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FUNDAMENATAL SOLUTION IN GENERALIZED FUNCTION SPACE IN NEGATIVE RESISTANCE OSCILLATORY CIRCUIT OF DYNATRON OSCILLATOR

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ABSTRACT

Using the method of variation of parameters and a special case of S.M. Joshi Distributional transform, fundamental solutions in generalized function space are utilized to analyze the electronic circuit of Dynatron Oscillator. The appropriate conditions for sustained and decaying oscillations are obtained by the new method.

If the decrease in the applied voltage induced increase in the current, in electronic circuitary arrangement, the resistance offered by this arrangement is called negative resistance. For crimple in a Vacuum tetrode the secondary commission displays the negative resistance. Herold [2] developed the idea (see Fig.1) that the sustained oscillations of high frequency stability can be achieved by shunting a tuned circuit with the negative resistance (here tetrode). The oscillator operators over the negative resistance region *EF* of tetrode plate characteristics. In this region the screen grid voltage becomes much higher than plate voltage for a larger number, than that of primary electrons, of secondary electrons may be produced by the plate and collected by the screen grid. Plate voltages e_{b_1} and e_{b_2} are kept suitably below the screen voltage to offer the negative resistance, due to secondary electron emmission from the plate. The positive resistance of the circuit is neutralized by the negative resistance offered by the tube.

The commercial application of negative resistance is made in dynatron, killitron and transitron oscillators. the dynatron (see Fig. 2), for example, oscillator operates in the negative resistance region of tetrode.

The present paper is concerned with analyzing mathematically the above circuit using the powerful method of fundamental solutions.

Using Kirchoff's law in the equivalent circuit of dynatron oscillator, differential equation for current I_t, after some simplification, takes the form

(1) $[D^2 + \mu D + \nu^2]\, I_t = 0,$ where D stands for the operator d/dt,

(2) $$\mu = \frac{\gamma_p RC - L}{\gamma_p \,.\, RC} \quad \text{and} \quad \nu^2 = \frac{\gamma_p - R}{\gamma_p \,.\, LC}\,.$$

Although one can use the method of variation of parameters [4] to solve (1) to study the nature of oscillatory current flowing in a dynatron oscillator as has been done in the following section yet that does not at least mathematically, give the whole picture, we therefore obtain the solution with the help of distributional transform

(3) $$(S^{a_0}_{b_0} f)\,(x) = \langle f(\xi), e^{r\,\xi x} \rangle,$$

which is a special case of distributional S.M.Joshi tranform [3]

(4) $(s_b^a f)(x) = < f(\xi), e^{i\xi x} > 0.$

Notation and terminology. Here

(5) $< f, \phi >$

is taken to mean the application of the functional f to the test function ϕ, and $\mathcal{T}$ is the space of basic functions i.e. the space of infinitely differentiable functions which together with all their derivatives approach zero more rapidly than any power of $1/|x|$ as $x \to \infty$.
The generalized function (or the distributions as they are called) Space on T i.e. the set of all linear continuous functionals on $\mathcal{T}$ is denoted by T' as dual space of T.

The linearity of is $\mathcal{T}$ signified by the relation

(6) $< f, \alpha_1\phi_1 + \alpha_2\phi_2 > = \alpha_1 < f, \phi_1 > + \alpha_2 < f, \phi_2 >$

where as the continuity of the generalized function f implies that, if the sequence $\phi_1, \phi_2, ..., \phi_v, ...$ converges to zero in $\mathcal{T}$ then the sequence $< f, \phi_1 >, < f, \phi_2 >, ...$ also converges to zero in T'. For regular functionals

(7) $< f, \phi > = \int f(x)\, \phi(x)\, dx.$

Functionals that can not be represented as (7), are called *singular*.

For example the delta functional represented by

(8) $< \delta(x), \phi(x) > = \phi(0)$

is singular and so is the so called "translated" [1] delta functional is defined by

(9) $< \delta(x - x_0), \phi(x) > = \phi(x_0).$

Sustained oscillations

To obtain solution of (1) by the method of fundmental solution let it has the driving impulse $-g(t, t')$ originating from a source point at the instant t'. We now concentrate on the solution of the differential equation

(10) $[D^2 + \mu D + \gamma^2]\, I_t = -g(t, t').$

The wellknown variation of parameter method then gives the solution as

(11) $I_t = [A(t) \cos \rho t + B(t) \sin \rho t] \exp(\sigma t),$ if $4v^2 > \mu^2$

and

(12) $I_t = [A(t) \cosh \rho t + B(t) \sinh \rho t] \exp(\sigma t),$ if $4v^2 < \mu^2.$

where

(13) $\rho^2 = \frac{1}{2}(\mu^2 - 4v^2)$ and $\sigma = -\frac{1}{2}\mu$

The variation of parameter method then requires, to obtain the parameters, that

(14) $$[\rho\{\frac{dB(t)}{dt} - (\tfrac{1}{2}\mu^2 - v^2) A(t)\} \cos \rho t$$
$$- \{\rho \frac{dA(t)}{dt} + (\tfrac{1}{2}\mu^2 - v^2) B(t)\} \sin \rho t] \exp(\sigma t) = -g(t, t')$$

Solution could then be obtained from (11) and (14). But in practice it is difficult to be obtained unless we take

(15) $\mu^2 = 2\nu^2$ or in other words

(16) $$\left[\frac{\gamma_p RC - L}{\gamma_p LC}\right]^2 = 2\left[\frac{\gamma_p - R}{\gamma_p LC}\right].$$

This is exactly the condition of oscillations in Dynatron oscillators.

