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(Dedicated to Professor J.N. Kapur on his 70th Birthday)

FREE CONVECTIVE MHD FLOW OF A DUSTY VISCOELASTIC LIQUID IN POROUS MEDIUM PAST AN OSCILLATING INFINITE POROUS PLATE

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ABSTRACT

Free convective laminar flow of an electrically conducting incompressible dusty viscoelastic liquid (Walters liquid model-B) embedded with non-conducting dust particles through a porous medium past an infinite porous plate subject to a slightly sinusoidal transverse velocity distribution has been analysed. The mathematical analysis is presented for the hydromagnetic boundary layer flow without taking into account the induced magnetic field. Expressions for velocity field, temperature distribution, skin- friction and heat transfer have been obtained and discussed with the help of graphs.

1. INTRODUCTION. The study of the flow through porous media is of principal interest due to its applications in geophysics in the assessment of geothermal resources, in design of under ground energy storage systems, oil recovery, soil sciences, astrophysics, nuclear power reactor engineering and so on. Lighthill [1] has studied laminar skin-friction and heat transfer to fluctuating in stream velocity. Stability laminar flow of a dusty gas has been studied by Saffman [5]. Mukherjee and Mukherjee [3] have investigated a problem on unsteady flow of dusty viscous fluid due to time dependent tangential stress applied at the surface. Free convection effect on the Stoke's problem for an infinite vertical plate in a dusty fluid has been discussed by Ramamurthy [4]. Mishra [2] has investigated a problem on MHD flow of an oscillating plate in absence of pressure gradient. Sharma and Sharma [6] have studied free convection non-Newtonian flow past an infinite plate with suction and constant heat flux. Unsteady free convection MHD dusty flow between heated porous vertical plates has been studied by Subeydar [10]. Recently, Singh and Rana [7] have discussed three dimensional flow and heat transfer through a porous medium. More, recently Singh et al [8] have investigated heat transfer in three dimensional MHD flow past a porous plate.

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The object of the present paper is to analyse the effect of free convective laminar flow of an electrically conducting incompressible dusty viscoelastic liquid (Walters liquid model-B) embedded with non-conducting dust particles through a porous medium past an infinite porous plate subject to a slightly sinusoidal transverse velocity distribution. Velocity field, temperature distribution, skin-friction and heat transfer have been obtained and discussed with the help of graphs.

2. FORMULATION OF THE PROBLEM. Let us consider the laminar flow of an electrically conducting, incompressible, dusty, viscoelastic (Walters liquid model-B) liquid in a porous medium past an infinite porous oscillating flat plate. The plate executes oscillations in its own plane with frequency n decreasing exponentially with time. Let x-axis is along the flow of liquid in the plane of the plate and y-axis perpendicular to it. The dust particles are assumed to be solid, spherical, non-conducting and symmetrically distributed in the flow region. A constant velocity v_0 ($v < 0$ for suction and $a > 0$ for injection) normal to the plate is applied.

A uniform magnetic field B_0 is applied perpendicular to the flow region. Since the liquid is viscoelastic, the induced magnetic field has been neglected in comparison with the applied magnetic field. It is assumed that the temperature of the plate remains steady at T_W and free stream temperature at T. Then the governing boundary layer equations for a two-dimensional unsteady, laminar, incompressible dusty viscoelastic liquid (Walters liquid model-B) can be expressed as

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g_x \beta (T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 u - \frac{\nu}{k} u + \frac{K_1 N_0}{\rho} (v - u) - k \left(\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\partial^3 u}{\partial y^3} \right)$$

$$m \frac{\partial v}{\partial t} = K_1 (u - v) \quad \text{... (2.2)}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad \text{... (2.3)}$$

The boundary conditions are :

$$u = v = v_0 (1 + \epsilon e^{-nt}), \quad T = T_W (1 + \epsilon e^{-nt}) \text{ at } y = 0 \quad \text{... (2.4)}$$

$$u = v \Rightarrow v_0, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty$$

where u is the velocity of the liquid, v is the velocity of the dust particles, c_p is the specific heat at constant pressure of the liquid, β is the coefficient of volume expansion, g_x is the acceleration due to gravity, ν is the kinematic viscosity of the liquid, K is the elastic constant, ρ is the density of the liquid, t is the time, λ is the thermal conductivity of the liquid, T is the temperature in boundary layer, B_0 is the induction of magnetic field, k is the permeability of the medium, T_∞ is the temperature of the liquid for away from the plate, σ is the electrical conductivity of the liquid, m is the mass of dust particules, N_0 is the number density of dust particles and v_0 is the suction velocity.

