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SOME WEAKER FORMS OF FUZZY CONTINUOUS MAPPINGS

Sunder Lal and Pushpendra Singh
Department of Mathematics, Institure of Basic Science
Agra University, Khandari; Agra - 282002, U.P

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ABSTRACT

Some weaker forms of fuzzy continuous mappings have been introduced and studied. Results pertaining to some of their preservation properties are also discussed.

1. INTRODUCTION

Using semi open sets in topological spaces, several weaker forms of continuity have been studied in the literature. For example, Bhamini [2] defined a mapping $f : X \to Y$ to be almost semi continuous if the inverse image of regularly open sets in Y are semi open in X. She [2] calls f to be weakly semi continuous if for every open set U of Y, $f^{-1}(U) \subset S-\text{int}\, f^{-1}(clU)$. Here S-int denotes the semi interior operator. Singal and Yadav [7] call f to be slightly semicontinuous if $f^{-1}(U)$ is semi open in X for every clopen subset U of Y. Here we study these mappings in the fuzzy setting. However, we use a more suggestive terminology for these mappings. We call a mapping $f : X \to Y$ to be $I(\alpha, \beta)$ if the inverse image of a set of type β is a set of type α. For example, a mapping $f : X \to Y$ is $I(o, clo)$ if the inverse image of each clopen set in Y is open in X. Bhamini's almost semi continuous mapping in our terminology is I (so, ro) and slightly semi continuous mapping of Singal and Yadav is I (so, clo). Here we study some of these mapping in the fuzzy setting.

2. PRELIMINARIES

Throughout this paper X and Y mean fuzzy topological spaces. I denotes the closed unit interval. The definitions of fuzzy sets, fuzzy topological spaces and other concepts about functions can be found in[3,8,9].

Let U be a fuzzy set of X. U is said to be fuzzy semi open set of X if there exists a fuzzy open set V such that $V \leq U \leq \text{cl}V$. Complement of a fuzzy semi open set is called fuzzy semi closed. S-int U (Semi interior of U) is defined as the suprimum of all fuzzy semi-open sets contained in U and S-cl$U$ (semi closure of U) as the infimum of all fuzzy semi closed sets containing U.

A fuzzy set U of a fuzzy space X is called a fuzzy regular open set of X if int cl$U$ = U, and a fuzzy regular closed set of X if *cl int* U = U.

A fuzzy set U of X is called fuzzy clopen (resp. fuzzy semi clopen) set of X if U is fuzzy open and fuzzy closed (resp. fuzzy semi open and fuzzy semi closed). A fuzzy set U of X is said to be fuzzy δ^* – open if U can be written in the form $U = \vee U_i$, where U_i are fuzzy clopen sets.Complement of a fuzzy δ^* – openset is called fuzzy δ^* closed.

A mapping $f: X \to Y$ from a fuzzy space X to fuzzy space is called a, (i) **fuzzy open mapping** [1], if $f(A)$ is fuzzy open set of Y, for each fuzzy open set A of X; (ii) **fuzzy closed mapping** [1], if $f(A)$ is fuzzy closed set of Y, for each fuzzy closed set A of X; (iii) **fuzzy presemi open mapping**, if $f(A)$ is fuzzy semiopen set of Y for each fuzzy semi open A of X.

A fuzzy space X is said to be (i) fuzzy semi compact (**resp. fuzzy slightly compat**) if every fuzzy semi open (resp. fuzzy clopen) cover of X has a finite subcover; (ii) fuzzy **strongly *S*-closed** if every fuzzy semi open cover of X has a finite subfamily such that semi closures of whose members cover X.

A fuzzy space X is said to be (i) **fuzzy semi T_0** , if every fuzzy set A of X can be written in the form $A = \bigvee_{i \in I} \bigwedge_{j \in J_i} U_{ij}$, where U_{ij} are fuzzy semi open or fuzzy semi closed sets; (ii) **fuzzy semi T_1**, if every fuzzy set A of X can be written in the form $A = \bigvee_{i \in I} U_i$, where U_i are fuzzy semi closed sets; (iii) **fuzzy semi T_2**, if every fuzzy set A of X can be written in the form $A = \bigvee_{i \in I} \bigwedge_{j \in J_i} U_{ij} = \bigvee_{i \in I} \bigwedge_{j \in J_i} S-\text{cl}U_{ij}$, where U_{ij} are fuzzy semi open sets, (iv) fuzzy S-regular, if every fuzzy open set U can be written in the form $U = \bigvee_{i \in I} U_i$, where U_i are fuzzy semi open sets with $S-\text{cl}\, U_i \le U$; (v) fuzzy S-normal, if for any fuzzy closes set K and fuzzy open set U such that $K \le U$, there exists a fuzzy V such that

$$K \le S - \text{int}\, V \le S - clV \le U.$$

A fuzzy space (X, τ) is called a fuzzy semi regular space [1] if the collection of all fuzzy regular open sets of X forms a base for fuzzy topology.

