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(Dedicated to Professor J.N. Kapur on his 70th Birthday)

INTEGRALS ASSOCIATED WITH GAUSS'S HYPERGEOMETRIC SERIES, MULTIVARIABLE *H*-FUNCTION AND A GENERAL CLASS OF POLYNOMIALS

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ABSTRACT

A new class of integrals associated with hypergeometric series, the multivariable H-function and a general class of polynomials are evaluated. The results obtained are of general character and include the integrals given by Sharma and Rathie [9] and Arora and Rathie [1] etc.

1. INTRODUCTION

The object of this paper is to evaluate some integrals involving product of hypergeometric series,the multivariable H-function due to Srivastava and Panda [11] and a general class of polynomials due to Srivastava [10]. The integrals evaluated in this paper extend the results of Sharma and Rathie [9] and Arora and Rathie [1] etc.

The multivariable H-function introduced and studied by Srivastava and Panda [11] will be defined and represented in the following contracted form [12, pp.251-252, Eqns.] $(C.1)-(C.3)$]:

$$H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j:A_j^{(1)},\ldots,A_j^{(r)})_{1,p}:(c_j^{(1)},C_j^{(i)})_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ (b_j:B_j^{(1)},\ldots,B_j^{(r)})_{1,q}:(d_j^{(1)},D_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{matrix}\right]$$

$$=\frac{1}{(2\pi w)^r}\int_{L_1}\cdots\int_{L_r}\psi(\xi_i,\ldots,\xi_r)\prod_{i=1}^{r}\phi_i(\xi_i)\prod_{i=1}^{r}z_i^{\xi_i}\,d\xi_1\ldots d\xi_r \qquad \ldots (1.1)$$

where $w=\sqrt{-1}$,

$$\phi_i(\xi_i)=\frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}\xi_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}\xi_i)}{\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}\xi_i)\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}\xi_i)}\quad \forall\ (i=1,\ldots,r)$$

$$\ldots (1.2)$$

$$\psi(\xi_1, ..., \xi_r) = \frac{\prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} A_j^{(i)} \xi_i)}{\prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} B_j^{(i)} \xi_i) \prod_{j=n+1}^{p} \Gamma(a_j - \sum_{i=1}^{r} A_j^{(i)} \xi_i)} \quad ... (1.3)$$

A detailed account of the multivariable H-function can be found in the monograh by Srivastava et al. [12].

Following Srivastava [10], the general class of polynomials is defined as

$$S_\beta^\alpha[x] = \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!} A_{\beta,u} (x)^u; \ \beta = 0, 1, 2, ... \quad ... (1.4)$$

where α is an arbitrary positive integer and the co-efficients $A_{\beta,u}$ $(\beta, u > 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{\beta,u}$, $S_\beta^\alpha[x]$ yeilds a number of known polynomials as its special cases [13,pp.158-161]. For convenience, the H-function of r complex variables defined by (1.1) will be denoted by the contracted notation $H[z_1, ..., z_r]$.

2.INTEGRALS

The following results are to be established here:

First Integral :

$$\int_0^1 t^{\rho-1}(1-t)^\rho [1 + at + b(1-t)]^{-2\rho-1} {}_2F_1[\nu,\delta; \tfrac{1}{2}(\nu+\delta+2); \frac{t(1+a)}{1+at+b(1-t)}]$$
$$. S_\beta^\alpha [xR^k] H[z_1 R_1^{\lambda_1}, ..., z_r R_r^{\lambda_r}] dt$$
$$= \frac{2^{\nu+\delta-2\rho-1}\Gamma(\frac{\nu}{2}+\frac{\delta}{2}+1)}{(\nu-\delta)(1+a)^\rho (1+b)^{\rho+1} \Gamma(\nu)\,\Gamma(\delta)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!} A_{\beta,u} (x)^u$$
$$.\left\{\Gamma\left(\frac{\nu}{2}+\frac{1}{2}\right)\Gamma\left(\frac{\delta}{2}\right) H^{0, n+3; m_1, n_1; ... ; m_r, n_r}_{p+3, q+3; p_1, q_1; ... ; p_r, q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} I_1 \\ \\ I_2 \end{matrix}\right]\right.$$
$$\left. - \Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\delta}{2}+\frac{1}{2}\right) H^{0, n+3; m_1, n_1; ... ; m_r, n_r}_{p+3, q+3 : p_1, q_1; ... ; p_r, q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} J_1 \\ \\ J_2 \end{matrix}\right]\right\} \quad ... (2.1)$$

where

$$R = R_i = \frac{4(t+at)(1+b)(1-t)}{[1+at+b(1-t)]^2} \; ; \forall \; i \; \varepsilon \; \{1, 2, ..., r\} \qquad ... (2.2)$$

$$I_1 = (1-\rho-ku; \lambda_1, ..., \lambda_r) \; ; (1-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2} \; ; \lambda_1, ..., \lambda_r);$$
$$\left(\frac{\nu}{2}-\frac{\delta}{2}-\rho-ku; \lambda_1, ..., \lambda_r\right); \left(a_j; A_j^{(1)}, ..., A_j^{(r)}\right)_{1,p};$$
$$(c_j^{(1)}, C_j^{(1)})_{1,p_1} \; ; ...; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \qquad .... (2.3)$$

