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MULTIDIMENSIONAL FRACTIONAL DERIVATIVES OF THE MULTIPLE HYPERGEOMETRIC FUNCTIONS OF SEVERAL VARIABLES

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ABSTRACT

In the present paper, multidimensional fractional derivatives of the multiple hypergeometric functions of several variables have been obtained.

1. INTRODUCTION

Motivated by the earlier work of Srivastava and Goyal [11], Srivastava, Chandel and Vishwarkarma [12] derived various fractional derivatives involving the generalized multiple hypergeometric function of Srivastava and Daoust [10] and the multivariable H-function of Srivastava and Panda [13]. They also discussed the special cases of these results involving the multiple hypergeometric functions of several variables and their confluent forms defined by Lauricella [8], Exton [5],[6]), Chandel [1], Chandel and Gupta [2], and Karlsson [7].

In the present paper, we apply the same techniques in order to derive the **multidimensional** fractional derivatives involving the multiple hypergeometric functions $F_A^{(n)}, F_B^{(n)}, F_C^{(n)}$ and $F_D^{(n)}$ of Lauricella [8] and their confluent forms $\Xi_1^{(n)}, \Phi_2^{(n)}, \Psi_2^{(n)}, \Phi_3^{(n)}, {}^{(k)}_{(1)}E_D^{(n)}, {}^{(k)}_{(2)}E_D^{(n)}$, of Exton ([5], [6]) ${}^{(k)}_{(1)}E_C^{(n)}$ of Chandel [1], ${}^{(k)}F_{AC}^{(n)}, {}^{(k)}F_{BD}^{(n)}, {}^{(k)}F_{AD}^{(n)}$ of Chandel and Gupta [2], and their confluent forms ${}^{(k)}_{(1)}\Phi_{AC}^{(n)}, {}^{(k)}_{(2)}\Phi_{AC}^{(n)}, {}^{(k)}_{(1)}\Phi_{AD}^{(n)}, {}^{(k)}_{(1)}\Phi_{BD}^{(n)}$ and ${}^{(k)}F_{CD}^{(n)}$ of Karlsson [7].

2. FRACTIONAL DERIVATIVES

Making an appeal to the formula [9,p.67]

$$(2.1)\qquad D_x^{\mu}\{x^{\lambda}\} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)}\, x^{\lambda-\mu},\ \ \mathrm{Re}(\lambda) > -1,$$

we derive the folloing fractional derivatives involving the above multiple hypergeometric functions :

$$(2.2)\qquad D_{x_1}^{\lambda_1-\mu_1} \dots D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ F_A^{(n)}[a, \mu_1, \dots, \mu_n;\right.$$

$$c_1, \ldots, c_n; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_A^{(n)} [a, \lambda_1, \ldots, \lambda_n; c_1, \ldots, c_n; z_1x_1, \ldots z_nx_n],$$

$$|z_1x_1| + \ldots + |z_nx_n| < 1, \ \text{Re}(\lambda_i) > 0, \ \ i = 1, \ldots, n.$$

(2.3) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \Big\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} F_A^{(n)} [a, b_1, \ldots, b_n; \lambda_1, \ldots \lambda_n; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_A^{(n)} [a, b_1, \ldots, b_n; \mu_1, \ldots, \mu_n; z_1x_1, \ldots, z_nx_n],$$

$$|z_1x_1| \ 1 \ldots + |z_nx_n| < 1, \ \text{Re}(\lambda_i) > 0, \quad i = 1, \ldots, n.$$

(2.4) $$D_{x_1}^{\lambda_1 \ \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \Big\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} F_B^{(n)} [\mu_1, \ldots, \mu_n, b_1, \ldots, b_n; c; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{J=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_B^{(n)} [\lambda_1, \ldots, \lambda_n, b_1, \ldots, b_n; c; z_1x_1, \ldots, z_nx_n],$$

$$max \Big\{ |z_1x_1|, \ldots, |z_nx_n| \Big\} < 1, \ \text{Re}(\lambda_j) > 0, \ \ j = 1, \ldots, n.$$

(2.5) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \Big\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} . F_B^{(n)} [a_1, \ldots, a_n, \mu_1, \ldots, \mu_n; c; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_B^{(n)} [a_1, \ldots a_n, \ \lambda_1, \ldots, \lambda_n; c; z_1x_1, \ldots, z_nx_n],$$

$$max \Big\{ |z_1x_1|^{1/2}, \ldots, |z_nx_n|^{1/2} \Big\} < 1, \quad \text{Re}\,(\lambda_i) > 0, i = 1, \ldots, n.$$