A fuzzy space X is called **product related** [1] to a fuzzy space Y if for any fuzzy set U of X and V of Y whenever $A' < U$ and $B' < V$ implies $(A' \times 1) \vee (1 \times B') \ge U \times V$, where A is fuzzy open set of X and B is fuzzy open set of Y, there exists a fuzzy open set A_1 of X and a fuzzy open set B_1 of Y such that $A_1' \geq U$ or $B_1' \geq V$ and $(A_1' \times 1) \vee (1 \times B_1')$ $= (A' \times 1) \vee (1 \times B')$.

3. FUZZY $I(so, ro)$ MAPPINGS

3.1 Definition : A mapping $f: X \longrightarrow Y$ from a fuzzy space X to a fuzzy space Y is said to be fuzzy $I(so, ro)$ if inverse image of every fuzzy regular open set of Y is fuzzy semi open set of X.

3.2 Remark : Fuzzy $I(so, ro)$ mappings are a generalization of the concept of almost semi continuous mapping studied by Bhamini [2].

3.3 Theorem : For a mapping $f: X \to Y$ from a fuzzy space X to a fuzzy space Y, the following are equivalent : (a) f is fuzzy $I(so, ro)$; (b) the inverse image of every fuzzy closed set of Y is fuzzy semi closed set of X; (c) $f^{-1}(U)) \leq S - \text{int}\ (f^{-1}\ (\text{int cl}U))$ for every fuzzy open set U of Y; (d) $S - cl\ (f^{-1}\ (cl\ \text{int}\ V)) \leq f^{-1}\ (V)$ for every fuzzy closed set V of Y.

Proof : $(a) \Leftrightarrow (b)$, because for any fuzzy set U of Y, we have $f^{-1}(U') = f^{-1}(U))'$. $(a) \Rightarrow (c)$. Let U be a fuzzy open set of Y. Then $U \leq$ int clU and hence $f^{-1}(U) \leq f^{-1}(\text{ int } clU)$. Since int clU is a fuzzy regular open set of $Y, f^{-1}(\text{ int } clU)$ is a fuzzy semi open set of X. Thus $f^{-1}(U) \leq f^{-1}(\text{ int } clU) = S - \text{int} f^{-1}(\text{ int } clU)$. $(c) \Leftrightarrow (d)$. Taking complement of both sides of (c) we get (d), and vice versa. $(d) \Rightarrow (b)$. Let U be a fuzzy regular closed set of Y. By (d) we have

$$S - cl\ (f^{-1}\ (cl\ \text{ int }\ U)) = S - cl\, f^{-1}(u) \leq f^{-1}(U).$$

Thus $S - cl\, f^{-1}(U) = f^{-1}(U)$, which is fuzzy semi closed.

3.4 theorem. If $f: X \to Y$ be a fuzzy $I(so, ro)$ mapping, then for every fuzzy open set U of Y, $S - cl\, f^{-1}(U) \leq f^{-1}(clU)$

Proof : Let $f: X \to Y$ be a fuzzy $I(so, ro)$ mapping from a fuzzy space X to a fuzzy space Y. Then $f^{-1}(U) \leq f^{-1}\ (clU)$ for every fuzzy set U of Y. If U is fuzzy open, then clU is fuzzy regular closed. Hence $f^{-1}(clU)$ is fuzzy semi closed set of X. Thus we have $S - cl\, f^{-1}\ (U) \leq f^{-1}\ (clU)$.

