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CERTAIN RESULAS FOR A GENERAL CLASS OF POLYNOMIALS, KONHAUSER BIORTHOGONAL POLYNOMIALS AND THE MULTIVARIABLE *H*-FUNCTION

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ABSTRACT

In this paper first we evaluate an integral involving the product of a general classes of polynomials, Konhauser biorthogonal polynomials and the multivariable *H*-function. This integral is then employed to establish an expansion formula for the product of general class of polynomials and the multivariable *H*-function in a series of biorthogonal polynomials. The results established here are quite general in character and a number of (known and new results which follow as special cases of our results are discussed briefly.

1. Introduction

Konhauser ([5], p. 303, 304) has considered following pair of biorthogonal polynomials

$$Z_n^{\alpha}(x;k)=\frac{\Gamma(\alpha+kn+1)}{n!}\sum_{j=0}^{n}(-1)^j\binom{n}{j}\frac{x^{kj}}{\Gamma(\alpha+kj+1)} \qquad ...(1.1)$$

and

$$Y_n^{\alpha}(x;k)=\frac{1}{n!}\sum_{w=0}^{n}\frac{x^w}{w!}\sum_{j=0}^{w}(-1)^j\binom{w}{j}\left(\frac{\alpha+j+1}{k}\right)_n \qquad ...(1.2)$$

which were actually suggested by the Laguerre polynomials, k being a positive integer. Indeed for $k=1$, each of these polynomials reduces to the Laguerre polynomials $L_n^{(\alpha)}(x)$, and their special cases when $k=2$ were encountered earlier by Spencer and Fano [9] in certain analytical calculations involving the penetration of Gamma rays through matter and were studied subsequently by Preiser [6]. (See also Srivastava [10] for comments on the claimed generalizations of the Konhauser polynomials by Raizanda [7, p. 110].)

Srivastava ([11], p.1, Eq. (1)) introduced a general class of polynomials

$$S_N^M[x] = \sum_{j=0}^{[N/M]} \frac{(-N)_{Mj}}{j!} A_{N,j} x^j, N = 0, 1, 2, \ldots \quad \ldots (1.3)$$

where M is arbitrary positive integer and $A_{N,j}$ $(N, j \geq 0)$ are arbitrary constants, real or complex.

By suitably specializing the coefficients $A_{N,j}$, the general class of polynomials can be reduced to classical orthogonal polynomials, Bessel polynomials and generalized hypergeometric polynomials (see, for example, Srivastava [11] and Srivastava and Singh [12]).

The multivariable H-function occuring in this paper has been defined by Srivastava and Panda ([13], p. 271, Eq. (4.1)). We shall use the following contracted notation (Srivastava et al. [14], p. 251, Eq. (C.1))

$$H[z_1, \ldots, z_u] = H^{O, N : m_1, n_1; \ldots ; m_u, n_u}_{P, Q : p_1, q_1; \ldots; p_u, q_u} \left[\begin{matrix} z_1 \\ \vdots \\ z_u \end{matrix} \right| \left. \begin{matrix} (a_j; \alpha_j', \ldots, \alpha_j^{(u)})_{1,P} : (c_j', \gamma_j')_{1,p_1} ; \ldots ; (c_j^{(u)}, \gamma_j^{(u)})_{1,p_u} \\ (b_j; \beta_j', \ldots, \beta_j^{(u)})_{1,Q} : (d_j', \delta_j')_{1,q_1} ; \ldots ; (d_j^{(u)}, \delta_j^{(u)})_{1,q_u} \end{matrix} \right] \quad \ldots (1.4)$$

to denote the H-function of u complex variables $z_1, \ldots, z_u$. Here all the Greek letters are assumed to be positive real numbers for standardization purposes, the definition of the multivariable H-function will, however, be meaningful even if some of these quantities are zero. The details of these quantities are zero. The details of this function can be found in the papers and book referred to above.

