Jñānābha, Vol 24, 1994
(Dedicated to Professor J.N. Kapur on his 70th Birthday)

MAGNETOHYDRODYNAMIC TEMPERATURE DISTRIBUTION OF TWO IMMISCIBLE VISCOUS LIQUIDS BETWEEN TWO PARALLEL PLATES

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(Received : November 28, 1993)

ABSTRACT

This paper is concerned with unsteady flow of two conducting, incompressible,viscous, immiscible liquids between two parallel plates in the presence of uniform magnetic field applied perpendicular to the flow region. In the analysis, it has been assumed that the upper plate is moving with transient velocity while the lower plate is fixed. Velocity of the liquids and temperature at the plates have been obtained, following Soundalgekar et al. (1990). The effect on magnetic field, Reynolds number on the velocities of the lower and upper liquids and the temperature distribution of the lower and upper liquids for different values of Eckert number have been studied with the help of tables.

1. Introduction

Heat transfer in unsteady flows of immiscible viscous liquids in porous channel assumes importance due to its important applications in ground water hydrology, aero-dynamics and petroleum industry. However, this aspect has received little attention in the literature. Hartmann (1937) has done the pioneer work in the field of magnetohydrodynamics.

Temperature distribution in poiseuille flow between two parallel flat plates has been studied by Dube (1970). Mathur et al. (1972), Sacheti and Bhatt (1973) have considered the unsteady flow between two plates when lower plate is oscillating and upper plate is moving uniformely in their own planes in the presence and absence of a magnetic field respectively. Mathur and Jain (1981) have studied the magnetohydrodynamic temperature distribution for an unsteady laminar flow between two parallel plates.Sacheti et al. (1989) have discussed heat transfer in steady flow of immiscible fluids in a channel bounded below by a naturally permeable wall. Bhargava and Sacheti (1989) have investigated heat transfer in generalised Couette flow of

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two immiscible Newtonian fluids through a porous channel. Pal and Pal (1989) have investigated the transient temperature distribution in skin and subcutanceous tissues with improved variations in bio-physical parameters under various environmental conditions. Recently, Soundalgekar et al. (1990) have considered heat transfer in MHD unsteady stagnation point flow with variable wall temperature.

In the present paper, unsteady flow of two conducting, incompressible, viscous liquids between two parallel plates in presence of uniform magnetic field has been studied. Velocity of the liquids and temperature at the plates have been obtained, following Soundalgekar et al. (1990). The effect on magnetic field, Reynolds number on the velocities of the lower and upper liquids and the temperature distribution of the lower and upper liquids for different values of Eckert number have been studied with the help of tables.

2. Formulation of the Problem

Let us consider the laminar flow of two incompressible immiscible viscous, electrically, conducting liquids between two parallel plates. A constant magnetic field B_0 is applied perpendicular to the flow region. The depth betwen the plates is $2d$. The x-axis is assumed in the line flow along the central line which passes through the midway of the parallel plates and the y-axis is taken perpendicular to it. It is further assumed that the lower plate is fixed and the upper plate is moving with transient velocity U. The governing equations of motion and energy for the present configuration are :

$$\frac{\partial u_i}{\partial t} = \nu_i \frac{\partial^2 u_i}{\partial y^2} - \frac{\sigma_i}{\rho_i} B_0^2 u_i \qquad \ldots (2.1)$$

$$\frac{\partial T_i}{\partial t} = \frac{K}{\rho_i c_p} \frac{\partial^2 T_i}{\partial y^2} + \frac{\mu_i}{\rho_i c_p} \left(\frac{\partial u_i}{\partial y}\right)^2 \quad (i = 1, 2) \qquad \ldots (2.2)$$

where in these equations, ν_i is the kinematic coefficient of viscosity, u_i is the velocity of the liquids, σ_i is the electrical conductivity of the liquids, ρ_i is the density of the liquids, c_p is the specific heat at constant pressure, K is the thermal conductivity, t is the time μ_i is the coefficient of viscosity, B_0 is the magnitude of the magnetic field and the temperture is denoted by T_i.