Clearly, every fuzzy $I(0, ro)$ mapping as well as every fuzzy $I(so, o)$ mapping is fuzzy $I(so, ro)$. But the converse need not be true as is shown by the following examples :

3.5 Example : Let U_1, U_2 and U_3 be fuzzy sets of I defined as follows :

for each $x \in I$,

$$U_1(x) = x, \qquad 0 \leq x \leq 1;$$

$$U_2\ (x) = 1 - x, \qquad 0 \leq x \leq 1;$$

$$U_3(x) = x, \qquad 0 \le x \tfrac{1}{2};$$
$$= 0, \qquad \tfrac{1}{2} < x \le 1.$$

Consider fuzzy topologies $\tau_1 = \{0, U_1 \wedge U_2, 1\}$ and $\tau_2 = \{0, U_1, U_2, U_1 \wedge U_2, U_1 \vee U_2, 1\}$ on I and the mapping $f: (I, \tau_1) \to (I, \tau_2)$ defined by $f(x) = x$, for each $x \in I$. It is clear that $U_1, U_2, U_1 \vee U_2$ and $U_1 \wedge U_2$ being fuzzy open and fuzzy closed are fuzzy regular open sets in (I, τ_2). It is obvious that inverse image of every fuzzy regular open set in (I, τ_2) is fuzzy semi open in (I, τ_1), because in (I, τ_1), $cl\,(U_1 \wedge U_2) = 1$ and $U_1 \wedge U_2 \le f^{-1}(U_1) \le 1$, $U_1 \wedge U_2 \le f^{-1}(U_2) \le 1$, $U_1 \wedge U_2 \le f^{-1}(U_1 \wedge U_2) \le 1$. Hence the mapping is fuzzy $I(so, ro)$. The mapping is not fuzzy $I(o, ro)$, because none of $f^{-1}(U_1), f^{-1}(U_2)$ and $f^{-1}(U_1 \vee U_2)$ is open in (I, τ_1).

3.6 Example : We refer to fuzzy sets as defined in Example 3.5 Consider the fuzzy topologies $\tau_1 = \{0, U_1 \wedge U_2, 1\}$ and $\tau_2 = \{0, U_1, U_2, U_3, U_1 \vee U_2, U_1 \wedge U_2, 1\}$ on I and the mapping $f: (I, \tau_1) \to (I, \tau_2)$ defined by $f(x) = x$ for every $x \in I$ It is clear that the mapping is fuzzy $I(so, ro)$. Also, because O is the only fuzzy open set contained in $f^{-1}(U_3), f^{-1}(U_3) = U_3$ is not fuzzy semi open set of (I, τ_1). Hence f is not a fuzzy $I(so, o)$ mapping.

3.7 Remark : Fuzzy $I(o, ro)$ and fuzzy $I(so, o)$ mappings have been studied under the names fuzzy almost continuous and fuzzy semi continuous [1].

3.8 Theorem : If $f: X \to Y$ is a fuzzy $I(so, ro)$ Mapping from a fuzzy space X to fuzzy semi regular space Y. Then f is fuzzy $I(so, o)$.

Proof : Let U be a fuzzy open set of Y. Then $U = \bigvee_{i \in I} U_i$, where U_i are fuzzy regular open sets of Y; since Y is fuzzy semi regular. Now, using Theorem 3.3 (c), we get

$$f^{-1}(U) = f^{-1}(\bigvee_{i \in I} U_i) = \bigvee_{i \in I} f^{-1}(U_i)$$

$$\le \bigvee_{i \in I} S-\text{int}\, clU_i) = \bigvee_{i \in I} S-\text{int} f^{-1}(U_i)$$

$$\le S-\text{int}\,(\bigvee_{i \in I} f^{-1}(U_i) = S-\text{int} f^{-1}(U),$$

which shows that $f^{-1}(U)$ is fuzzy semi open set of X. Hence f is fuzzy $I(so, o)$.

3.9 Theorem : Let $f : X \to Y$ be a fuzzy $I(so, so)$ and fuzzy pre semi open mapping and let $g : Y \to Z$ be any mapping. Then $gof : X \to Z$ is fuzzy $I(so, ro)$ if g is fuzzy $I(so, ro)$.

Proof : The if part is obvious. To prove the only if part, let $gof : X \to Z$ be a fuzzy I (so, ro) mapping. Let U be a fuzzy regular open set of Z. then $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is fuzzy semi open set of X and hence $f(f^{-1}(g^{-1}(U)) = g^{-1}(U)$ is fuzzy semi open set of Y. Therefore g is fuzzy $I(so, ro)$.

3.10 Remark : Fuzzy $I(so, so)$ mappings have been studied under the name fuzzy irresolute by Mukherjee and Sinha [6].

3.11 Theorem : Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be fuzzy $I(so, ro)$ mappings. If Y_1 is product related to Y_2, then the product mapping $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is fussy $I(so, ro)$.