$$I_2 = (b_j \; ; B_j^{(}1); ... \; ; B_j^{(r)})_{1,q}; (d_j^{(1)}, D_j^{(1)})_{1,q_1}; ... \; ; (d_j^{(r)}, D_j^{(r)})_{1,q_r};$$
$$(1-\rho-ku+\frac{\nu}{2}-\frac{\delta}{2}; \lambda_1, ..., \lambda_r); (\frac{\nu}{2}-\rho-ku; \lambda_1, ..., \lambda_r);$$
$$(\frac{1}{2}+\frac{\delta}{2}-\rho-ku; \lambda_1, ..., \lambda_r). \qquad ... (2.4)$$

$$J_1 = (1-\rho-ku; \lambda_1, ..., \lambda_r); (1-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2}; \lambda_1, ..., \lambda_r);$$
$$(\frac{\delta}{2}-\frac{\nu}{2}-\rho-ku; \lambda_1, ..., \lambda_r) \; ; (a_j \; ; A_j^{(1)} \; , ..., A_j^{(r)})_{1,p};$$
$$(c_j^{(1)}, C_j^{(1)})_{1,p_1} \; ; ... \; ; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \qquad ... (2.5)$$

$$J_2 = (b_j \; ; B_j^{(1)}, ..., B_j^{(r)})_{1,q} \; ; (d_j^{(1)}, D_j^{(1)})_{1,q_1}; ...; (d_j^{(r)}, D_j^{(r)})_{1,q_r};$$
$$(1-\rho-ku-\frac{\nu}{2}+\frac{\delta}{2} \; ; \lambda_1, ..., \lambda_r); (\frac{\delta}{2}-\rho-ku; \lambda_1, ..., \lambda_r);$$
$$(\frac{1}{2}+\frac{\nu}{2}-\rho-ku; \lambda_1, ..., \lambda_r). \qquad ... (2.6)$$

The (sufficient) conditions of validity of (2.1) are given below :

(*i*) The constants a and b are such that nons of the expressions $1+a, 1+b, 1+at+(1-t)$, where $0 \leq t \leq 1$, is zero. ... (2.7)

(*ii*) $\mathrm{Re}(\rho) > 0, \mathrm{Re}(2(\rho-\nu-\delta) > 0, \; k \geq 0.$ ... (2.8)

(*iii*) $\mathrm{Re}(\rho+ku+\sum_{i=1}^{r} \lambda_i \xi_i) > 0, \; \lambda_j \geq 0$ ... (2.9)

and $\xi_i = \min_{i \leq j \leq m_i} [\mathrm{Re}(d_j^{(i)}/D_j^{(i)})]$ where $u = 0, 1, ..., [\beta/\alpha]$

$\forall \; i \; \varepsilon \; \{1, 2, ..., r\} \; j = 1, 2, ..., m_r.$

(*iv*) $\Omega_i > 0, \; |\arg z_i| < \frac{1}{2}\pi\,\Omega_i \; ; \forall \; i \; \varepsilon \; \{1, 2, ..., r\}$ ... (2.10)

where

$$\Omega_i = -\sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)}$$

$$+ \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} > 0; \forall\, i\, \varepsilon \{1, 2, ..., r\}$$

Second Integral :

$$\int_0^1 t^{\rho-1}(1-t)^{\rho-2}\,[\,1+at+b(1-t)]^{-2\rho+1}\,{}_2F_1[\nu, \delta; \tfrac{1}{2}(\nu+\delta); \frac{t(1+a)}{1+at+b(1-t)}]\; S_\beta^\alpha\,[xR^k] H[z_1R_1^{\lambda_1}, ..., z_rR_r^{\lambda_r}]\,dt$$

$$= \frac{2^{\nu+\delta-2\rho}\,\Gamma(\frac{\nu}{2}+\frac{\delta}{2})}{(1+a)^\rho(1+b)^{\rho-1}\,\Gamma(\nu)\,\Gamma(\delta)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!} A_{\beta,u}(x)^u$$

$$\cdot\left\{\Gamma(\frac{\nu}{2}+\frac{1}{2})\,\Gamma(\frac{\delta}{2})\,H^{0,\,n+3\,;\,m_1,\,n_1;\,...;\,m_r,\,n_r}_{p+3,\,q+3;\,p_1,\,q_1;\,...;\,p_r,\,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\middle| \begin{matrix} E_1 \\ \\ E_2\end{matrix}\right]\right.$$

$$\left. + \Gamma(\frac{\nu}{2})\,\Gamma(\frac{\delta}{2}+\frac{1}{2})\,H^{0,\,n+3;\,m_1,\,n_1\,;\,...\,;\,m_r,\,n_r}_{p+3,\,q+3;\,p_1,\,q_1\,;\,...\,;\,p_r,\,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\middle| \begin{matrix} G_1 \\ \\ G_2\end{matrix}\right]\right\} \quad ...\,(2.11)$$

where

$R, R_1, ..., R_r$ are defined by (2.2).