Raizada ([7], p.64, Eq. (2.1.2)) has introduced the generalised polynomial set defined by the following Rodirigues type formula

$$S_n^{\alpha, \beta, \tau}[x; r, s, q, A, B, k, l] = (Ax + B)^{-\alpha} (1 - \tau x^r)^{-\beta/\tau}$$

$$T_{k,l}^{m+n} [(Ax + B)^{\alpha + qn} (1 - \tau x^r)^{(-\beta/\tau) + sn}] \quad \ldots (1.5)$$

where the differential operator $T_{k,l}$ is defined as

$$T_{k,l} \equiv x^l (k + xD_x) \quad \ldots (1.6)$$

The explicit form of this generalized polynomial set is ([7], p. 71, Eq. (2.3.4))

$$S_n^{\alpha, \beta, \tau}[x; r, s, q, A, B, m, k, l] = B^{qn} x^{l(m+n)} (1 - \tau x^r)^{sn} l^{m+n}$$

$$\sum_{p=0}^{m+n} \sum_{t=0}^{p} \sum_{\delta=0}^{m+n} \sum_{i=0}^{\delta} \frac{(-1)^\delta (-p)_t (-\delta)_i}{p!\,\delta!\,i!} \frac{(\alpha)_\delta}{t!} \frac{(\alpha - qn)_i}{(i - \alpha - \delta)_i}$$

$$(-\frac{\beta}{\tau} - sn)_p \, (\frac{i+k+rt}{l})_{m+n} \, (\frac{-\tau x^r}{1-\tau x^r})^p \, (\frac{Ax}{B})^\delta \qquad \ldots (1.7)$$

Taking $A = 1, B = 0$ and $\tau = 0$ in (1.7), we arrive at the following polynomial set :

$$\lim_{\tau \to 0} S_n^{\alpha, \beta, \tau} [x; r, s\, q, 1, 0, m\, k, l]$$

$$= S_n^{\alpha, \beta, 0} [x; r, q, 1, 0, m, k, l]$$

$$= x^{qn + l(m+n)} \, l^{m+n} \sum_{p=0}^{m+n} \sum_{t=0}^{p} \frac{(-p)_t}{p\,!\,t\,!} \left(\frac{\alpha + qn + k + rt}{l}\right)_{m+n} (\beta x^r)^p \qquad \ldots (1.8)$$

The above-mentioned polynomial set (1.8) is also general in nature and contains known polynomials due to Gould and Hopper [2], Singh and Srivastava [8], Chatterjee [1] and Krall and Frink [3] as its special cases.

2. Main Integral : In this section we evaluate the following general integral

$$\int_\xi^\infty (x-\xi)^{\mu-1} e^{-\eta x} \, Y_n^\alpha(x; k) \, S_\lambda^{\nu, \zeta, 0} (y(x-\zeta)^\rho ; r, q, 1, 0, m, k, l)$$

$$S_N^M [z(x-\xi)^\sigma] \, H[z_1(x-\xi)^{\sigma_1}, \ldots, z_h (x-\xi)^{\sigma_h}] \, dx$$

$$= e^{-\eta\xi} \, e^{m+\lambda} \, y^{q\lambda + l(m+\lambda) + rp} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{[N/M]} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f (-N)_{M\delta} (-w)_i}{p\,!\,f\,!\,\delta\,!\,i\,!\,n\,!\,w\,!} A_{N,\delta} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda} (-1)^n z^\delta \zeta^p \xi^w$$

$$H_{i,\delta,p,t}^{\mu'} [z_1, \ldots, z_h] \qquad \ldots (2.1)$$

where $\mu' = \mu + q\lambda\rho + \rho l(m+\lambda) + \sigma\delta + \rho r p$ $\qquad \ldots (2.2)$