Proof: Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that Y_1 is product related to Y_2. Let $U \equiv \vee (U_i \times U_j)$ be a fuzzy open set of $Y_1 \times Y_2$, where U_i and U_j are open sets of Y_1 and Y_2 respectively. Since f_1 and f_2 are fuzzy $I(so, ro)$ mappings, we have

$$
\begin{aligned}
(f_1 \times f_2)^{-1}(U) &= \vee \{f_1^{-1}(U_i) \times f_2^{-1}(U_j)\} \\
&\leq \vee \{S - \text{int}\, f_1^{-1}(\text{int}\, cl\, U_i) \times S - \text{int}\, f_2^{-1}(\text{int}\, cl\, U_j)\} \\
&\leq \vee \{S - \text{int}\, (f_1^{-1}(\text{int}\, cl\, U_i) \times f_2^{-1}(\text{int}\, cl\, U_j))\} \\
&\leq S - \text{int}\, \{\vee (f_1 \times f_2)^{-1}(\text{int}\, cl\, U_i \times \text{int}\, cl\, U_j)\} \\
&= S - \text{int}\, \{\vee f_1 \times f_2)^{-1}(\text{int}\, cl\, (U_i \times U_j))\} \\
&\leq S - \text{int}\, \{(f_1 \times f_2)^{-1}(\text{int}\, cl\, (\vee (U_i \times U_j)))\} \\
&= S\, \text{int}\, (f_1 \times f_2)^{-1}(\text{int}\, clU).
\end{aligned}
$$

Thus by theorem 3.3 (c), $f_1 \times f_2$ is fuzzy $I(so, ro)$.

3.12 Theorem : Let X and Y be fuzzy spaces such that X is product related to Y and let $f : X \to Y$ be a mapping. If the graph mapping $g : X \to X \times Y$ of f is fuzzy $I(so, ro)$, then f is also fuzzy $I(so, ro)$.

Proof : Suppose that $g : X \to X \times Y$ is fuzzy $I(so, ro)$ mapping and U is a fuzzy open set of Y. We have

$$
\begin{aligned}
f^{-1}(U) &= 1 \wedge f^{-1}(U) = g^{-1}(1 \times U) \\
&\leq S - \text{int}\, g^{-1}(\text{int}\, cl\, (1 \times U)) \\
&= S - \text{int}\, g^{-1}(\text{int}\, (1 \times clU)) \\
&= S - \text{int}\, g^{-1}(1 \times \text{int}\, clU)
\end{aligned}
$$

$= S - \text{int} f^{-1} (\text{int } clU)$.

Hence by Theorem 3.3 (c), f is fuzzy I (*so, ro*).

4. FUZZY WEAKLY SEMI CONTINUOUS MAPPINGS

4.1 Definition : A mapping $f: X \to Y$ from a fuzzy space X to a fuzzy space Y is said to be fuzzy weakly semi continuous if for every fuzzy open set U of Y,

$$f^{-1} (U) \leq S - \text{int} f^{-1} (clU).$$

4.2 Remark : Fuzzy weakly semi continuous mappings are the genralization of weakly semi continuous mappings intorduced by Bhamini [2].

4.3 Definition [1] : A mapping $f: X \to Y$ from a fuzzy space X to a fuzzy space Y is called fuzzy weakly continuous mapping if for each fuzzy open set U of Y, $f^{-1} (U) \leq \text{int} f^{-1} (clU)$.

Clearly, every fuzzy weakly continuous as well as every fuzzy I(*so, ro*) mapping is fuzzy weakly semi continuous. For the converse, we have

4.4 Example : Let $X = \{x, y, z\}$ and U_1, U_2 be fuzzy sets of X defined as follows :

$U_1(x) = 0.2, U_1(y) = 0.4, U_1(z) = 0.3;$

$U_2(x) = 0.4, U_2(y) = 0.5, U_2(z) = 0.3;$

Let $\tau_1 = \{0, U_2, 1\}$ and $\tau_2 = \{0, U_1, 1\}$. Let $f: (X, \tau_1) \to (X, \tau_2)$ be the identity, mapping. By easy computations it is clear that in (X, τ_2), $cl\ U_1 = U_1'$, $\text{int } clU_1 = U_1$ and in (X, τ_1), $S - \text{int} f^{-1} (U_1') = U_2'$ and $S - \text{int} f^{-1} (U_1) = 0$. Hence $f^{-1} (U_1) \leq S - \text{int} f^{-1} (clU_1)$, which shows that mapping is fuzzy weakly semi continuous. However, $f^{-1} (U_1) > \text{int} f^{-1} (\text{int } clU_1)$showing that the mapping is not fuzzy I(*so, ro*).