$$E_1 = (2-\rho-ku\,;\lambda_1, ..., \lambda_r); (2-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2}\,;\lambda_1, ..., \lambda_r);$$

$$(1+\frac{\nu}{2}-\frac{\delta}{2}-\rho-ku\,;\lambda_1, ..., \lambda_r); (a_j\,; A_j^{(1)}, ..., A_j^{(r)})_{1,p}\,;$$

$$(c_j^{(1)}, C_j^{(1)})_{1,p_1}\,; ...\,; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \quad ...\,(2.12)$$

$$E_2 = (b_j\,; B_j^{(1)}, ..., B_j^{(r)})_{1,q}; (d_j^{(1)})_{1,q_1}\,; ...; (d_j^{(r)}, D_j^{(r)})_{1,q_r};$$

$$(2\quad \rho-ku+\frac{\nu}{2}-\frac{\delta}{2}; \lambda_1, ..., \lambda_r); (1+\frac{\nu}{2}-\rho-ku; \lambda_1, ..., \lambda_r);$$

$$(\frac{3}{2}+\frac{\delta}{2}-\rho-ku; \lambda_1, ..., \lambda_r) \quad ...\,(2.13)$$

$$G_1 = (2-\rho-ku\,;\lambda_1, ..., \lambda_r); (2-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2}\,;\lambda_1, ..., \lambda_r);$$

$$(1+\frac{\delta}{2}-\frac{\nu}{2}-\rho-ku\,;\lambda_1,\ldots,\lambda_r);(a_j\,;A_j^{(1)},\ldots,A_j^{(r)})_{1,p}\,;$$

$$(c_j^{(1)},C_j^{(1)})_{1,p_1}\,;\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \qquad \ldots(2.14)$$

$$G_2=(b_j\,;B_j^{(1)},\ldots,B_j^{(r)})_{1,q};(d_j^{(1)},D_j^{(1)})_{1,q_1}\,;\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r};$$

$$(2-\rho-ku-\frac{\nu}{2}+\frac{\delta}{2};\lambda_1,\ldots,\lambda_r);(1+\frac{\delta}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r);$$

$$(\frac{3}{2}+\frac{\nu}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r) \qquad \ldots(2.15)$$

The (sufficient) conditions of validity of (2.11) are given below :

(*i*) The conditions (2.7) and (2.10) hold.

(*ii*) $\mathrm{Re}(\rho)>1$, $\mathrm{Re}(2\rho-\nu-\delta)>2$; $k\geq 0$.

$$(iii)\ \mathrm{Re}(\rho+ku+\sum_{i=1}^{r}\lambda_i\,\xi_i)>1;\ \lambda_j\geq 0\ \forall\ \mathrm{j}, \qquad \ldots(2.16)$$

where

$$\xi_i=\min_{1\leq j\leq m_i}\,[\mathrm{Re}(d_j^{(i)}/D_j^{(i)})]\ \ \forall\ i\ \varepsilon\ \{1,2,\ldots,r\} \qquad \ldots(2.17)$$

Third Integral :

$$\int_0^{\pi/2} e^{w(2\rho+1)\theta}\,(\sin\theta)^{\rho}\,(\cos\theta)^{\rho-1}\,{}_2F_1[\nu,\delta;\tfrac{1}{2}(\nu+\delta+2)\,e^{w\theta}\cos\theta]$$

$$.\,S_\beta^\alpha[xT^k]H[z_1T_1^{\lambda_1},\ldots,z_rT_r^{\lambda_r}]d\theta$$

$$=\frac{e^{(w(\rho+1)/2)\pi}\,\Gamma(\frac{\nu}{2}+\frac{\delta}{2}+1)}{2^{2\rho-\nu-\delta+1}\,\Gamma(\nu)\,\Gamma(\delta)\,\Gamma(\nu-\delta)}\sum_{u=0}^{[\beta/\alpha]}\frac{(-\beta)_{\alpha u}}{(u)\,!}A_{\beta,u}(x)^u$$

$$\Big\{\Gamma\,(\nu+1)/2)\,\Gamma\,(-\delta/2)\,H^{0,n+3;m_1,n_1;\ldots;m_r,n_r}_{p+3,q+3;p_1,q_1,\ldots;p_r,q_r}\left[\begin{array}{c|c} z_1 & I_1\\ \vdots & \\ z_r & I_2\end{array}\right]$$

$$-\Gamma(\frac{\nu}{2})\,\Gamma(\frac{\delta}{2}+\frac{1}{2})H^{0,n+3;m_1,n_1;\ldots;m_r,n_r}_{p+3,q+3;p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c|c} z_1 & J_1\\ \vdots & \\ z_r & J_2\end{array}\right]\Big\} \qquad \ldots(2.18)$$

where I_1, I_2, J_1 and J_2 are defined by (2.3), (2.4), (2.5), and (2.6) respectively and $w=\sqrt{-1}$.

Also

$$T = T_i = \frac{4e^{2w\theta} \sin\theta \cos\theta}{e^{w\pi/2}} ; \ \forall\, i \,\varepsilon\, \{1, 2, ..., r\} \qquad ... (2.19)$$

The (sufficient) conditions for the validity of the integral (2.18) are given below :

(i) $\mathrm{Re}(\rho) > 0$, $\mathrm{Re}(2\rho - \nu - \delta) > 0$, $k \geq 0$;

(ii) $\mathrm{Re}(\rho + ku + \sum_{i=1}^{r} \lambda_i \xi_i) > 0$; $\lambda_i \geq 0 \ \forall j$;

where ξ_i is defined by (2.17).