We introduce the following non-dimensional quantities -

$$y^* = \frac{yv_0}{\nu}, \quad u^* = \frac{u}{v_0}, \quad v^* = \frac{v}{v_0}, \quad t^* = \frac{v_0^2}{4\nu},$$

$$K^* = \frac{Kv_0^2}{\nu^2}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad \frac{\mu^c p}{\lambda} = p \text{ (Prandtl number)},$$

$$\frac{\nu^2}{v_0^2 k} = \frac{1}{k^*} \text{ (Permeability parameter)},$$

$$\frac{\sigma\nu B_0^2}{\rho v_0^2} = M \text{ (Hartmann number)},$$

$$\frac{\nu g_x (T_w - T_\infty)\beta}{v_0^3} = G \text{ (Grashoff number)}.$$

Using these non-dimensional quantities, the equations (2.1), (2.2) and (2.3) after ignoring the stars over them reduces to

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - (M + \frac{1}{k})u + \frac{l}{w}(v - u) - \frac{1}{4}\frac{\partial u}{\partial t} - K(\frac{1}{4}\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\partial^3 u}{\partial y^3}) = -GT \quad \text{.... (2.5)}$$

$$w\frac{\partial v}{\partial t} = u - v \quad \text{... (2.6)}$$

and

$$\frac{\partial^2 T}{\partial y^2} + p\frac{\partial T}{\partial y} - \frac{P}{4}\frac{\partial T}{\partial t} = 0 \quad \text{... (2.7)}$$

where $l = \frac{m N_0}{\rho}$ (mass concentration of dust particles), and $w = mv^2{}_0/4\nu K_1$.

The non-diemensional boundary conditions are

$$u = v = 1 + \epsilon\, e^{-nt}, \quad T = 1 + \epsilon\, e^{-nt} \text{ at } y = 0$$
$$u = v \to 1, \quad T \to 0 \text{ as } y \to \infty \quad \text{... (2.8)}$$

3. SOLUTION OF THE PROBLEM.

Following Lighthill [1], we assume

$$T(y, t) = [1 - f_1(y)] + \in e^{-nt} [1 - f_2(y)] \quad \text{... (3.1)}$$

$$u(y, t) = g(y) + \in e^{-nt} g_2(y) \quad \text{... (3.2)}$$

$$v(y, t) = F_1(y) + \in e^{-nt} F_2(y) \quad \text{... (3.3)}$$

Substituting (3.1) - (3.3) in the equations (2.6) and (2.7), after comparing the harmonic terms we get

$$f_1'' + pf_1' = 0 \quad \text{... (3.4)}$$

$$\frac{1}{P} f_2'' + f_2' + \frac{n}{4} f_2 = \frac{n}{4} \quad \text{... (3.5)}$$

$$g_1 = F_1 \quad \text{... (3.6)}$$

$$F_2 = \frac{g_2}{(1 - nw)}. \quad \text{... (3.7)}$$

Using relations (3.6), (3.7) and (3.1) - (3.3) in the equation (2.5), we obtain

$$K g_1''' + g_1'' + g_1' - (M + \frac{1}{k}) g_1 = (f_1 - 1) G \quad \text{... (3.8)}$$

$$Kg_2''' + (1 + \frac{Kn}{4}) g_2'' + g_2' - (M + \frac{1}{k} - \frac{n}{4} - \frac{ln}{1 - wn}) g_2 = (f_2 - 1) G \quad \text{... (3.9)}$$

The transformed boundary conditions are :

$$f_1 = f_2 = 0,\ g_1 = a_2 = 1,\ F_1 = F_2 = 1 \text{ at } y = 0 \quad \text{... (3.10)}$$

$$f_1 = f_2 \to 1,\ f = g_1 \to 1,\ g_2 = F_2 \Rightarrow 0 \text{ as } y \to \infty$$