4.5 Example : We take fuzzy sets U_1, U_2 and fuzzy topologies as defined in previous example, and $f: (X, \tau_2) \to (X, \tau_1)$ the identity mapping. It is clear that in (X, τ_1), $clU_2 = U_2'$, and in (x, τ_2), $\text{int} f^{-1} (U_2') = U_1$ and $S - \text{int} f^{-1} (U_2') = U_2'$. Hence $f^{-1}(U_2) \leq S - \text{int} f^{-1} (U_2')$ showing that the mapping is fuzzy weakly semi continuous. However, $f^{-1} (U_2) > \text{int} f^{-1} (U_2')$, hence the mapping is not fuzzy weakly continuous.

4.6 Theorem : Let $f: (X, \tau_1) \to (X, \tau_2)$ be a fuzzy weakly semi continuous mapping from a fuzzy space X to a fuzzy regular space Y. Then f is fuzzy I(*so, o*).

Proof : Let U be a fuzzy open set of Y. Since Y is fuzzy regular space $U = \vee_{i \in I} U_i$, where U_i are fuzzy open sets of Y with $cl\ U_i \leq U$. Now, since f is fuzzy weakly semi continuous, we have.

$$f^{-1}(U) = f^{-1}(\vee_{i \in I} U_i) = \vee_{i \in I} f^{-1}(U_i)$$

$$\leq \vee_{i \in I} S - \text{int} f^{-1}(cl\ U_i)$$

$$\leq S - \text{int}(\vee_{i \in I} f^{-1}(clU_i)) = S - \text{int} f^{-1}(U),$$

which shows that $f^{-1}(U)$ is a fuzzy semi open set of X. Thus, f is fuzzy $I(so, o)$.

4.7 Theorem : Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that Y_1 is product related to Y_2. Then, the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of fuzzy weakly semi continuous mapping $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is fuzzy weakly semi continuous.

Proof : Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be fuzzy weakly semi continuous mappings such that fuzzy space Y_1 is product related to Y_2. Let $U \equiv \vee (U_i \times U_j)$ be a fuzzy open set of $Y_1 \times Y_2$, where U_i and U_j are fuzzy open sets of Y_1 and Y_2 respectively. We have, by fuzzy weakly semi continuity of f_1 and f_2

$$(f_1 \times f_2)^{-1}(U) = \vee \{f_1^{-1}(U_i) \times f_2^{-1}(U_j)\}$$

$$\leq \vee \{S - \text{int} f_1^{-1}(clU_i) \times S - \text{int} f_2^{-1}(clU_j)\}$$

$$\leq \vee \{S - \text{int}(f_1^{-1}(cl\ U_i) \times f_2^{-1}(clU_j))\}$$

$$\leq S - \text{int}\{\vee f_1 \times f_2)^{-1}(cl\ U_i \times cl\ U_j)\}$$

$$= S - \text{int}\{\vee (f_1 \times f_2)^{-1}(cl\ (U_i \times U_j))\}$$

$$= S - \text{int}(f_1 \times f_2)^{-1}(clU).$$

Thus by the definition of fuzzy weakly semi continuity, $f_1 \times f_2$ is fuzzy weakly semi continuous.

4.8 Theorem : Let X and Y be product related fuzzy spaces and $f : X \to Y$ be a mapping.

If the graph function $g : X \to X \times Y$ of f is fuzzy weakly semi continuous, then f is also fuzzy weakly semi continuous.

Proof : Let X and Y be fuzzy spaces such that X is product related to Y and let $f : X \to Y$ be a mapping. Suppose that the graph function $g : X \to X \times Y$ of f is fuzzy weakly semi continuous. Then for any fuzzy open set U of Y, we have

$$f^{-1}(U) = 1 \wedge f^{-1}(U) = g^{-1}(1 \times U) \leq S - \text{int}\, g^{-1}(cl\,(1 \times U)$$
$$= S - \text{int}\, g^{-1}(1 \times clU) = S - \text{int}\, f^{-1}(clU).$$

Thus by the definition, f is fuzzy weakly semi continuous.