and

$$H_{i,\delta,p,t}^{\mu'} [z_1, \ldots, z_h]$$

$$= H^{0, B+2 : A_1, B_1 ; \ldots ; A_h, B_h}_{C+2, D+1 : C_1, D_1 ; \ldots ; C_h, D_h} \left[\begin{matrix} z_1 \\ \vdots \\ z_h \end{matrix} \middle| \begin{matrix} (1-\mu'; \sigma, \ldots, \sigma_h), \\ (\frac{1+i+\alpha-\mu'}{k} + n ; \frac{\sigma_1}{k}, \ldots, \frac{\sigma_h}{k}), \end{matrix} \right.$$

$$\frac{1+i+\alpha-\mu'}{k} ; \frac{\sigma_1}{k}, \ldots, \frac{\sigma_h}{k}), (a_j, \alpha_j', \ldots, \alpha_j^{(h)})_{1,C} ;$$

$$(b_j; \beta_j', \ldots, \beta_j^{(h)})_{1,D} :$$

$$\left.\begin{matrix} (c_j',\gamma_j')_{1,c_1};\ldots;(c_j^{(h)},\gamma_j^{(h)})_{1,C_h} \\ (d_j',\delta_j')_{1,D_1};\ldots;(d_j^{(h)},\delta_j^{(h)})_{1,D_h} \end{matrix}\right] \quad \ldots (2.3)$$

The (sufficient) conditions of validity of (2.1) are

(i) k, s and r are positive integers.

(ii) $\mathrm{Re}(\eta) > 0$, $\mathrm{Re}(\alpha + 1) > 0$,

$$\mathrm{Re}(\mu + \sigma\delta) + \sum_{j=1}^{h} \sigma_j \min_{1 \le i \le A_j} \mathrm{Re}[d_i^{(j)}/\delta_i^{(j)}] > 0, \ (\delta = 1, \ldots, [N/M])$$

(iii) $\Omega_j > 0$, $|arg\, z_i| < 1/2\, \Omega_i \pi$

where

$$\Omega_j = -\sum_{i=B+1}^{C} \alpha_i^{(j)} - \sum_{i=1}^{D} \beta_i^{(j)} + \sum_{i=1}^{A_j} \delta_i^{(j)} - \sum_{i=A_j+1}^{D_j} \delta_i^{(j)}$$

$$+ \sum_{i=1}^{B_j} \gamma_i^{(j)} - \sum_{i=B_j+1}^{C_j} \gamma_i^{(j)} \quad (j = 1, 2, \ldots, h) \qquad \ldots (2.4)$$

Proof

To evaluate the integral (2.1), we first express the multi-variable H-function involved in the left-hand side in terms of multiple Mellin-Barnes type contour integral with the help of (1.4), general class of polynomials and generalised polynomial set in series given by (1.3) and (1.8) respectively and change the order of integrations, wich is permissible under the conditions stated with (2.3), we get the left-hand side of (2.1) as

$$\Delta = \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{[N/M]} \frac{(-p)_f}{p!\,f!} \zeta^p \frac{(-N)_{M\delta}}{\delta!} A_{N,\delta} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda}$$

$$l^{m+\lambda} z^{\delta} y^{q\lambda + l(m+\lambda) + rp}$$

$$\frac{1}{(2\pi i)^h} \int_{L_1} \cdots \int_{L_h} \theta(t_1) \ldots \theta(t_h)\, \psi(t_1, \ldots, t_h)$$

$$\left\{ \int_{\xi}^{\infty} (x - \xi)^{\mu + q\lambda\rho + l\rho(m+\lambda) + \sigma\delta + \rho r p + \sum_{j=1}^{h} \sigma_j t_j - 1} \right.$$

$$\left. e^{-\eta x}\, Y_n^{\alpha}(x; k)\, dx \right\} z_1^{t_1} \ldots z_h^{t_h}\, dt_1 \ldots dt_h \qquad \ldots (2.5)$$

Now for evaluation of the inner x-integral, using the {[10], p. 43, Eq. (3.9)), changing the order of integration and summation involved therein and expressing the multiple contour integral as the multi-variable H-function, we easily get the right-had side of (2.1).