Fourth Integral :

$$\int_0^{\pi/2} e^{w(2\rho - 1)\theta} (\sin\theta)^{\rho - 2} (\cos\theta)^{\rho - 1} \, {}_2F_1[\nu, \delta; (\frac{\nu + \delta}{2}) ; e^{w\theta} \cos\theta]$$

$$. S_\beta^\alpha[xT^k] H[z_1 T_1^{\lambda_1}, ..., z_r T_r^{\lambda_r}] d\theta$$

$$= \frac{e^{w\pi(\rho - 1)/2}\, \Gamma(\frac{\nu}{2} + \frac{\delta}{2})}{2^{2\rho - \nu - \delta}\, \Gamma(\nu)\, \Gamma(\delta)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{(u)!} A_{\beta, u} (x)^u$$

$$. \left\{ \Gamma(\frac{\nu}{2} + \frac{1}{2}) \Gamma(\frac{\delta}{2})\, H^{0, n+3; m_1, n_1; ..., m_r, n_r}_{p+3, q+3; p_1, q_1; ...; p_r, q_r} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} E_1 \\ \\ E_2 \end{matrix} \right] \right.$$

$$\left. + \Gamma(\frac{\nu}{2}) \Gamma(\frac{\delta}{2} + \frac{1}{2}) H^{0, n+3; m_1, n_1; ...; m_r, n_r}_{p+3, q+3; p_1, q_1; ...; p_r, q_r} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} G_1 \\ \\ G_2 \end{matrix} \right] \right\} \qquad ... (2.20)$$

where $T, T_1, ..., T_r$ are defined by (2.19) and E_1, E_2, G_1 and G_2 are defined by (2.12), (2.13), (2.14) and (2.15) respectively.

The (sufficient) conditions for the validity of the integral (2.20) are given below :

(*i*) $\mathrm{Re}(\rho) > 1$, $\mathrm{Re}(2\rho - \nu - \delta) > 2$, $k \geq 0$.

(*ii*) $\mathrm{Re}(\rho + ku - 2 + \sum_{i=1}^{r} \lambda_i \xi_i) > 0$; $\lambda_i \geq 0 \ \forall j$ .... (2.21)

where ξ_i is defined by (2.17).

(*iii*) The (2.16) also hold.

Fifth Integral :

$$\int_0^{\pi/2} e^{w(2\rho + 1)\theta} (\sin\theta)^{\rho - 1} (\cos\theta)^{\rho} \, {}_2F_1[\nu, \delta; \tfrac{1}{2}(\nu + \delta + 2);$$

$$e^{w(\theta - \pi/2)} \sin\theta] \, S_\beta^\alpha [XT^k] \, H\, [z_1 T_1^{\lambda_1}, ..., z_r T_r^{\lambda_r}] \, d\theta$$

$$= \frac{e^{w\pi\rho/2}\,\Gamma(\nu/2+\delta/2+1)}{2^{2\rho-\nu-\delta+1}\,(\nu-\delta)\,\Gamma(\nu)\,\Gamma(\delta)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!} A_{\beta,u}\,(x)^u$$

$$\left\{\Gamma(\frac{\nu}{2}+\frac{1}{2})\,\Gamma(\frac{\delta}{2})\,H^{0,\,n+3;\,m_1,\,n_1;\,\ldots;\,m_r,\,n_r}_{p+3,\,q+3;\,p_1,\,q_1;\,\ldots;\,p_r,\,q_r}\left[\begin{array}{c|c} z_1 & I_1 \\ \vdots & \\ z_r & I_2 \end{array}\right]\right.$$

$$\left. -\,\Gamma(\frac{\nu}{2})\,\Gamma(\frac{\delta}{2}+\frac{1}{2})\,H^{0,\,n+3;\,m_1,\,n_1;\,\ldots;\,m_r,\,n_r}_{p+3,\,q+3;\,p_1,\,q_1;\,\ldots;\,p_r,\,q_r}\left[\begin{array}{c|c} z_1 & J_1 \\ \vdots & \\ z_r & J_2 \end{array}\right]\right\} \qquad \ldots (2.22)$$

where $T, T_1, T_2, \ldots, T_r$ are defined by (2.19).

and I_1, I_2, J_1 and J_2 are defined by (2.3), (2.4), (2.5) and (2.6) respectively.

The (sufficient) conditions for the validity of the integral (2.22) are given below :

(*i*) $\text{Re}(\rho) > 0$; $\text{Re}(2\rho-\nu-\delta) > 0$; $k \geq 0$.

(*ii*) The condition (2.16) holds.

Sixth Integral :

$$\int_0^{\pi/2} e^{w(2\rho-1)\theta}\,(\sin\theta)^{\rho-1}\,(\cos\theta)^{\rho-2}\;{}_2F_1[\nu,\delta;\tfrac{1}{2}(\nu+\delta);$$

$$e^{w(\theta-\pi/2)}\sin\theta]\;S^{\alpha}_{\beta}\,[XT^k]\;H\,[z_1T_1^{\lambda_1},\ldots,z_rT_r^{\lambda_r}]d\theta$$

$$= \frac{e^{iw\pi\rho/2}\,\Gamma(\frac{\nu}{2}+\frac{\delta}{2})}{2^{2\rho-\nu-\delta}\,\Gamma(\nu)\,\Gamma(\delta)} \sum_{u-0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!}\;A_{\beta,u}\,(x)^u$$

$$.\left\{\Gamma(\frac{\nu}{2}+\frac{1}{2})\,\Gamma(\frac{\delta}{2})\,H^{0,\,n+3;\,m_1,\,n_1;\,\ldots;\,m_r,\,n_r}_{p+3,\,q+3,\,p_1,\,q_1;\,\ldots;\,p_r,\,q_r}\left[\begin{array}{c|c} z_1 & E_1 \\ \vdots & \\ z_r & E_2 \end{array}\right]\right.$$

$$\left. +\,(\frac{\nu}{2})\,\Gamma(\frac{\delta}{2}+\frac{1}{2})\,H^{0,\,n+3;\,m_1,\,n_1;\,\ldots;\,m_r,\,n_r}_{p+3,\,q+3,\,p_1,\,q_1;\,\ldots;\,p_r,\,q_r}\left[\begin{array}{c|c} z_1 & G_1 \\ \vdots & \\ z_r & G_2 \end{array}\right]\right\} \qquad \ldots (2.23)$$

where T, T_i are defined by (2.19) and E_1, E_2, G_1 and G_2 are defined by (2.12),(2.13),(2.14) and (2.15) respectively.