(2.6) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \Big\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} F_C^{(n)} [a, b; \lambda_1, \ldots, \lambda_n; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_C^{(n)} [a, b; \mu_1, ..., \mu_n; z_1 x_1, ..., z_n x_n],$$

$$|z_1 x_1|^{1/2} + ... + |z_n x_n|^{1/2} < 1, \ \mathrm{Re}(\lambda_i) > 0, \ i = 1, ..., n.$$

(2.7) $$D_{x_1}^{\lambda_1 - \mu_1} ... D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} F_D^{(n)} [a, \mu_1, ..., \mu_n; c; z_1 x_1, ..., z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} F_D^{(n)} [a, \lambda_1, ... \lambda_n; c; z_1 x_1, ..., z_n x_n],$$

$$max \left\{ |z_1 x_1|, ..., |z_n x_n| \right\} < 1, \ \mathrm{Re}(\lambda_i) > 0, \ \ i = 1, ..., n.$$

(2.8) $$D_{x_1}^{\lambda_1 - \mu_1} ... D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j-1}^{n} x_j^{\lambda_j - 1} \, {}^{(k)}_{(1)}E_D^{(n)} [a, \lambda_1, ..., \lambda_n; c, c'; z_1 x_1, ..., z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}^{(k)}_{(i)}E_D^{(n)} [a, \lambda_1, ... \lambda_n; c, c'; z_1 x_1, ..., z_n x_n],$$

$$Re\ (\lambda_i) > 0, \ |z_i x_i| < r_i, \ \ i = 1, ..., n; \ \ r_1 = ... = r_k,$$

$$r_{k+1} = ... = r_n, r_k + r_n = 1.$$

(2.9) $$D_{x_1}^{\lambda_1 - \mu_1} ... D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \, {}^{(k)}_{(2)}E_D^{(n)} [a, a', \mu_1, ..., \mu_n \ c; z_1 x_1, ..., z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}^{(k)}_{(2)}E_D^{(n)} [a\ a', \lambda_1, ... \lambda_n; c; z_1 x_1, ..., z_n x_n],$$

$$Re\ (\lambda_i) > 0, \ |z_i x_i| < r_i, \ \ i = 1, ..., n; \ \ r_1 = ... = r_k,$$

$$r_{k+1} - ... r_n, r_k \cdot r_n - r_k + r_n.$$

(2.10) $$D_{x_1}^{\lambda_1 - \mu_1} ... D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \, {}^{(k)}_{(1)}E_C^{(n)} [a, a', \lambda_1, ..., \lambda_n; \ ; z_1 x_1, ..., z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; {}^{(k)}_{(1)}E_C^{(n)} [a\, a', b\, \mu_1, \ldots, \mu_n; z_1x_1, \ldots, z_nx_n],$$

$Re\ (\lambda_i) > 0, \ \ |z_ix_i| < r_i, \ \ i = 1, \ldots, n; \ \ (\sqrt{r_1} + \ldots + \sqrt{r_k})^2 + (\sqrt{r_{k+1}} + \ldots + \sqrt{r_n})^2 = 1.$

$$(2.11) \quad D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_1}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \; {}^{(k)}F_{AC}^{(n)} [a, b, b_{k+1}, \ldots, b_n; \lambda_1, \ldots, \lambda_n; z_1x_1, \ldots, z_nx_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; {}^{(k)}F_{AC}^{(n)} [a\ , b, b_{k+1}, \ldots, b_n\ ; \mu_1, \ldots \mu_n; z_1x_1, \ldots, z_nx_n],$$

$Re\ (\lambda_j) > 0, \ j = 1, \ldots, n.$

$$(2.12) \quad D_{x_{k+1}}^{\lambda_{k+1} - \mu_{k+1}} \ldots D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=k+1}^{n} x_j^{\lambda_j - 1} \; {}^{(k)}F_{AC}^{(n)} [a, b, \mu_{k+1}, \ldots, \mu_n; c_1, \ldots, c_n; z_1, \ldots, z_k, z_{k+1}x_{k+1}, \ldots, z_nx_n] \right\}$$

$$= \prod_{j=k+1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; {}^{(k)}F_{AC}^{(n)} [a\ , b, \lambda_{k+1}, \ldots, \lambda_n\ ; c_1, \ldots, c_n; z_1, \ldots, z_k, z_{k+1}\, x_{n+1}, \ldots, z_nx_n],$$