5. FUZZY *I*(*so*, *clo*) MAPPINGS

Slightly semi continuous mappings in general topology have been introduced by Singal and Yadav [7]. Here we generalize this notion to the fuzzy setting.

5.1 Definition : A mapping $f : X \to Y$ from a fuzzy topological space X to a fuzzy space Y is said to be fuzzy $I(so, clo)$ if inverse image of every fuzzy clopen set of Y is fuzzy semi open set of X.

5.2 Theorem : For a mapping $f : X \to Y$ from a fuzzy space X to a fuzzy spce Y, the following are equivalent :

(a) f is fuzzy $I(so, clo)$;

(b) Inverse image of every fuzzy clopen set of Y is fuzzy semi closed set of X.

(c) Inverse image of every fuzzy clopen set of Y is fuzzy semi clopen set of X.

(d) Inverse image of every fuzzy δ^*-open set of Y is fuzzy semi open set of X.

(e) Inverse image of every fuzzy δ^*-closed set of Y is fuzzy semi closed set of X.

Proof : It is obvious.

The following implications hold.

Fuzzy $I(0, clo) \Rightarrow$ Fuzzy $I(so, clo) \Rightarrow$ Fuzzy weakly semi continuity

We now show that none of the above implications is reversible.

5.2 Example : Let U_1 and U_2 be fuzzy sets of I defined as follows : for each $x \in I$,

$$U_1(x) = 2x, \qquad 0 \leq x \leq 1/2$$
$$= 1, \qquad 1/2 \leq x < 1;$$
$$U_2(x) = 2x, \qquad 0 \leq x \leq 1/4$$
$$= 1 - 2x, \qquad 1/4 \leq x < 1/2,$$
$$= 0, \qquad 1/2 \leq x \leq 1.$$

Consider fuzzy topologies $\tau_1 = \{0, U_2, 1\}$ and $\tau_2 = \{0, U_1, U_1',$ $U_1 \vee U_1', U_1 \wedge U_1', 1\}$ on I and the mapping $f : (I, \tau_1) \to (I, \tau_2)$ defined by $f(x) = x$, for every $x \in I$. It is clear that $U_1, U_1', U_1 \vee U_1'$ and $U_1 \wedge U_1'$ are fuzzy clopen sets in (I, τ_2). In (I, τ_1)

we have $clU_2 = U'_2, U_2 \leq f^{-1}(U_1) \leq U_2', U_2 \leq f^{-1}(U_1') \leq U_2', f^{-1}(U_1 \vee U_1') = U_2'$ and $f^{-1}(U_1 \wedge U_1') = U_2$. Thus the inverse image of every fuzzy clopen set in (I, τ_2) is fuzzy semi open in (I, τ_1). Hence f is fuzzy $I(so, clo)$. However, the inverse image of fuzzy clopen set U_1 in (I, τ_2) is not fuzzy open in (I, τ_1). Hence the mapping is not fuzzy $I(o, clo)$.

5.3 Remark : $I(o\ clo)$ maps in general topology have been studied under the name slightly continuous maps [5].

5.4 Example : Let $X = \{x, y, z\}$, and U_1, U_2, V_1, V_2 and V_3 be fuzzy sets of X defined as follows :

$U_1(x) = 0.1,\quad U_1(y) = 0.4,\quad U_1(z) = 0.2;$

$U_2(x) = 0.3,\quad U_2(y) = 0.3,\quad U_2(z) = 0.3;$

$V_1(x) = 0.4,\quad V_1(y) = 0.5,\quad V_1(z) = 0.3;$

$V_2(x) = 0.6,\quad V_2(y) = 0.5,\quad V_2(z) = 0.7;$

$V_3(x) = 0.8,\quad V_3(y) = 0.6,\quad V_3(z) = 0.8;$

Let $\tau_1 = \{0, U_2, 1\}$ and $\tau_2 = \{0, U_1, V_1, V_2, V_3, 1\}$.

If we define $f(X, \tau_1) \to (X, \tau_2)$ to be identity mapping, then it is easy to see that the inverse image of fuzzy clopen sets V_1 and V_2 in (I, τ_2) is fuzzy semi open in (I, τ_1). Thus the mapping f is fuzzy $I(so, clo)$. It is clear that in (I, τ_2), $cl\ U_1 = V_3'$ and in (I, τ_1), $S - \text{int} f^{-1}(V_3') = 0$. Hence U_1 is a fuzzy open set in (I, τ_2) with $f^{-1}(U_1) > S - \text{int} f^{-1}(clU_1)$. This shows that the mapping f is not fuzzy weakly semi continuous.