The soluiton of equations (3.4) and (3.5) under the boundary condtions (3.10), we have

$$f_1 = 1 - \exp(-Py) \quad \text{... (3.11)}$$

$$f_2 = 1 - \exp(-PH_2 y) \quad \text{... (3.12)}$$

where $H_2 = \frac{1}{2}\left[1 + (1 - \frac{n}{p})^{1/2}\right]$

Following Soundalgekar [9], we assume

$$g_1 = g_{01} + Kg_{11} + 0(K^2) \quad \text{... (3.13)}$$

$$g_2 = g_{02} + Kg_{12} + 0(K^2) \quad \text{... (3.14)}$$

$$F_1 = F_{01} + KF_{11} + 0(K^2) \quad \text{... (3.15)}$$

$$F_2 = F_{02} + KF_{12} + 0(K^2) \quad \text{... (3.16)}$$

Substituting (3.13) - (3.16) and using (3.11) and (3.12) in the equations (3.6) - (3.9), we obtain

$$f_{01} = g_{01}, \quad f_{11} = g_{11} \qquad \text{... (3.17)}$$

$$F_{02} = \frac{g_{02}}{(1 - wn)} \qquad \text{... (3.18)}$$

$$f_{12} = \frac{g_{12}}{(1 - wn)} \qquad \text{... (3.19)}$$

$$g_{01}'' + g_{01}' - (M + \frac{1}{k}) g_{01} = - G e^{-py} \qquad \text{... (3.20)}$$

$$g_{11}'' + q_{11}' - (M + \frac{1}{k}) g_{11} = - g_{01}''' \qquad \text{... (3.21)}$$

$$g_{02}'' + q_{02}' - (M + \frac{1}{K} - \frac{n}{4} - \frac{ln}{(1 - wn)} g_{02} = - Ge^{-PH_2 y} \qquad \text{... (3.22)}$$

$$G_{12}'' + g_{12}' - (M + \frac{1}{k}\frac{n}{4} - \frac{ln}{(1 - 2n)}) g_{12} = - \frac{n}{4} g_{02}'' - g_{02} \qquad \text{... (3.23)}$$

The corresponding boundary conditions are

$$g_{01} = 1, \quad g_{11} = 0, \quad g_{02} = 1, \quad g_{12} = 0,$$

$$F_{01} = 1, \quad F_{11} = 0, \quad f_{02} = 1, \quad F_{12} = 0, \quad \text{at } y = 0 \qquad \text{... (3.24)}$$

$$g_{01} \to 1, \; F_{01} \to g_{11} = g_{02} = g_{12} = F_{02} = F_{11} = F_{12} \to 0$$

$$\text{as } y \to \infty.$$

On solving equations (3.17) to (3.23) under the boundary conditions (3.24), after substituting in the equations (3.13) - (3.16), we have

$$g_1 = 1 + Gb_1(e^{-H_4 y} - e^{-py}) + KGb_1^2 p^3(e^{-H_4 y} - e^{-Py}) \qquad \text{... (3.25)}$$

$$g_2 = (1 + Gb_3 + KGb_3 b_5)e^{-H_6 y} - b_3 G(1 + Kb_5)\, e^{-PH_2 y} \qquad \text{... (3.26)}$$

$$F_1 = 1 + Gb_1(e^{-H_4 y} - e^{-py}) + KGb_1^2 p^3(e^{-H_4 y} - e^{-py}) \qquad \text{... (3.27)}$$

$$F_2 = (1 + Gb_7 + KGb_9 b_3)\, e^{-H_6 y} - (Gb_7 + KGb_3 b_9)e^{-PH_2 y} \qquad \text{... (3.28)}$$

Hence on putting the values of g_1, g_2, F_1, F_2, f_1 and f_2 in the equations (3.1), (3.2) and (3.3) we obtain the velocity of the liquid, particle and temperature of the liquid

$$u = 1 + Gb_1(1 + Kb_1 p^3)\,(e^{-H_4 y} - e^{-py}) + \in (1 + Gb_3 + KGb_3 b_5)$$

$$e^{-(H_6 y + nt)} - \in Gb_3(1 + Kb_5)\, e^{-(PH_2 y + nt)} \qquad \text{... (3.29)}$$

$$v = 1 + Gb_1\,(1 + Kb_1 p^3)(e^{-H_4 y} - e^{-py}) + \in (1 + Gb_7 + KGb_3 b_9)$$