The (sufficient) conditions of validity of the integral (2.23) are given below :

(*i*) $\text{Re}(\rho) > 1$, $\text{Re}(2\rho-\nu-\delta) > 2$; $k \geq 0$.

(*ii*) The conditions (2.16) and (2.21) also hold.

Seventh Integral :

$$\int_0^1 t^{\sigma-1}(1-t)^{\sigma-\mu-1}\,{}_2F_1[\nu,-\nu;\mu;t]\,S_\beta^\alpha[xt^k(1-t)^k]$$

$$.\,H[z_1R_1;\ldots;z_rR_r]dt$$

$$=\frac{2^{-2\nu-1}\Gamma(\mu)}{\Gamma(\mu-\nu)}\sum_{u=0}^{[\beta/\alpha]}\frac{(-\beta)_{\alpha u}}{u\,!}A_{\beta,u}(x)^u\left\{\frac{\Gamma(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2})}{\Gamma(\frac{\mu}{2}+\frac{\nu}{2}+\frac{1}{2})}\right.$$

$$.\,H^{0,n+3;m_1,n_1;\ldots;m_r,n_r}_{p+3,q+2;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\middle|\begin{matrix}L_1\\ \\ L_2\end{matrix}\right]-\frac{\Gamma(\frac{\mu}{2}-\frac{\nu}{2})}{\Gamma(\frac{\mu}{2}+\frac{\nu}{2})}\,.$$

$$\left.\,.\,H^{0,n+3;m_1,n_1;\ldots;m_r,n_r}_{p+3,q+2;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\middle|\begin{matrix}L_1^*\\ \\ L_2^*\end{matrix}\right]\right\} \quad \ldots (2.24)$$

where $R_i=[t(1-t]^{\lambda_i}\,;\;\forall\, i\,\varepsilon\,\{1,2,\ldots,r\}$ ... (2.25)

$$L_1=(1-\sigma-ku;\lambda_1,\ldots,\lambda_r)\,;\,(1-\sigma+\mu-ku;\lambda_1,\ldots,\lambda_r);$$

$$(1-\sigma+\frac{\mu}{2}+\frac{\nu}{2}-ku;\lambda_1,\ldots,\lambda_r);(a_j\,;A_j^{(1)}),\ldots,A_j^{(r)})_{1,p}$$

$$.\,(c_j^{(1)},C_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \quad \ldots (2.26)$$

$$L_2=(b_j;B_j^{(1)},\ldots,B_j^{(r)})_{1,q};(d_j^{(1)},D_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r};$$

$$(1-2\sigma+\mu+\nu-2ku;2\lambda_1,\ldots,2\lambda_r);(1-\sigma+\frac{\mu}{2}-\frac{\nu}{2}-ku;\lambda_1,\ldots,\lambda_r) \quad \ldots (2.27)$$

$$L_1^*=(1-\sigma-ku;\lambda_1,\ldots,\lambda_r)\,;\,(1-\sigma+\mu-ku;\lambda_1,\ldots,\lambda_r);$$

$$(\tfrac{1}{2}-\sigma-ku+\frac{\mu}{2}+\frac{\nu}{2}\,;\lambda_1,\ldots,\lambda_r);(a_j\,;A_j^{(1)},\ldots,A_j^{(r)})_{1,p}\,;$$

$$.\,(c_j^{(1)},C_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \quad \ldots (2.28)$$

$$L_2^*=(b_j;B_j^{(1)},\ldots,B_j^{(r)})_{1,q};(d_j^{(1)},D_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r};$$

$$(1-2\sigma+\mu+\nu-2ku;2\lambda_1,\ldots,2\lambda_r);(\tfrac{1}{2}-\sigma+\frac{\mu}{2}-\frac{\nu}{2}-ku;\lambda_1,\ldots,\lambda_r) \quad \ldots (2.29)$$

The (sufficient) conditions of validity of (2.24) are given below:

(*i*) $\text{Re}(\sigma) > 0, \text{Re}(\mu) > 0, \text{Re}(\sigma - \mu) > 0, k \geq 0.$ ... (2.30)

(*ii*) $\text{Re}(\sigma + ku + \sum_{i=1}^{r} \lambda_i \xi_i) > 0, \text{Re}(\sigma - \mu + ku + \sum_{i=1}^{r} \lambda_i \xi_i) > 1;$ ... (2.31)

$$\lambda_j \geq 0 \ \forall j; u = 0, 1, ..., (\beta/\alpha) \text{ and } j = 1, 2, ..., m_r.$$

and

$$\xi_i = \min_{1 \leq j \leq m_i} [\text{Re}(\frac{d_j^{(i)}}{D_j^{(i)}})], \forall i \in 1, 2, ..., r) \qquad ... (2.32)$$

(*iii*) The condition (2.10) also holds.