$Re\ (\lambda_i) > 0, \ i = k + 1, \ldots, n\ ;$

$(|z_1|^{1/2} + \ldots + |z_k|^{1/2})^2 + |z_{k+1}x_{k+1}| + \ldots + |z_nx_n| < 1.$

$$(2.13) \quad D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \; {}^{(k)}F_{AD}^{(n)} [a, \lambda_1, \ldots, \lambda_n; c\ ; c_{k+1}, \ldots, c_n\ ; z_1x_1, \ldots, z_nx_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; {}^{(k)}F_{AD}^{(n)} [a\ , \lambda_1, \ldots, \lambda_n\ ; c; c_{k+1}, \ldots, c_n; z_1x_1, \ldots, z_nx_n],$$

$Re\ (\lambda_i) > 0, \ i = 1, \ldots, n.$

$$\max\{|z_1x_1|, ..., |z_kx_k|\} + |z_{k+1}x_{k+1} + ... + |z_nx_n| < 1$$

(2.14) $$D_{x_{k+1}}^{\lambda_{k+1}-\mu_{k+1}} ... D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=k+1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}F_{AD}^{(n)}[a, b_1, ..., b_n; c;\right.$$

$$\left.\lambda_{k+1}, ..., \lambda_n; z_1, ..., z_k, z_{k+1}x_{k+1}, ..., z_nx_n]\right\}$$

$$= \prod_{j=k+1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j-1}\ {}^{(k)}F_{AD}^{(n)}[a, b_1, ..., b_n; c, \mu_{k+1}, ..., \mu_n; z_1, ..., z_k,$$

$$z_{k+1}x_{k+1}, ..., z_nx_n],$$

$$\max\{|z_1|, ..., |z_k|\} + |z_{k+1}x_{k+1}| + ... + |z_nx_n| < 1,$$

$Re(\lambda_i) > 0,\ i = 1, ..., n.$

(2.15) $$D_{x_{k+1}}^{\lambda_{k+1}-\mu_{k+1}} ... D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=k+1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}F_{BD}^{(n)}[a, \mu_{k+1}, ..., \mu_n, b_1,\right.$$

$$\left. ..., b_n; c; z_1, ..., z_k, z_{k+1}x_{k+1}, ..., z_nx_n]\right\}$$

$$= \prod_{j=k+1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j-1}\ {}^{(k)}F_{BD}^{(n)}[a, \lambda_{k+1}, ..., \lambda_n, b_1, ..., b_n; c; z_1, ..., z_k,$$

$$z_{k+1}x_{k+1}, ..., z_nx_n],$$

$$\max\{|z_1|, ..., |z_k|, |z_{k+1}x_{k+1}|, ..., |z_nx_n|\} < 1,$$

$Re(\lambda_i) > 0,\ i = k+1, ..., n.$

(2.16) $$D_{x_1}^{\lambda_1-\mu_1} ... D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}F_{BD}^{(n)}[a, a_{k+1}, ..., a_n,\right.$$

$$\left.\mu_1, ..., \mu_n; c; z_1x_1, ..., z_nx_n]\right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j-1}\ {}^{(k)}F_{BD}^{(n)}[a, a_{k+1}, ..., a_n, \lambda_1, ..., \lambda_n; c; z_1x_1, ..., z_nx_n],$$

$$max\ \{|z_1x_1|, ..., |z_nx_n|\} < 1, \qquad Re(\lambda_i) > 0,\ i = 1, ..., n.$$

(2.17) $$D_{x_1}^{\lambda_1-\mu_1} ... D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}F_{CD}^{(n)}[a, b, \mu_1, ..., \mu_k; c,\right.$$

$$\left.\lambda_{k+1}, \ldots, \lambda_n; z_1x_1, \ldots, z_nx_n]\right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; {}^{(k)}F_{CD}^{(n)} [a, b, \lambda_1, \ldots, \lambda_k; c, \mu_{k+1}, \ldots, \mu_n; z_1x_1, \ldots, z_nx_n],$$

$Re\,(\lambda_i) > 0,\ i = 1, \ldots, n.$

(2.18) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_n}^{\lambda_n - \mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \; \Xi_1^{(n)} [\mu_1, \ldots, \mu_n, b_1, \ldots, b_{n-1}; \right.$$

$$\left. c; z_1x_1, \ldots, z_nx_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; \Xi_1^{(n)} [\lambda_1, \ldots, \lambda_n, b_1, \ldots, b_{n-1}; c; z_1x_1, \ldots, z_nx_n],$$