The boundary conditions for the lower liquid are

$$\left.\begin{array}{llll} u_1 = U_0, & T_1 = T_d & \text{at} & y = 0 \\ u_1 = 0, & T_1 = T_0 & \text{at} & y = -d \end{array}\right\} \qquad \ldots (2.3)$$

The boundary conditions for the upper liquid are

$$\left.\begin{array}{llll} u_2 = U_0, & T_2 = T_d & \text{at} & y = 0 \\ u_2 = U_0 U, & T_2 = T_0 & \text{at} & y = d \end{array}\right\} \quad \ldots (2.4)$$

We introduce the following non-dimensional variables-

$$u_i^* = \frac{u_i}{U_0}, \quad y^* = \frac{y}{d}, \quad t^* = \frac{t}{d}, \quad T_i^* = \frac{T_i - T_0}{T_d - T_0}.$$

Using the above non-dimensional variables, the equations (2.1) and (2.2) after neglecting the asterisks over them are reduced to

$$\frac{\partial u_i}{\partial t} = \frac{1}{R_i} \frac{\partial^2 u_i}{\partial y^2} - M_i^2 u_i \quad \ldots (2.5)$$

and

$$\frac{\partial T_i}{\partial t} = \frac{1}{R_i E_{c_i}} \frac{\partial^2 T_i}{\partial y^2} + \frac{P_r}{R_i} \left(\frac{\partial u_i}{\partial y} \right)^2 \quad \ldots (2.6)$$

where,

$$\frac{\nu_i}{d} = \frac{1}{R_i} \text{ (Reynolds number)}, \quad \frac{K}{\mu_i c_p} = \frac{1}{E_{c_i}} \text{ (Eckert number)}.$$

$$B_0 \left(\frac{\sigma_i^d}{\rho_i} \right)^{1/2} = M_i \text{ (Hartmann number)}, \quad \frac{U_0^2}{(T_d - T_0) c_p} = P_r \text{ (Prandtl number)}$$

The non-dimensional boundary conditions for lower liquid are

$$\left.\begin{array}{llll} u_1 = 1, & T_1 = 1 & \text{at} & y = 0 \\ u_1 = 0, & T_1 = 0 & \text{at} & y = -1 \end{array}\right\} \quad \ldots (2.7)$$

The non-dimensional boundary conditions for the upper liquid are

$$\left.\begin{array}{llll} u_2 = 1, & T_2 = 1 & \text{at} & y = 0 \\ u_2 = U, & T_2 = 0 & \text{at} & y = 1 \end{array}\right\} \quad \ldots (2.8)$$

3. Solution of the Problem

Following Soundalgekar et al. (1990), we assume

$$\left.\begin{array}{l} u_i = u_i^{(0)}(y) + \in e^{-nt} u_i^{(1)}(y) \\ T_i = T_i^{(0)}(y) + \in e^{-nt} T_i^{(1)}(y) \end{array}\right\} \quad \ldots (3.1)$$

Substituting the values of u_i and T_i in the equations (2.5) and (2.6), we obtain.

$$\frac{\partial^2 u_i^{(0)}}{\partial y^2} - M_i^2 R_i u_i^{(0)}(y) = 0 \quad \ldots (3.2)$$

$$\frac{\partial^2 u_i^{(1)}}{\partial y^2} - R_i\,(M_i^2 - n)\, u_i^{(1)}(y) = 0 \qquad \ldots (3.3)$$

$$\frac{\partial^2 T_i^{(0)}}{\partial y^2} + P_r E_{c_i} \left(\frac{\partial u_i^{(0)}}{\partial y}\right)^2 = 0 \qquad \ldots (3.4)$$

$$\frac{\partial^2 T_i^{(1)}}{\partial y^2} + nR_i E_{c_i} T_i^{(1)}(y) = -2P_r E_{c_i} \frac{\partial u_i^{(0)}}{\partial y} \cdot \frac{\partial u_i^{(1)}}{\partial y} \qquad \ldots (3.5)$$

Taking $i = 1$ and $i = 2$, the solutions of equations (3.2) and (3.3) under the boundary conditions (2.7) and (2.8), are

$$u_1^{(0)} = \frac{\sinh b_1 (1 + y)}{\sinh b_1} \qquad \ldots (3.6)$$

$$u_2^{(0)} = \frac{\sinh b_2 (1 - y) + U \sinh b_2 y}{\sinh b_2} \qquad \ldots (3.7)$$

$$u_1^{(1)} = u_2^{(1)} = 0 \qquad \ldots (3.8)$$

where $b_1 = M_1 \sqrt{R_1}$, $b_2 = M_2 \sqrt{R_2}$.