Eighth Integral :

$$\int_0^1 t^{\sigma-1} (1-t)^{\sigma-\mu-1} {}_2F_1[\nu, -1-\nu; \mu; t] S_\beta^\alpha [xt^k (1-t)^k] . H[z_1 R_1; ...; z_r R_r] dt$$

$$= \frac{2^{-2\nu-2} \Gamma(\mu)}{\Gamma(\mu-\nu)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u!} A_{\beta, u} (x)^u \left\{ \frac{2\Gamma(\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2})}{\Gamma(\frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2})} \right.$$

$$.H^{0, n+4; m_1, n_1; ...; m_r, n_r}_{p+4, q+3; p_1, q_1; ...; p_r, q_r} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} L_3 \\ \\ L_4 \end{matrix} \right] - \frac{\mu \Gamma(\frac{\mu}{2} - \frac{\nu}{2})}{\Gamma(\frac{\mu}{2} + \frac{\nu}{2} + 1)}$$

$$\left. .H^{0, n+3; m_1, n_1; ...; m_r, n_r}_{p+3, q+2; p_1, q_1; ...; p_r, q_r} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} L_1^* \\ \\ L_2^* \end{matrix} \right] \right\} \qquad ... (2.33)$$

where $R_1, ..., R_r$ are defined by (2.25) and L_1^* and L_2^* are defined by (2.28) and (2.29) respectively.

$$L_3 = (1 - \sigma - ku; \lambda_1, .., \lambda_r); (1 - \sigma + \mu - ku; \lambda_1, ..., \lambda_r);$$

$$(\frac{\mu}{2} - \sigma - ku; \lambda_1, .., \lambda_r); (1 - \sigma + \frac{\mu}{2} + \frac{\nu}{2} - ku; \lambda_1, ..., \lambda_r);$$

$$(a_j; A_j^{(1)}, ..., A_j^{(r)})_{1,p}; (c_j^{(1)}, C_j^{(1)})_{1,p_1}; ..., (c_j^{(r)}, C_j^{(r)})_{1,p_r} \qquad ... (2.34)$$

$$L_4 = (b_j; B_j^{(1)}; ...; B_j^{(r)})_{1,q}; (d_j^{(1)}, D_j^{(1)})_{1,q_1}; ...; (d_j^{(r)}, D_j^{(r)})_{1,q_r};$$

$$(1-2\sigma+\mu+\nu-2ku;\ 2\lambda_1,\ \ldots,\ 2\lambda_r);\ (1-\sigma+\frac{\mu}{2}-ku;\ \lambda_1,\ \ldots,\ \lambda_r)$$

$$.(\frac{\mu}{2}-\frac{\nu}{2}-\sigma-ku;\ \lambda_1,\ \ldots,\ \lambda_r). \qquad \ldots (2.35)$$

The (sufficient) conditions of validity of (2.33) are given below :

(*i*) The conditions (2.10), (2.30) and (3.31) also hold.

Ninth Integral :

$$\int_0^1 t^{\sigma-1}(1-t)^{\sigma-\mu-1}\ {}_2F_1[\nu,\ 1-\nu;\ \mu;\ t]\ S_\beta^\alpha\ [x\ t^k(1-t)^k]$$

$$.H[z_1R_1;\ \ldots;\ z_rR_r]dt$$

$$=\frac{2^{-2\nu}\Gamma(\mu)}{\Gamma(\mu-\nu)}\sum_{u=0}^{[\beta/\alpha]}\frac{(-\beta)_{\alpha\mu}}{u\ !}A_{\beta,\ u}\ (x)^u$$

$$\left\{\frac{\Gamma(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2})}{\Gamma(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2})}H^{0,\ n+3\ ;\ m_1,\ n_1;\ \ldots;\ m_r,\ n_r}_{p+3,\ q+2;\ p_1,\ q_1;\ \ldots;\ p_r,\ q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\middle|\begin{matrix}L_5\\ \\ L_6\end{matrix}\right]\right.$$

$$\left.+\frac{\Gamma(\frac{\mu}{2}-\frac{\nu}{2})}{\Gamma(\frac{\mu}{2}+\frac{\nu}{2})}H^{0,\ n+3;\ m_1,\ n_1;\ \ldots;\ m_r,\ n_r}_{p+3,\ q+2;\ p_1,\ q_1;\ \ldots;\ p_r,\ q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\middle|\begin{matrix}L_5^*\\ \\ L_6^*\end{matrix}\right]\right\} \qquad \ldots (2.36)$$

where $R_1, R_2, \ldots, R_r$ defined by (2.25).

$$L_5=(2-\sigma-ku;\ \lambda_1,\ \ldots,\ \lambda_r);\ (1-\sigma+\mu-ku;\ \lambda_1,\ \ldots,\ \lambda_r);$$

$$(1-\sigma+\frac{\mu}{2}+\frac{\nu}{2}-ku;\ \lambda_1,\ \ldots,\ \lambda_r);\ (a_j;\ A_j^{(1)},\ \ldots A_j^{(r)})_{1,\ p};$$

$$(c_j^{(1)}, C_j^{(1)})_{1,\ p_1};\ \ldots;\ (c_j^{(r)},\ C_j^{(r)})_{1,\ p_r} \qquad \ldots (2.37)$$

$$L_6=(b_j;\ B_j^{(1)},\ \ldots,\ B_j^{(r)})_{1,\ q};\ (d_j^{(1)}, D_j^{(1)})_{1,\ q_1};\ \ldots;\ (d_j^{(r)},\ D_j^{(r)})_{1, q_r};$$