$$- e^{-(H_6 y + nt)} - \in G(b_7 + Kb_3 b_9)\, e^{-(PH_2 y + nt)} \qquad \text{... (3.30)}$$

$$T = e^{-py} + \in e^{-(PH_2 y + nt)} \qquad \text{... (3.31)}$$

where,

$$H_4 = \frac{1}{2}\left[1 + \left\{1 + 4\left(M + \frac{1}{K}\right)\right\}^{1/2}\right]$$

$$H_6 = \frac{1}{2}\left[1 + \left\{1 + 4\left(M + \frac{1}{K} - \frac{n}{4} - \frac{n}{1 - wn}\right)\right\}^{1/2}\right]$$

$$b_1 = \left[P(P-1) - \left(M + \frac{1}{K}\right)\right]^{-1}$$

$$b_3 = \left[PH_2(PH_2 - 1) - \left(M + \frac{1}{k} - \frac{n}{4} - \frac{ln}{1 - wn}\right)\right]^{-1}$$

$$b_5 = b_3 P^2 H_2^2 \left(PH_2 - \frac{n}{4}\right)$$

$$b_7 = b_3 \mid (1 - 2n)$$

$$b_9 = b_7 b_5.$$

4. SKIN-FRICTION AND HEAT TRANSFER.

Skin-friction τ_1 for the liquid is given by

$$\tau_1 = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= b_1 G(1 + Kb_1 p^3)(P - H_4) - \in \left|(1 + Gb_3 + Kb_3 b_5 G)H_6 - (Gb_3 + KGb_3 b_5) PH_2\right] e^{-nt} \qquad \text{... (4.1)}$$

Skin-friction τ_2 of the dust particles is given by

$$\tau_2 = \left(\frac{\partial v}{\partial y}\right)_{y=0}$$

$$\tau_2 = Gb_1 (1 + Kb_1 p^3)(P - H_4) - \in \left[(1 + b_7 G + KGb_3 b_9) H_6 - (b_7 + Kb_3 b_9) GPH_2\right] e^{-nt} \qquad \text{... (4.2)}$$

Heat transfer in terms of Nusselt number N_a is given by

$$N_a = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= P(1 + \in H_2) e^{-nt} \qquad \text{... (4.3)}$$

5. DISCUSSION. Velocity profiles of the liquid and dust particles for varius values of magnetic field ($M = 0.2$, 1.2 and 2.2) at $P = .71$, $G - 5.0$, $l = 0.3$, $w = 0.2$, $K = 1.0$, $\in = 0.4$, $t = 1.0$, $n = 0.1$ and $k = 0.25$ are shown in figure 1. From the figure it is obvious that the velocity of the liquid and particles decreases as the intensity of magnetic field M increases. We also observe that the velocity of the

FIG.1 VELOCITY PROFILES OF THE LIQUID AND DUST PARTICLES FOR VARIOUS VALUES OF MAGNETIC FIELD (HARTMANN NUMBER M)

(P = 0·71, G = 5·0, l = 0·3, w = 0·2, K = 1·0, ∈ = 0·4, t = 1·0, n = 0·1 and k = 0·25)