Taking $i = 1$ and $i = 2$, the solution of equation (3.4) and (3.5) with the help of equations (3.6) and (3.7) under the boundary conditions (2.7) and (2.8), are

$$T_1^{(0)} = 1 - \frac{P_r E_{c_1} b_1^2}{2\sinh^2 b_1}\left[\frac{y^2}{2} + \frac{\cosh 2b_1 (1+y)}{4b_1^2}\right] + \left[1 - \frac{P_r E_{c_1} b_1^2}{2\sinh^2 b_1}\left\{\frac{1}{2} + \frac{1}{4b_1^2} - \frac{\cosh 2b_1}{4b_1^2}\right\}\right] y + \frac{P_r E_{c_1} b_1^2 \cosh 2b_1}{8b_1^2 \sinh^2 b_1} \qquad \ldots (3.9)$$

$$T_2^{(0)} = 1 - \frac{P_r E_{c_2} b_2^2}{\sinh^2 b_2}\left[\frac{\cosh 2b_2 y}{4b_2^2}\left\{\frac{U^2}{2} - U\cosh b_2\right\} + \frac{y^2}{2}\left\{\frac{U^2}{2} + \frac{1}{2} - U\cosh b_2\right\} + \frac{\cosh 2b_2 (1-y)}{8b_2^2} - \frac{U \sinh b_2 \sinh 2b_2 y}{4b_2^2}\right]$$

$$- y - \frac{b_2 P_r E_{c_2}}{\sinh^2 b_2}\left[\frac{\cosh 2b_2}{4b_2^2}\left\{\left(\frac{1}{2} - \frac{U^2}{2} + U\cosh b_2\right) y - \frac{1}{2}\right\}\right.$$

$$+\frac{U}{4b_2^2}\left\{\left(\frac{U}{2}-\frac{1}{2U}-\cosh b_2\right)y-\left(\frac{U}{2}-\cosh b_2\right)\right\}$$

$$\left.-\frac{y}{2}\left\{\frac{U^2}{2}+\frac{1}{2}-U\cosh b_2\right\}+y\,\frac{U\sinh 2b_2\sinh b_2}{4b_2^2}\right] \qquad \ldots (3.10)$$

and $$T_1^{(1)}=T_2^{(1)}=0 \qquad \ldots (3.11)$$

On putting the values of $u_1^{(0)}, u_2^{(0)}, u_1^{(1)}, u_2^{(1)}, T_1^{(0)}, T_2^{(0)}, T_1^{(1)}$ and $T_2^{(2)}$ in equation (3.1), we obtain

$$u_1=\frac{\sinh b_1(1+y)}{\sinh b_1} \qquad \ldots (3.12)$$

$$u_2=\frac{\sinh b_2(1-y)+U\sinh b_2 y}{\sinh b_2} \qquad \ldots (3.13)$$

$$T_1=1-\frac{P_rE_{c_1}b_1^2}{2\sinh^2 b_1}\left[\frac{y^2}{2}+\frac{\cosh 2b_1(1+y)}{4b_1^2}\right]+$$

$$\left[1-\frac{P_rE_{c_1}b_1^2}{2\sinh^2 b_1}\left\{\frac{1}{2}+\frac{1}{4b_1^2}-\frac{\cosh 2b_1}{4b_1^2}\right\}\right]y+\frac{P_rE_{c_1}b_1^2\cosh 2b_1}{8b_1^2\sinh^2 b_1} \qquad \ldots (3.14)$$

and

$$T_2=1-\frac{P_rE_{c_2}b_2^2}{2\sinh^2 b_2}\left[\frac{\cosh 2b_2 y}{4b_2^2}\left\{\frac{U^2}{2}-u\cosh b_2\right\}\right.$$

$$+\frac{y^2}{2}\left\{\frac{U^2}{2}+\frac{1}{2}-U\cosh b_2\right\}+\frac{\cosh 2b_2(1-y)}{8b_2^2}$$

$$\left.-\frac{U\sinh b_2\sinh 2b_2 y}{4b_2^2}\right]-y-\frac{b_2^2P_rE_{c_2}}{\sinh^2 b_2}\left[\frac{\cosh 2b_2}{4b_2^2}\right.$$

$$\left\{\left(\frac{1}{2}-\frac{U^2}{2}+U\cosh b_2\right)y-\frac{1}{2}\right\}+\frac{U}{4b_2^2}\left\{\left(\frac{U}{2}-\frac{1}{2U}-\cosh b_2\right)y\right.$$

$$\left.-\left(\frac{U}{2}-\cosh b_2\right)\right\}-\frac{y}{2}\left\{\frac{U^2}{2}+\frac{1}{2}-U\cosh b_2\right\}$$

$$+y\frac{U\sinh 2b_2 \sinh b_2}{4b_2^2}\Bigg] \quad \ldots (3.15)$$

Table-I

Velocity of the lower liquid defined in equation (3.12) for different values of Hartmann number M_1.