5.5 Theorem : If the graph function of a function is fuzzy $I(so,lo)$, then the function itself is fuzzy $I(so, clo)$.

Proof : Let $f: X \to Y$ be a mapping from a fuzzy space X to a fuzzy space Y such that the graph function $g: X \to X \times Y$ is fuzzy $I(so, clo)$. Let U be a fuzzy clopen set of Y. Then $1 \times U$ is a fuzzy clopen set of $X \times Y$. For each $x \in X$ we have.

$$g^{-1}(1 \times U)(x) = (1 \times U)\, g(x) = (1 \times U)(x, f(x))$$

$$= \min(1(x), U(f(x))) = 1 \wedge f^{-1}(U)(x) = f^{-1}(U)(x),$$

which is fuzzy semi open, because g is fuzzy $I(so, clo)$. Hence f is also fuzzy $I(so, clo)$.

5.6 Remark : Since the intersection of two fuzzy clopen sets is fuzzy clopen, the family of all fuzzy clopen sets of a fuzzy topological space (X, τ) forms a base for a fuzzy topology τ^* on X.

5.7 Definition : Fuzzy topology τ^* generated by the set of all fuzzy clopen sets of (X, τ) is called the fuzzy 0- dimensionalization of τ. A fuzzy topological space (X, τ) is fuzzy 0-dimensional if $\tau^* = \tau$.

5.8 Theorem : A mapping $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy $I(so, clo)$ if $f : (X, \tau_1) \to (Y, \tau_2^*)$ is fuzzy $I(so, o)$.

The proof is obvious from Theorem 5.2 (e).

5.9 Corollary : If $f : X \to Y$ is a fuzzy $I(so, clo)$ mapping and Y is a fuzzy 0-dimensional space,then f is fuzzy $I(so, o)$.

5.10 Theorem : If $f : X \to Y$ is a fuzzy $I(so, so)$ fuzzy presemi open mapping from a fuzzy space X on to fuzzy space Y and $g : Y \to Z$ is any mapping, then gof is fuzzy $I(so, clo)$ iff g is fuzzy $I(so, clo)$.

Proof : Suppose that $gof : X \to Z$ is fuzzy $I(so, clo)$ mapping. Let U be a fuzzy clopen set of Z. Then $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is fuzzy semi open set of X. Since f is fuzzy pre semi open and surjective, hence $f(f^{-1}(g^{-1}(U))) = g^{-1}(U)$is fuzzy semi open set of Y. Thus g is fuzzy $I(so, clo)$.

The converse is obvious.

5.11 Theorem : A fuzzy $I(so, clo)$ image of a fuzzy strongly S-closed space is fuzzy slightly compact.

Proof : Let $f : X \to Y$ be a fuzzy $I(so, clo)$ mapping from a fuzzy strongly S-closed space X on to a fuzzy space Y. Let $\{U_i\}_{i \in I}$ be a fuzzy clopen cover of Y. Since f is fuzzy $I(so,clo)$ $\{f^{-1}(U_i)\}$ is a fuzzy semi clopen cover of X. Since the space is fuzzy strongly S-closed, there exists a finite subset K of I such that

$$\bigvee_{i \in K} f^{-1}(U_i) = 1_X.$$

From the surjectivity of f, we have

$$f(\bigvee_{i \in K} f^{-1}(U_i)) = \bigvee_{i \in K} f(f^{-1}(U_i)) = \bigvee_{i \in K} U_i = 1_Y.$$

Thus Y is fuzzy slightly compact.

Since fuzzy semi compact space is fuzzy strongly S-closed, the following corollary is immediate :

5.12 Corollary : A fuzzy $I(so, clo)$ image of a fuzzy semi compact space is fuzzy slightly compact.

5.13 Theorem : Let $f : X \to Y$ be fuzzy $I(so, clo)$ injective mapping from a fuzzy space X to a fuzzy 0-dimensional space Y. Then X is fuzzy semi P whenever Y is fuzzy P, where $P \in \{T_0, T_1, T_2\}$ [4].

Proof : Since Y is fuzzy O-dimensional and f is fuzzy $I(so, clo)$ then by Corollary 5.9, f is fuzzy $I(so, o)$.