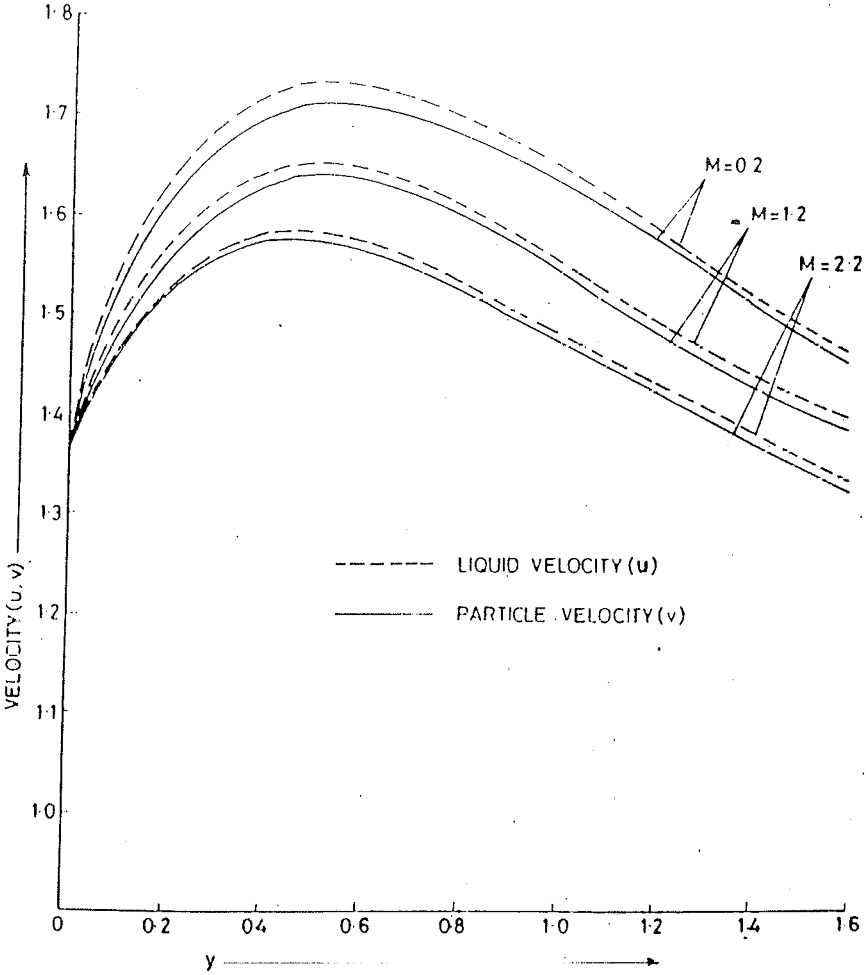


FIG.2 VELOCITY PROFILES OF THE LIQUID AND DUST PARTICLE FOR VARIOUS VALUES OF POROSITY PARAMETER k ($P=0.71$, $G=5.0$, $l=0.3$, $w=0.2$, $K=1.0$, $\epsilon=0.4$, $t=1.0$, $n=0.1$ and $M=1.2$)