3. Special cases of (2.1)

(i) Taking $z = M = 1, \sigma = g, A_{N,\delta} = \frac{\Gamma(1+\beta+gN)}{N!\,\Gamma(1+\beta+g\delta)}$

and using the following relationship

$S_N^1[(x-\xi)^g] \rightarrow Z_N^\beta[x-\xi); g)$ in (2.1), we get the follwoing interesting integral :

$$\int_\xi^\infty (x-\xi)^{\mu-1} e^{-\eta x} Y_n^\alpha(x;k) Z_N^\beta((x-\xi);g)$$

$$S_\lambda^{\nu,\zeta,0}(y(x-\xi)^\rho; r, q, 1, 0, m, k, l) H[z_1(x-\xi)^{\sigma_1}, \ldots, z_h(x-\xi)^{\sigma_h}]\, dx$$

$$= e^{-\eta\xi} l^{m+\lambda} y^{q\lambda+(m+n)+rp} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{N} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f(-N)_\delta(-w)_i\,\Gamma(1+\beta+gN)}{p!\,f!\,\delta!\,n!\,w!\,N!\,\Gamma(1+\beta+g\delta)} \left(\frac{\nu+q\lambda+k+rf}{l}\right)_{m+\lambda}$$

$$(-1)^n \zeta^p \xi^w H^{\mu''}_{i,\delta,p,t}[z_1, \ldots, z_h] \quad \ldots (3.1)$$

where $\mu'' = \mu + q\lambda l + \rho l(m+\lambda) + g\delta + \rho rp$. ... (3.2)

(ii) Taking $M = 1, N = 0, A_{N,\delta} = 1$ in (2.1), then $S_0^1[z(x-\xi)^\sigma]$ reduce to one and the integral (2.1) takes the following form

$$\int_\xi^\infty (x-\xi)^{\mu-1} e^{-\eta x} Y_n^\alpha(x;k) S_\lambda^{\nu,\zeta,0}(y(x-\xi)^\rho; r, q, 1, 0, m, k, l)$$

$$H[z_1(x-\xi)^{\sigma_1}, \ldots, z_h(x-\xi)^{\sigma_h}]\, dx$$

$$= e^{-\eta\xi} l^{m+\lambda} y^{q\lambda+l(m+\lambda)+rp} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f(-w)_i}{p!\,f!\,n!\,i!\,w!} \left(\frac{\nu+q\lambda+k+rf}{l}\right)_{m+\lambda} (-1)^n \zeta^p \xi^w H^{\mu'''}_{i,o,p,t}[z_1, \ldots, z_h] \quad \ldots (3.3)$$

where $\mu''' = \mu + q\lambda\rho + \rho l(m+\lambda) + \rho rp$.

4. Expansion theorem

Suppose

(i) k and r are positive integers,

(ii) $\sigma_j > 0, \sigma > 0, \mu > 0$; $\text{Re}(\alpha) > -1$,

$$\text{Re}(\beta+\sigma\delta) + \sum_{j=1}^{h} \sigma_j \min_{1<i\leq A_j} \text{Re}(d_i^{(j)}/\delta_i^{(j)}) > -1$$

$$(\delta = 1, \ldots, [N/M])$$

(iii) $\Omega_j > 0$ $|arg\, z_j| < 1/2\, \Omega_j \pi$ $(j = 1, \ldots, h)$

(Ω_j is defined by (2.4)),

then

$$x^{\mu-1} S_{\lambda}^{\nu,\zeta,0}(y x^{p} ; r, q, 1, 0, m, k, l)\, S_N^M [z x^{\sigma}]\, H[z_1 x^{\rho_1}, \ldots, z_h x^{\sigma_h}]$$

$$= \sum_{n=0}^{\infty} \Bigg[l^{m+\lambda} y^{q\lambda + l(m+\lambda)+rp} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{[N/M]} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f (-N)_{M\delta} (-w)_i}{p\,!\,f\,!\,\delta\,!\,w\,!\,i\,!} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda} A_{N,\delta}\, \zeta^p z^{\delta} \frac{(-1)^n}{\Gamma(1+\alpha+kn)}$$

$$H_{i,\delta,p,t}^{\mu'+\alpha} [z_1, \ldots, z_h] \Bigg] Z_n^{\alpha}(x; k) \qquad \ldots (4.1)$$