Suppose Y is fuzzy T_0. Let A be a fuzzy set of X. Then $f(A) = \vee_{i \in I} \wedge_{j = \in J_i} U_{ij}$, where U_{ij} are fuzzy open or fuzzy closed sets of Y. Since f is fuzzy $I(so, o)$ and injective, we have

$$A = f^{-1}(f(A)) = f^{-1}(\vee_{i \in I} \wedge_{j \in J_i} U_{ij}) = \vee_{i \in I} \wedge_{j \in J_i} f^{-1}(U_{ij}),$$

where $f^{-1}(U_{ij})$ are fuzzy semi open or fuzzy semi closed sets of X. Thus X is fuzzy semi T_0.

Next, let Y be fuzzy T_1. If A is a fuzzy set of X, then $f(A)$ can be written in the form $f(A) = \vee_{i \in I} U_i$, where U_i are fuzzy closed sets of Y. Since f is fuzzy $I(so, o)$ injection.

We have

$$A = f^{-1}(f(A))) = f^{-1}(\vee_{i \in I} U_i) = \vee_{i \in I} f^{-1}(U_i),$$

where $f^{-1}(U_i)$ are fuzzy semi closed sets of X. Thus X is fuzzy semi T_1.

Finally, suppose that Y is fuzzy T_2 space. Let A be a fuzzy set of X. Then $f(A)$ being fuzzy set of Y can be written as

$f(A) = \vee_{i \in I} \wedge_{j \in J_i} U_{ij} = \vee_{i \in I} \wedge_{j \in J_i} cl\ U_{ij}$, where U_{ij} are fuzzy semi open sets of Y.

Since f is injective and $I(so, 0)$, we have

$$A = f^{-1}(f(A)) = f^{-1}(\vee_{i \in I} \wedge_{J \in J_i} U_{ij}) = f^{-1}\left(\vee_{i \in I} \wedge_{J \in J_i} cl\ U_{ij}\right)$$

$$= \vee_{i \in I} \wedge_{j \in J_i} f^{-1}(cl\ U_{ij}) \geq \vee_{i \in I} \wedge_{J \in J_i} S - cl\, f^{-1}(U_{ij})$$

where $f^{-1}(U_{ij})$ are fuzzy open sets of X

Hence $A = A = \vee_{i \in I} \wedge_{j \in J} f^{-1}(U_{ij}) = \vee_{i \in I} \wedge_{j \in J_i} S - cl\, f^{-1}(U_{ij})$.

Thus X is fuzzy semi T_2.

5.14 Theorem : Let $f: X \to Y$ be a fuzzy $I(so, clo)$ injection from a fuzzy space X to fuzzy 0-dimensional space Y.

(a) If Y is fuzzy regular [4] and f is fuzzy open (or fuzzy closed), then X is fuzzy S-regular.

(b) If Y is fuzzy normal [4] and f is fuzzy closed, then X is fuzzy S-normal.

Proof : (a) Let A be a fuzzy open set of X. Since f is fuzzy open mapping , $f(A)$ is fuzzy open set of Y. Since Y is fuzzy regular, we have $f(A) = \vee_{i \in I} U_i$, where U_i are fuzzy open sets of Y with $cl\, U_i \leq f(A)$. By Corollary 5.9, f is fuzzy $I(so, o)$ and injective. We therefore have

$$A = f^{-1}(f(A)) = f^{-1}(\vee_{i \in I} U_i) = \vee_{i \in I} f^{-1}(U_i),$$

where $f^{-1}(U_i)$ are fuzzy semi open sets of X with $S = cl\, f^{-1}(U_i) \leq f^{-1}(cl\, U_i) \leq A$. Thus X is fuzzy S-regular.

(b) Let K and U be, respectively fuzzy closed and fuzzy open sets of X such that $K \leq U$. Then $f(K)$ is fuzzy closed and $f(U')'$ is fuzzy open set of Y such that $f(K) \leq f(U')'$. Since Y is fuzzy normal, there exists a fuzzy set V of Y such that

$$f(K) \leq \text{int } V \leq cl\, V \leq f(U')'.$$

By Corollary 5.9, f is fuzzy $I(so, o)$and injective, we have

$$K = f^{-1}(f(K)) \leq f^{-1}(\text{int } V) \leq S - \text{int}\, f^{-1}(V) \leq S - cl\, f^{-1}(V)$$

$$\leq f^{-1}(clV) \leq f^{-1}(f(U')') = U.$$

Thus X is fuzzy S-normal.

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