less than those of the two dimensional flow.

Computations have been carried out for various values of the parameters M, b, j and λ. However, the results are presented only for some representative values of these parameters. Figure 1 and Figure 2 present the comparison with the results of the previous investigators.

The results corresponding to the accelerating free stream velocity $\phi(t^*) = 1 + \in t^*$, $\in > 0$ are presented.

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