Proof : Let

$$x^{\mu-1} S_{\lambda}^{\nu,\zeta,0}(yx^{\rho}; r, q, 1, 0, m, k, l)\, S_N^M [zx^{\sigma}]\, H[z_1 x^{\sigma_1}, \ldots, z_h x^{\sigma_h}]$$

$$= \sum_{n=0}^{\infty} k_n Z_n^{\alpha}(x; k) \qquad \ldots (4.2)$$

Multiplying both sides of (4.2) by $e^{-x} x^{\alpha} Y_v^{\alpha}(x; k)$ and intergrating with respect to x from 0 to ∞, we get

$$\int_0^{\infty} e^{-x} x^{\mu+\alpha-1} Y_k^{\alpha}(x\,; k)\, S_{\lambda}^{\nu,\zeta,0}(yx^{\rho}; r, q, l, 0, m, k, l)$$

$$S_N^M [zx^{\sigma}]\, H[z_1 x^{\sigma_1}, \ldots, z_h x^{\sigma_h}]\, dx$$

$$= \sum_{n=0}^{\infty} k_n \int_0^{\infty} e^{-x} x^{\alpha} Z_n^{\alpha}(x; k)\, Y_v^{\alpha}(x;k)\, dx \qquad \ldots (4.3)$$

using the integral (2.1) and the following orthogonal property ([5], p. 303)

$$\int_0^{\infty} x^{\alpha} e^{-x} Y_v^{\alpha}(x; k)\, Z_n^{\alpha}(x; k)\, dx = \frac{\Gamma(\alpha + kn + 1)}{n\,!} \delta_{v,n} \qquad \ldots (4.4)$$

(where $\mathrm{Re}(\alpha) > -1$, k, v, n are positive integers. Also $\delta_{v,n}$ is the well-known Kronecker delta function)

in (4.3), we find that

$$K_n = l^{m+\lambda} y^{q\lambda + l(m+\lambda)+rp} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{[N/M]} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f (-N)_{M\delta} (-w)_i}{p\,!\,f\,!\,\delta\,!\,i\,!\,w\,!} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda} A_{N,\delta}\, \zeta^p \frac{(-1)^n z^{\delta}}{\Gamma(1+\alpha+kn)}$$

$$H_{i,\delta,p,t}^{\mu'+\alpha} [z_1, \ldots, z_h] \qquad \ldots (4.5)$$

where $H^{\mu'+\alpha}_{i,\delta,p,t}[z_1, ..., z_h]$ can be defined similarly to (2.3).

Substituting the value of K_n from (4.5) in (4.2), we arrive at the required expansion formula (4.1).

5. Special Cases of (4.1) : Using the substitutions in (4.1) as mentioned with the special cases of (2.1), we arrive easily at the following expansion formulas

$$x^{\mu-1} Z_N^{\beta}(x; g)\, S_{\lambda}^{\nu,\zeta,0}(y\, x^{\rho}; r, q, 1, 0, m, k, l)\, H\,[z_1 x^{\sigma_1}, ..., z_h\, x^{\sigma_h}]$$

$$= l^{m+\lambda}\, y^{q\lambda + l(m+\lambda) + rp} \sum_{n=0}^{\infty} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{\delta=0}^{N} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f (-N)_\delta (-w)_i}{p\,!\, f\,!\, \delta\,!\, w\,!\, i\,!} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda} \frac{\Gamma(1+\beta+gN)}{N\,!\,\Gamma(1+\beta+g\delta)} \frac{(-1)^n}{\Gamma(1+\alpha+kn)}\, \zeta^p$$