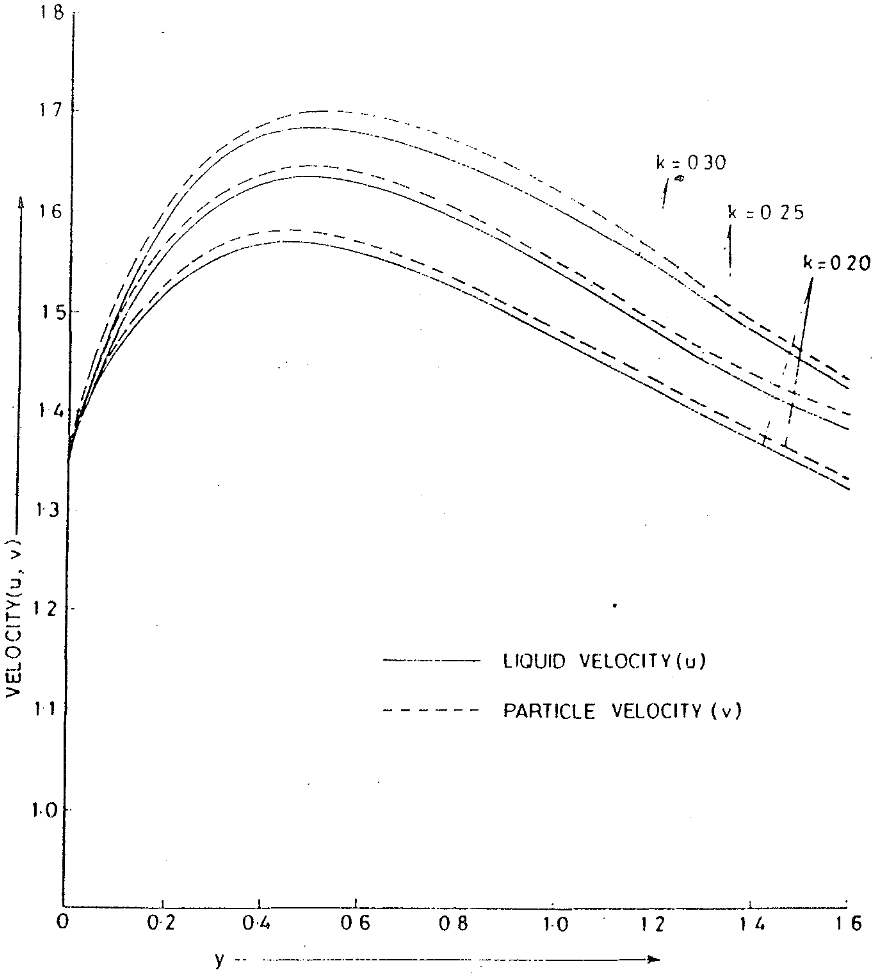


FIG.3 VARIATION OF SKIN-FRACTION FACTOR WITH TIME FOR DIFFERENT VALUES OF GRASHOFF NUMBER G,
(P=0.71, l=0.3, w=0.2, K=1.0, Є=0.4, n=0.1, k=0.25 and M=1.2)

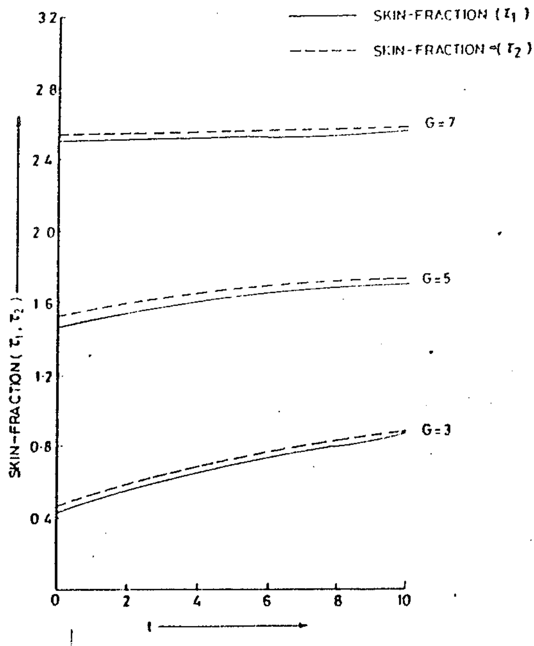
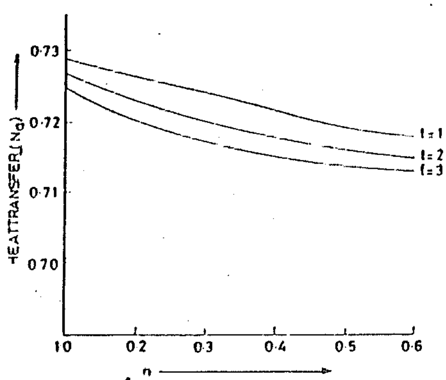


FIG.4 VARIATION OF HEAT TRANSFER WITH FREQUENCY FOR DIFFERENT VALUES OF TIME t

(P=0.71, Є=0.4)



liquid and particles increases as y increases from $y = 0.0$ to $y = 0.4$ after that the velocity of the liquid particles decreases from $y = 0.4$ to $y = 1.6$ for all given magnitudes of magnetic intensity.

Figure 2 shows the velocity profiles of the liquid and dust particles for various values of porosity parameter ($k = .20$, .25 and .30) at $P = .71$, $G = 5.0$, $l = 0.3$, $w = 0.2, K = 1.0$, $\in = 0.4$, $t = 1.0$, $n = 0.1$ and $M = 1.2$. From the figure, we observe that the valocity of the liquid and dust particles increases as the porosity parameter k increases. Besides, we observe that the velocity of the liquid and particles increases as y increases from $y = 0.0$ to $y = .4$ after that it decreases from $y = .4$ to $y = 1.6$ for all the given values of the porosity parameter.

Variation of skin-friction factor with time for different values of Grashoff number ($G = 3.0, 5.0$ and 7.0) at $P = .71$, $l = 0.3$, $w = 0.2$, $K = 1.0$, $\in = 0.4$, $n = 0.1$, $k = 0.25$ and $M = 1.2$ are shown in figure 3. From the figure, it is obvious that the skin-friction factor of the liquid and dust particles increases as the Grashoff number increases. Besides, we observe that on increasing y the variation of skin-friction factor increases.

Figure 4 indicates the variation of heat transfer with frequency for different values of time ($t = 1.0, 2.0$ and 3.0) at $P = .71$, $\in = 0.4$. From the figure, we observe that the heat transfer decreases as time increases. It is also obvious that the heat transfer decreases as frequency n increases.

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