$$(1-2\sigma+\mu+\nu-2ku;\ 2\lambda_1,\ \ldots,\ 2\lambda_r);\ (1-\sigma+\frac{\mu}{2}-\frac{\nu}{2}-ku;\ \lambda_1,\ \ldots,\ \lambda_r)$$

$$\ldots (2.38)$$

$$L_5^*=(2-\sigma-ku;\ \lambda_1,\ \ldots,\ \lambda_r);\ (1-\sigma+\mu-ku;\ \lambda_1,\ \ldots,\ \lambda_r);$$

$$(\tfrac{1}{2}-\sigma+\frac{\mu}{2}+\frac{\nu}{2}+\frac{\nu}{2}-ku;\ \lambda_1,\ \ldots,\ \lambda_r);\ (a_j;\ A_j^{(1)},\ \ldots,\ A_j^{(r)})_{1,\ p};$$

$$(c_j^{(1)}, C_j^{(1)})_{1,\ p_1},\ \ldots,\ (c_j^{(r)}, C_j^{(r)})_{1,\ p_r} \qquad \ldots (2.39)$$

$$L_6^* = (b_j; B_j^{(1)}, \ldots, B_j^{(r)})_{1,q}; (d_j^{(1)}, D_j^{(1)})_{1,q_1}; \ldots; (d_j^{(r)}, D_j^{(r)})_{1,q_r};$$

$$(1 - 2\sigma + \mu + \nu - 2ku; 2\lambda_1, \ldots, 2\lambda_r); (\tfrac{3}{2} - \sigma + \frac{\mu}{2} - \frac{\nu}{2} - ku; \lambda_1, \ldots, \lambda_r) \quad \ldots (2.40)$$

The (sufficient) conditions of validity of (2.36) are given below:

(i) The conditions (2.10), (2.30) and (2.31) hold .

Proof of (2.1) : To evaluate the integral (2.1), we first express the multivariable H-function in terms of multiple Mellin-Barnes type contour integral (1.1), then on using (1.4) and interchanging the order of ξ_i-integrals and t-integral which is permissible under the conditions stated on account of the absolute (and uniform) convergence of the integrals and finally evaluating the t-integral with the help of known result [(9,pp.26,Eq. (2.1)], the result readily follows.

The remaining integrals (2.11), (2.18), (2.20), (2.22) and (2.23) can be proved in the same way by employing the integrals [9, pp.26-28, Eqs. (2.2) to (2.6)], respectively.The following integrals (2.24), (2.33) and (2.36), can be established by using the results [1,pp.84-86, Eqs. (5.1) to (5.3)] respectively.

3. SPECIAL CASES

(1) If we have $n = p = q = 0$, the multivariable H-function in(2.1) breaks up into products of r, H-functions and following integral is obtained :

$$\int_0^1 t^{\rho-1} (1-t)^\rho [1 + at + b(1-t)]^{-2\rho-1} {}_2F_1\left[\nu, \delta; \tfrac{1}{2}(\nu+\delta+2); \frac{t(1+a)}{1+at+b(1-t)}\right]$$

$$\cdot S_\beta^\alpha[xR^k] \prod_{i=1}^{r} H_{p_i, q_i}^{m_i, n_i}\left[z_i R_i^{\lambda_i} \left| \begin{matrix} (c_j^{(i)}, C_j^{(i)})_{1,p_i} \\ (d_j^{(i)}, D_j^{(i)})_{1,q_i} \end{matrix}\right.\right] dt$$

$$= \frac{2^{\nu+\delta-2\rho-1}\,\Gamma(\frac{\nu}{2}+\frac{\delta}{2}+1)}{(\nu-\delta)(1+a)^\rho (1+b)^{\rho+1}\,\Gamma(\nu)\,\Gamma(\delta)} \sum_{u=0}^{[\beta/\alpha]} \frac{(-\beta)_{\alpha u}}{u\,!} A_{\beta,u}\,(x)^u$$

$$\left\{\Gamma(\frac{\nu}{2}+\frac{1}{2})\,\Gamma(\frac{\delta}{2})\, H_{3,3;p_1,q_1;\ldots;p_r,q_r}^{0,3;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} I'_1 \\ \\ I'_2 \end{matrix}\right.\right]\right.$$

$$\left. - \Gamma(\frac{\nu}{2})\,\Gamma(\frac{\delta}{2}+\frac{1}{2})\, H_{3,3;p_1,q_1;\ldots;p_r,q_r}^{0,3;m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} J'_1 \\ \\ J'_2 \end{matrix}\right.\right]\right\} \quad \ldots (3.1)$$

where $R = R_1 = R_2 = \ldots = R_i$ are defined by (2.2)

$$I'_1 = (1-\rho-ku;\lambda_1,\ldots,\lambda_r);(1-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2};\lambda_1,\ldots,\lambda_r)$$

$$(\frac{\nu}{2}-\frac{\delta}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r);(c_j^{(1)},C_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \quad \ldots (3.2)$$

$$I'_2 = (d_j^{(1)},D_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r};(1-\rho-ku+\frac{\nu}{2}-\frac{\delta}{2};\lambda_1,\ldots,\lambda_r);$$

$$(\frac{\nu}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r);(\frac{1}{2}+\frac{\delta}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r) \quad \ldots (3.3)$$

$$J'_1 = (1-\rho-ku;\lambda_1,\ldots,\lambda_r);(1-\rho-ku+\frac{\nu}{2}+\frac{\delta}{2};\lambda_1,\ldots,\lambda_r)$$

$$(\frac{\delta}{2}-\frac{\nu}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r);(c_j^{(1)},C_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \quad \ldots (3.4)$$

$$J'_2 = (d_j^{(1)},D_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},D_j^{(r)})_{1,q_r};(1-\rho-ku-\frac{\nu}{2}+\frac{\delta}{2};$$

$$\lambda_1,\ldots,\lambda_r);(\frac{\delta}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r);(\frac{1}{2}+\frac{\nu}{2}-\rho-ku;\lambda_1,\ldots,\lambda_r)\ldots (3.5)$$