$$H^{\mu'+\alpha}_{i,\delta,p,t}[z_1, ..., z_n] \qquad ... (5.1)$$

and

$$x^{\mu-1} S_{\lambda}^{\nu,\zeta,0}(yx^{\rho}; r, q, 1, 0, k, m, l)\, H[z_1 x^{\sigma_1}, ..., z_h\, x^{\sigma_h}] =$$

$$= l^{m+\lambda}\, y^{\,q\lambda + l(m+\lambda)} \sum_{n=0}^{\infty} \sum_{p=0}^{m+\lambda} \sum_{f=0}^{p} \sum_{w=0}^{n} \sum_{i=0}^{w}$$

$$\frac{(-p)_f (-w)_i}{p\,!\, f\,!\, w\,!\, i\,!} \left(\frac{\nu + q\lambda + k + rf}{l}\right)_{m+\lambda} \frac{(-1)^n}{\Gamma(1+\alpha+kn)}\, \zeta^p$$

$$H^{\mu'+\alpha}_{i,0,p,t}[z_1, ..., z_h] \qquad ... (5.2)$$

The conditions of validity for (5.1) and (5.2) are easly obtainable from those mentioned with the main expansion theorem (4.1).

A number of other integrals and expansion formulas involving product of elementary special functions of one and more variables can be obtained from (2.1), (3.1), (3.2), (4.1), (5.1) and (5.2) as special cases. This can be done by specializing the parameters of the multi-variable H-function in a suitable manner.

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REFERENCES

[1] S.K. Chatterjea, Quelgues functions generatica de polynomes d' Hermite du point de l' algebra de Lie, *C.R. Acad. Sci. Paris Ser. A* **268** (1969), 600-602.

[2] H.W. Gould and A.T. Hopper Operational formulas connected with two generalizations of Hermite polynomials, *Duke Math. J* **29** (1969), 55.

[3] H.L. Krall and O. Frink, On a new class of polynomials: The Bessel polynomials, *Trans. Amer. math. Soc.* **65** (1949), 110-115.

[4] J.D.E. Konhauser, Some properties of biorthogonal polynomials, *J. Math. Anal. Appl.* **11** (1965), 242-260.

[5] J.D.E. Konhauser, Biorthogonal polynomials suggested by Laguerre polynomials, *Pacific J. Math.* **21** (1967), 303-304

[6] S. Preiser, An investigation of biorthogonal polynomials derivable from ordinary differential equations of the third order, *J. Math. Anal. Appl,* **4** (1962), 38-64

[7] S.K. Raizada, *A Study of Unified Representations of Special Functions of Mathematical Physics and Their use in Statistical and Boundary Value Problems,* Ph.D. thesis, Bundelkhand Univ., Jhansi, India, 1991.

[8] R.P. Singh and K.N. Srivastava, A note on generalization of Laguerre and Humbert polynomials, *Ricerca (Napoli)* (2) 14 (1963), *Settembre-dicember,* 11-21.

[9] L. Spencer, and U. Fano, Penetration and diffusion of X-Rays. Calculation of spatial distribution by polynomials expansion, *J. Res. Nat. Bur. Standards* **46** (1951), 446-461

[10] H.M. Srivastava, Some families of dual and triple series equations involving the Konhauser birothogonal polynomials, *Ganita* **43**, (1992), 75-84.

[11] H.M. Srivastava, A contour integral involving Fox's *H*-function, *Indian J.Math.*. **14**, (1972), 1-6.

[12] H.M. Srivastava and N.P. Singh, The integration of certain products of the multivariable *H*-function with a general class of polynomials, *Rend. Circ. Mat. Palermo* (2) **32**, (1983) 157-187.

[13] H.M. Srivastava and R. Panda, Some bilateral generating functions for a generalized hypergeometric polynomials, *J. Reine Angew. Math.* **283/284**, (1976), 265-274.

[14] H.M. Srivastava, K.C. Gupta and S.P. Goyal : The *H*-Functions of One and Two Variables with Applications, *South Assian Publishers, New Delhi,* 1982.