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G/G/1/N QUEUEING MODEL WITH LCFS-P/R SERVICE POLICY

By

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ABSTRACT

This paper is devoted to the G/G/1/N queueing model in which customers are served under last-come-first-served (LCFS) queue discipline and arbitrary restarting service policy (R). The input of the is stationary with rate λ. The service time of customers depends on the queue size (i.e. number of customers present in the system). For steady-state, queue size distribution has been obtained explicitly.

1.INTRODUCTION. Queueing system with last come-first served (LCFS) service discipline plays vital role in practice however in a limited area, for example : stocks are often refilled but also worked off at the top. In LCFS P service discipline an arriving customer gets service immediately and a pre-empted customer restarts service when all customers arriving after him leave the system. Some authors have made efforts to tackle the queueing problems having LCFS service discipline. Kelly [5] introduced LCFS-P/R queue discipline for M/G/1 system. Yamazaki [8] analysed the G/G/1 queue with LCFS service discipline and later on, he [9] obtained invariance relations for GI/G/1 queue with LCFS and preemptive resume restarting service policy. Santhikumar and Sumita [7] discussed GI/GI/1 model 1 by imposing LCFS-preemptive restarting policy and proved that the stationary system queue length distribution jsut after a departure instant is geometric. Fakinos [2] obtained the expressions for GI/GI/1 queue with LCFS- P/resume and service time distribution depending on the system queue length. Yamazaki [10] genralized it for GI/GI/1/K system with LCFS-P/H service discipline where H is a restarting plicy which may depend on the history of premption of the restarting customers. Miyazawa [6] extended the work of Yamazaki [10] by imposing simple rejection rule but without any restriction (i.e. arriving customers are accepted without depending on queue size). Fakios [3, 4] obtained some useful results for G/G/1 model under the last-come-first served service discipline when the customers where drawn from infinite population and interarrival times and service times under stationary stochastic processes. Dijk [1] studied a finite

LCFS buffer queueing model with batch input and non-exponential service.

In may practical applications, the customers are often drawn from finite population for example; telecommunication, computer and manufacturing systems etc. The present day demands motivate us to investigate a G/G/1/N model under LCFS/P service discipline For solution purpose the system is considered at arrival epochs and in continuous time. The steady state queue size distribution has been obtained in section 2. Section 3 deals with special case. In the last section 4, the conclusion has been drawn.

2. THE MODEL To describe our model we use the following notations :

$j =$	Numebr of custormers waiting behind each moving customer from position $n - 1$ to position n.
$b =$	Mean value of random variable x_n,
$c =$	Mean value of the busy period.
$\sigma =$	$1 - b/c$.
$N =$	Total number of customers present in the system.
$Q_K =$	Queue size immediately before Kth arrival epocks.
$Q(t) =$	Queue size at time t.
$Q(n) =$	Number of customer in queue n.
$r_n, p_n =$	Limiting probabilities distributions of queue size.
$r_n, p_n =$	Tails probabilities of r_n and p_n respectively.
$b^* =$	Mean actual service time at conditional probability.
$\bar{b}^* =$	Mean service time at unconditional probability.
$s_n =$	Mean system time at conditional probability.
$\bar{s}_n =$	Mean system time unconditional probability.
$w_n =$	Mean waiting time for type n customers.
$B_n(x) =$	Distribution function of service time.

The stochastic processes $\{Q(t) : t \geq 0\}$ and $\{Q_K : K = 1, 2, \dots, N\}$ have limiting probability distributions :

$$p_n = \lim_{t \to \infty} p\{Q(t) = n\}$$

and $$r_n = \lim_{K \to \infty} p\{Q(n) = n\}, (n = 0, 1, 2, \dots, N\}.$$

The corresponding tail probabilities are

$$\hat{p}_n = \sum_{i=n}^{N} p_i \qquad \dots (1)$$

$$r_n = \sum_{i=n}^{N} r_i \qquad \dots (2)$$

Under LCFS/P discipline the relation between limiting probability distributions p_n and r_n are given by (Yamazaki [10])

$$p_n = \rho_n r_{n-1}, \; n = 1, 2, \dots, N \qquad \dots (3)$$

where $\rho_n = \lambda b_n$ is traffic intensity.

and

$$r_n = \prod_{j=1}^{n} \sigma_j \Big/ \sum_{K=0}^{N} \prod_{j=1}^{K} \sigma_j \qquad \dots (4)$$

We assume that in equilibrium the arrival processes for queue n is a stationary point process with rate $\lambda_n = \lambda \hat{r}_{n-1}$. The mean service time ($\bar{b}_n^*$) and mean system time ($\bar{s}_n$) are given respectively by

$$\bar{b}_n^* = \sum_{j=0}^{N-n} b_{n+j}^* \frac{r_{n-1+j}}{\hat{r}_{n-1}} \qquad \dots (5)$$

$$\bar{s}_n = \sum_{j=0}^{N-n} s_{n+j} \frac{r_{n-1+j}}{\hat{r}_{n-1}} \qquad \dots (6)$$