This gives $\gamma_p = \frac{1}{2C}\left(1 - \frac{R^2}{L}\right)^{-1}$, or $= \frac{1}{2C}\left(\frac{R^2}{L} - 1\right)^{-1}$.

Under the assumption of negative plate resistance, which as explained earlier in the fundamental requirement in the Dynatron oscillator, only the later value is edmissible. Hence

(17) $w_0^2 R^2 C \leq 1,$

where as usual w_0 is the resonant frequency.

The dumping factor μDI_t helps to decay the oscillations.

The sustained oscillations, then, can only be obtained if

(18) $\mu = 0$ and $\gamma_p = L/CR.$

The frequency of oscillation is, then, given by

(19) $$f = \frac{LC}{2\pi} - \frac{1}{2}\left(1 - \frac{R}{\gamma_p}\right)^{1/2}.$$

Equation (1) under these conditions is that of simple Harmonic oscillator. However practically such an adjustment is difficult to obtain.

Fundamental Solution in τ'.

A fundamental solution in τ' is the solution of the equation

(20) $[D^2 + \mu D + \nu^2]\, F(t, t') = -\,\delta(t, t'),$

where $\delta(t, t')$ is the "translated" generalized delta function defined by [1], of unit impulse.

We take the timings t and t' when the oscillator operates at points E and F of Fig.2 respectively.

Once $F(t, t')$ is known the solution for oscillatory current equation (10) can be obtained [1] as

(21) $I_t = F(t) * g(t),$

where * denotes convolution operation in T'.

But taking Distributional transform (4) of both sides of (20) one obtains

$$F(t, t') = \frac{1}{\xi^2 + \mu\xi - \nu^2}, \qquad \text{where } F(t) = (s_{b_0}^{a_0} F).$$

Taking inverse [3] we easily obtain

(22) $F(t, t') = \frac{1}{2\pi} < (\rho^2 + i\mu\xi - \nu^2)^{-1}, exp\ t\ \xi(t - t')>$

Hence

(23) $F(t, t') = \{2(4\nu^2 - \mu^2)^{-1/2} \sin[(\nu^2 - \mu^2/4)^{1/2}(t - t')]\ expt\ (t - t')^{1/2}\}$

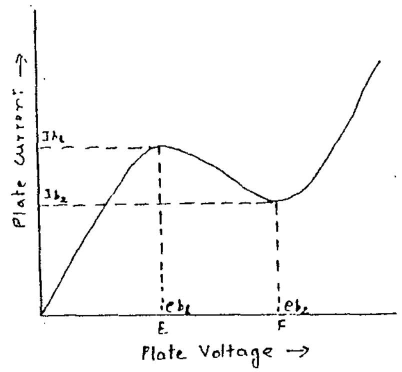
$$\{H(t - t')\},$$

where $H(t - t')$ is the Heavisides unit step Function.

Discussion

To obtain sustained oscillations it is necessary that γ_p be given as closely as given by (18) so that harmonically oscillating system of oscillating frequency 'f' (given by (19) may be function.

This is verified by the forms of solution (23) which shows that the oscillation dies out exponentially resulting in failure of dynatron oscillator.



(FIG 1) TETRODE CHARACTERISTIC

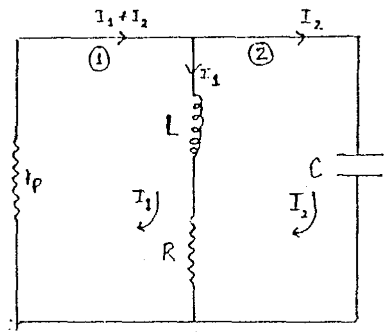


FIG 2(b) - EQUIVALENT CIRCUIT OF DYNATRON.

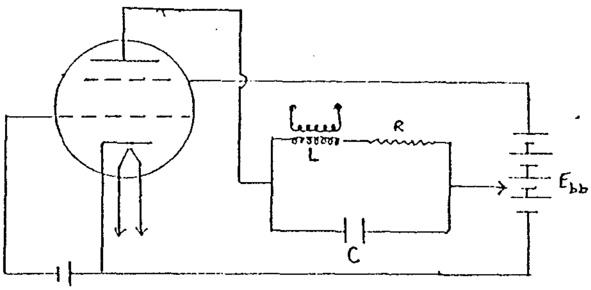


FIG 2 (a) DYNATRON OSCILLATOR

REFERENCES

[1] I Ger'Fand, & G.N. Shilov, *Generalized functions* Vol.1. Academic Press, New York and London.. (1964)

[2] E.W. Harold, I.R.E., Vol. 23. (1935)

[3] J.M.C. Joshi on a class of integral equations (involving S.M. Joshi functionals) over a C-like surface in D' (r+) Algebra (algebras).

[4] R.C. Whitten, and P.T. Chromic, *American, J. Phys.*, 43 (1975)