$Re\,(\lambda_i) > 0,\ i - 1, \ldots, n.$

(2.19) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_{n-1}}^{\lambda_{n-1} - \mu_{n-1}} \left\{ \prod_{j=1}^{n-1} x_j^{\lambda_j - 1} \; \Xi_1^{(n)} [a_1, \ldots, a_n, \mu_1, \ldots, \mu_{n-1}; \right.$$

$$\left. c; z_1x_1, \ldots, z_{n-1}x_{n-1}, z_n] \right\}$$

$$= \prod_{j=1}^{n-1} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; \Xi_1^{(n)} [a_1, \ldots, a_n, \lambda_1, \ldots, \lambda_{n-1}; c;$$

$$z_1x_1, \ldots, z_{n-1}x_{n-1}, z_n],$$

$Re\,(\lambda_i) > 0,\ i = 1, \ldots, n - 1.$

(2.20) $$D_{x_1}^{\lambda_1 - \mu_1} \ldots D_{x_{n-1}}^{\lambda_{n-1} - \mu_{n-1}} \left\{ \prod_{j=1}^{n-1} x_j^{\lambda_j - 1} \; \Phi_3^{(n)} [\mu_1, \ldots, \mu_{n-1}; \right.$$

$$\left. c; z_1x_1, \ldots z_{n-1}x_{n-1}, z_n] \right\}$$

$$= \prod_{j=1}^{n-1} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \; \Phi_3^{(n)} [\lambda_1, \ldots, \lambda_{n-1}; c; \quad z_1x_1, \ldots, z_{n-1}x_{n-1}, z_n],$$

$Re\,(\lambda_i) > 0,\ i = 1, \ldots, n - 1.$

(2.21) $$D_{x_1}^{\lambda_1-\mu_1}\dots D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ \Psi_2^{(n)}[a,\lambda_1,\dots,\lambda_n;z_1x_1,\dots,z_nx_n]\right\}$$
$$=\prod_{j=1}^{n}\frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)}\,x_j^{\mu_j-1}\ \Psi_2^{(n)}[a,\mu_1,\dots,\mu_n;z_1x_1,\dots,z_nx_n].$$
$Re\,(\lambda_i)>0,\ i=1,\dots,n.$

(2.22) $$D_{x_1}^{\lambda_1-\mu_1}\dots D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ \Phi_2^{(n)}[\mu_1,\dots,\mu_n;c;z_1x_1,\dots,z_nx_n]\right\}$$
$$=\prod_{j=1}^{n}\frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)}\,x_j^{\mu_j-1}\ \Phi_2^{(n)}[\lambda_1,\dots,\lambda_n;c;z_1x_1,\dots,z_nx_n],$$
$Re\,(\lambda_i)>0,\ i=1,\dots,n.$

(2.23) $$D_{x_1}^{\lambda_1-\mu_1}\dots D_{x_{n-1}}^{\lambda_{n-1}-\mu_{n-1}}\left\{\prod_{j=1}^{n-1} x_j^{\lambda_j-1}\ \Phi_D^{(n)}[a,\mu_1,\dots,\mu_{n-1},-;\ c;z_1x_1,\dots,z_{n-1}x_{n-1},z_n]\right\}$$
$$=\prod_{j=1}^{n-1}\frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)}\,x_j^{\mu_j-1}\ \Phi_D^{(n)}[a,\lambda_1,\dots,\lambda_{n-1},-;c;z_1x_1,\ \dots,z_{n-1}x_{n-1},z_n],$$
$Re\,(\lambda_i)>0,\ i=1,\dots,n-1.$

(2.24) $$D_{x_1}^{\lambda_1-\mu_1}\dots D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}_{(1)}\Phi_{AC}^{(n)}[a,b;\lambda_1,\dots,\lambda_n;z_1x_1,\dots,z_nx_n]\right\}$$
$$=\prod_{j=1}^{n}\frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)}\,x_j^{\mu_j-1}\ {}^{(k)}_{(1)}\Phi_{AC}^{(n)}[a,b;\mu_1,\dots,\mu_n;z_1x_1,\dots,z_nx_n],$$
$Re\,(\lambda_i)>0,\ i=1,\dots,n.$

(2.25) $$D_{x_1}^{\lambda_1-\mu_1}\dots D_{x_n}^{\lambda_n-\mu_n}\left\{\prod_{j=1}^{n} x_j^{\lambda_j-1}\ {}^{(k)}_{(2)}\Phi_{AC}^{(n)}[a,b_{k+1},\dots,b_n;\lambda_1,\ \dots,\lambda_n;z_1x_1,\dots,z_nx_n]\right\}$$
$$=\prod_{j=1}^{n}\frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)}\,x_j^{\mu_j-1}\ {}^{(k)}_{(2)}\Phi_{AC}^{(n)}[a,b_{k+1},\dots,b_n;\mu_1,\dots,\mu_n;z_1x_1,\dots,z_nx_n],$$