The (sufficient) conditions of validity of (3.1) are given below :

(i) The conditions (2.7),(2.8) and (2.9) hold.

(ii) $\Omega_i^* > 0$, $|\arg z_i| < \frac{1}{2}\pi\,\Omega_i^*$; $\forall\, i \in \{1, 2, \ldots, r\}$.

where

$$\Omega_i^* = \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{i=1}^{n_i} C_j^{(i)} - \sum_{i=n_i+1}^{p_i} C_j^{(1)} > 0 \quad \ldots (3.6)$$

Similarly from the remaining integrals, integrals involving product of r, H-functions can also be derived but for the sake of brevity,they are not presented here. A detailed account of the H- function is available from the monograph of Mathai and Saxena [7].

(2) If we take $r = 1$; $\beta = 0$ in (3.1) it gives

$$\int_0^1 t^{\rho-1}(1-t)^{\rho}\,[1+at+b(1-t)]^{-2\rho-1}\,{}_2F_1[\nu,\delta;\tfrac{1}{2}(\nu+\delta+2);$$

$$\frac{t(1+a)}{1+at+b(1-t)}]\,H_{p_1,q_1}^{m_1,n_1}\left[z_1R_1^1\,\middle|\,\begin{matrix}(c_j,C_j)_{1,p_1}\\(d_j,D_j)_{1,q_1}\end{matrix}\right]dt$$

$$= \frac{2^{\nu+\delta-2\rho-1}\,\Gamma(\frac{\nu}{2}+\frac{\delta}{2}+1)}{(\nu-\delta)(1+a)^{\rho}(1+b)^{\rho+1}\,\Gamma(\nu)\,\Gamma(\delta)}\,\{\Gamma(\frac{\nu}{2}+\frac{1}{2})\,\Gamma(\frac{\delta}{2})$$

$$. H^{m_1, n_1+3}_{p_1+3, q_1+3}\left[z_1\left|\begin{matrix} I_1'' \\ I_2'' \end{matrix}\right.\right] - \Gamma(\tfrac{\nu}{2})\,\Gamma(\tfrac{\delta}{2}+\tfrac{1}{2})\, H^{m_1, n_1^{+3}}_{p_1+3, q_1+3}\left[z_1\left|\begin{matrix} J_1'' \\ J_2'' \end{matrix}\right.\right]. \quad \ldots (3.7)$$

where

$$R_1 = \frac{4(t+at)(1+b)(1-t)}{[1+at+b(1-t)]^2}$$

$$I_1'' = (1-\rho, \lambda_1); (1-\rho+\frac{\nu}{2}+\frac{\delta}{2}; \lambda_1); (\frac{\nu}{2}-\frac{\delta}{2}-\rho, \lambda_1)(c_j, C_j)_{1, p_1} \quad \ldots. (3.8)$$

$$I_2'' = (d_j, D_j)_{1, q_1}; (1-\rho+\frac{\nu}{2}-\frac{\delta}{2}, \lambda_1); (\frac{\nu}{2}-\rho, \lambda_1); (\frac{1}{2}+\frac{\delta}{2}-\rho, \lambda_1) \quad \ldots (3.9)$$

$$J_1'' = (1-\rho, \lambda_1), (1-\rho+\frac{\nu}{2}+\frac{\delta}{2}; \lambda_1); (\frac{\delta}{2}, -\frac{\nu}{2}-\rho, \lambda_1)(c_j, C_j)_{1, p_1} \quad \ldots. (3.10)$$

$$J_2'' = (d_j, D_j)_{1, q_1}; (1-\rho-\frac{\nu}{2}+\frac{\delta}{2}, \lambda_1); (\frac{\delta}{2}-\rho, \lambda_1); (\frac{1}{2}+\frac{\nu}{2}-\rho, \lambda_1) \quad \ldots (3.11)$$

The (sufficient) conditions of validity of (3.7) are given below:

(i) The constants a and b are such that none of the expressions $1+a, 1+b, 1+at+b(1-t)$ where $0 \le t \le 1$ is zero.

(*ii*) $\mathrm{Re}(\rho) > 0, \mathrm{Re}(2\rho - \nu - \delta) > 0.$

(*iii*) $\mathrm{Re}(\rho + \lambda_1 \min_{1 \le j \le m_1} [\mathrm{Re}(d_j/D_j]) > 0; \lambda_1 \ge 0.$

(*iv*) $\Omega > 0, \quad |\arg z_1) < \frac{1}{2}\pi\Omega.$

where $\Omega = \sum_{j=1}^{n_1} C_j - \sum_{j=n_1+1}^{p_1} C_j + \sum_{j=1}^{m_1} D_j - \sum_{j=m_1+1}^{q_1} D_j > 0.$

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