In equilibrium for non-empty queue n

$$p(Q(n) > 0) = \lambda_n \bar{b}_n^* \qquad \dots (7)$$

so that (5) implies

$$p\{Q(1) \geq n\} = \lambda \hat{r}_{n-1} \sum_{j=0}^{N-n} b_{n+j}^* \frac{r_{n-1+j}}{\hat{r}_{n-1}}$$

Also $$\hat{p}_n = \sum_{j=0}^{N-n} \rho_{n+j}^* \, r_{n-1+j}, \qquad \dots (8)$$

$(n = 1, 2, \dots, N)$

We note that

$$p_n = \hat{p}_n - \hat{p}_{n+1}$$
$$= \rho_n^* \, r_{n-1} \qquad (n = 1, 2, \dots, N) \qquad \dots (9)$$

where $\rho_n^* = \lambda b_n^*$ is new traffic intensity.

Using Little's result to queue n, we get

$$E\,[Q\,(n)] = \lambda_n \bar{s}_n, \qquad \dots (10)$$

Also

$$E[Q(n)] = \sum_{k=1}^{N-n} p \quad \{Q(n) \geq K\}$$

$$= \sum_{K=1}^{N-n} p\{Q(1) \geq n + K - 1\}$$

$$= \sum_{K=1}^{N-n} \hat{p}_{n+K-1} \qquad \dots (11)$$

From equation (6) and (11), equation (10) yields

$$\sum_{j=0}^{N-n} \hat{p}_{n+j} = \lambda \sum_{j=0}^{N-n} s_{n+j} \quad r_{n-i+j} \qquad \dots (12)$$

so that

$$\hat{p}_n = s_n \, r_{n-1} \qquad \dots (13)$$

and

$$\hat{p}_n - \hat{p}_{n+1} = \lambda \, (s_n \, r_{n-1} - s_{n+1} \, r_n) \qquad \dots (14)$$

Now from equations (9) and (14), we get

$$r_n = \frac{(s_n - b_n^*)}{s_{n+1}} r_n \quad 1 \qquad \dots (15)$$

Denoting

$$\sigma_n = \frac{s_n - b_n^+}{s_{n+1}} = \frac{w_n}{s_{n+1}}, \quad (n = 1, 2, \dots, N) \qquad \dots (16)$$

Equation (15) reduces to

$$r_n = \sigma_n \, r_{n-1} = \left(\frac{w_n}{s_{n+1}} \right) r_{n-1}, \; (n = 1, 2, \dots, N) \dots (17)$$

From normalizing condition

$\sum_{K=0}^{N} r_K = 1$ and equation (17), we get

$$r_0 \sum_{K=0}^{N} \prod_{j=1}^{K} \sigma_j = 1 \qquad \dots (18)$$

Thus limiting probability distribution r_n is given by

$$r_n = \prod_{j=1}^{n} \sigma_j \Big/ \sum_{K=0}^{N} \prod_{j=1}^{K} \sigma_j, \qquad (n = 0, 1, 2, \dots, N) \qquad \dots (19)$$

Also equation (9) becomes

$$p_n = \rho_n^* \prod_{j=1}^{n-1} \sigma_j \Big/ \sum_{k=0}^{n} \prod_{j=1}^{k} \sigma_j, \quad (n = 0, 1, \dots, N) \qquad \dots (20)$$

The limiting probability when there is no customer in the system is given by

$$p_0 = 1 - \sum_{n=1}^{N} \rho_n^* \prod_{j=1}^{n-1} \sigma_j \Big/ \sum_{K=0}^{N} \prod_{j=1}^{K} \sigma_j \qquad \text{.... (21)}$$

3. SPECIAL CASE

In the special case when service requirement is independent of queue size, $B_n(x) = B(x)$, $b_n^* = b^*$, $s_n = s$, $\sigma_n = \sigma$ and $\rho_n^* = \rho^*$, $(n = 1, 2, ..., N)$, we have

$$p_0 = \frac{1 - \rho^*}{1 - p^{*N+1}} \qquad \text{... (22)}$$

$$r_0 = \frac{1 - \sigma}{1 - \sigma^{N+1}} \qquad \text{... (23)}$$

where $= 1 - b/c$

so that

$$\hat{r}_n = (1 - b^*/c^*)\ \hat{r}_{n-1} \qquad \text{... (24)}$$

and

$$r_n = r_0 (1 - b^*/c^*)^n\ b^*/c^* \qquad \text{... (25)}$$

From equation (9), we get

$$P_n = \frac{(1 - \sigma)}{1 - \sigma^{N+1}} \rho^*\ \sigma^{n-1}\ (n = 1, 2, ..., N) \qquad \text{... (26)}$$

4. CONCLUSION

The steady state queue size distribution obtained explicitly for considered G/G/1/N model is of great utilizaton due to its applications in finite buffer situations in industrial problems. The last-come first-served queue discipline with preemption and arbitrary restarting policy are common in inventory and production systems.

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