$Re\,(\lambda_i) > 0,\ i = 1, \ldots, n.$

(2.26) $$D_{x_{k+1}}^{\lambda_{k+1}-\mu_{k+1}} \ldots D_{x_n}^{\lambda_n-\mu_n} \left\{ \prod_{j=k+1}^{n} x_j^{\lambda_j - 1} \, {}_{(2)}^{(k)}\Phi_{AC}^{(n)} [a, \mu_{k+1}, \ldots, \mu_n; c_1, \ldots, c_n; z_1, \ldots, z_k, z_{k+1}x_{k+1}, \ldots, z_n x_n] \right\}$$

$$= \prod_{j=k+1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}_{(2)}^{(k)}\Phi_{AC}^{(n)} [a, \lambda_{k+1}, \ldots, \lambda_n; c_1, \ldots, c_n; z_1, \ldots z_k, z_{k+1}z_{k+1}, \ldots, z_n x_n],$$

$Re\,(\lambda_i) > 0,\ i = k+1, \ldots, n.$

(2.27) $$D_{x_1}^{\lambda_1-\mu_1} \ldots D_{x_n}^{\lambda_n-\mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \, {}_{(1)}^{(k)}\Phi_{AD}^{(n)} [a, \mu_1, \ldots, \mu_n; c; z_1x_1, \ldots, z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}_{(1)}^{(k)}\Phi_{AD}^{(n)} [a, \lambda_1, \ldots, \lambda_n; c; z_1x_1, \ldots, z_n x_n],$$

$\mathrm{Re}\,(\lambda_i) > 0,\ i = 1, \ldots, n.$

(2.28) $$D_{x_1}^{\lambda_1-\mu_1} \ldots D_{x_n}^{\lambda_n-\mu_n} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \, {}_{(1)}^{(k)}\Phi_{BD}^{(n)} [a, \mu_1, \ldots, \mu_n; c; z_1x_1, \ldots, z_n x_n] \right\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}_{(1)}^{(k)}\Phi_{BD}^{(n)} [a, \lambda_1, \ldots, \lambda_n; c; z_1x_1, \ldots, z_n x_n],$$

$Re\,(\lambda_i) > 0,\ i = 1, \ldots, n.$

(2.29) $$D_{x_{k+1}}^{\lambda_{k+1}-\mu_{k+1}} \ldots D_{x_n}^{\lambda_n-\mu_n} \left\{ \prod_{j=k+1}^{n} x_j^{\lambda_j - 1} \, {}_{(2)}^{(k)}\Phi_{BD}^{(n)} [a, \mu_{k+1}, \ldots, \mu_n, , b_1, \ldots, b_n; c; z_1, \ldots, z_k, z_{k+1}x_{k+1}, \ldots, z_n x_n] \right\}$$

$$= \prod_{j=k+1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}_{(2)}^{(k)}\Phi_{BD}^{(n)} [a, \lambda_{k+1}, \ldots, \lambda_n, b_1, \ldots, b_n; c; z_1, \ldots, z_k, z_{k+1}x_{k+1}, \ldots, z_n x_n],$$

$Re\,(\lambda_i) > 0,\ i = k+1, \ldots, n.$

(2.30) $$D_{x_1}^{\lambda_1-\mu_1} \ldots D_{x_n}^{\lambda_{n-1}-\mu_{n-1}} \left\{ \prod_{j=1}^{n} x_j^{\lambda_j - 1} \, {}_{(2)}^{(k)}\Phi_{BD}^{(n)} [a, a_{k+1}, \ldots, u_n, \right.$$

$$\mu_1, \ldots, \mu_n; c; z_1x_1, \ldots, z_nx_n]\Big\}$$

$$= \prod_{j=1}^{n} \frac{\Gamma(\lambda_j)}{\Gamma(\mu_j)} x_j^{\mu_j - 1} \, {}^{(k)}_{(2)}\Phi^{(n)}_{BD} [a, a_{k+1}, \ldots, a_n, \lambda_1, \ldots, \lambda_n;$$

$$c; z_1x_1, \ldots, z_nx_n],$$

$Re\ (\lambda_i) > 0,\ i = 1, \ldots, n.$

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