y	$M_1 = 1.0$	$M_1 = 3.0$	$M_1 = 5.0$
0	1.00000	1.00000	1.00000
– 0.1	0.81112	0.54880	0.36788
– 0.2	0.65499	0.30118	0.13544
– 0.3	0.52506	0.16526	0.04979
– 0.4	0.41619	0.09065	0.01832
– 0.5	0.32403	0.04966	0.00674
– 0.6	0.24487	0.02709	0.00248
– 0.7	0.17554	0.01459	0.00091
– 0.8	0.11325	0.00748	0.00034
– 0.9	0.05551	0.00316	0.00011
– 1.0	0.00000	0.00000	0.00000

Table-II

Velocity of the lower liquid defined in equation (3.12) for different values of Reynolds number R_1.

y	$R_1 = 2.0$	$R_1 = 4.0$	$R_1 = 6.0$
0	1.00000	1.00000	1.00000
– 0.1	0.85030	0.81112	0.77903
– 0.2	0.71763	0.65499	0.60503
– 0.3	0.59934	0.52506	0.46752
– 0.4	0.49305	0.41619	0.35819
– 0.5	0.39664	0.32403	0.27048
– 0.6	0.30818	0.24487	0.19907
– 0.7	0.22589	0.17554	0.13966
– 0.8	0.14812	0.11325	0.08868
– 0.9	0.07333	0.05551	0.04304
– 1.0	0.00000	0.00000	0.00000

Table-III

Velocity of the upper liquid defined in equation (3.13) for different values of Hartmann number M_2 at $U = 2.0$

y	$M_2 = 1.5$	$M_2 = 3.5$	$M_2 = 5.5$

0	1.00000	1.00000	1.00000
0.1	0.78015	0.47786	0.38528
0.2	0.63997	0.23152	0.14914
0.3	0.56513	0.11874	0.05955
0.4	0.54799	0.07449	0.02847
0.5	0.58681	0.07322	0.02537
0.6	0.68555	0.11427	0.04718
0.7	0.85491	0.22108	0.11535
0.8	1.11030	0.45555	0.29687
0.9	1.47960	0.95280	0.77009
1.0	2.00000	2.00000	2.00000

Table-IV

Velocity of the upper liquid defined in equation (3.13) for different values of Reynolds number R_2 at $U = 2.0$

y	$R_2 = 2.5$	$r_2 = 4.5$	$R_2 = 6.5$
0	1.00000	1.00000	1.00000
0.1	0.87478	0.78015	0.71608
0.2	0.79907	0.63997	0.53812
0.3	0.76844	0.56513	0.43988
0.4	0.78125	0.54799	0.40674
0.5	0.83828	0.58681	0.44384
0.6	0.94256	0.68555	0.52512
0.7	1.10020	0.85491	0.69415
0.8	1.32000	1.11030	0.96593
0.9	1.61430	1.47960	1.38080
1.0	2.00000	2.00000	2.00000

Table-V

Temperature distribution of the lower liquid defined in equation (3.14) for different values of Eckert number E_{c_1} at $P_r = 0.71$

y	$E_{c_1} = 0.10$	$E_{c_1} = 0.20$	$E_{c_1} = 0.30$
0.0	1.00000	1.00000	1.00000
– 0.1	0.90478	0.90956	0.91434
– 0.2	0.80744	0.81489	0.82234
– 0.3	0.70866	0.71733	0.72598
– 0.4	0.60886	0.61744	0.62659
– 0.5	0.50834	0.51673	0.52507
– 0.6	0.40730	0.41467	0.42197
– 0.7	0.30643	0.31183	0.31771
– 0.8	0.20415	0.20838	0.21253
– 0.9	0.10217	0.10442	0.10658
– 1.0	0.00004	0.00006	0.00008

Table-VI

Temperature distribution of the upper liquid defined in equation (3.15) for different values of Eckert number E_{c_2} at $P_r = 0.71$ and $u = 2.0$

y	$E_{c_2} = 0.15$	$E_{c_2} = 0.25$	$E_{c_1} = 0.35$
0	0.99593	0.99320	0.99045
0.1	0.67094	0.51770	0.36484
0.2	0.34495	0.04126	− 0.26212
0.3	0.02376	− 0.42745	− 0.87821
0.4	− 0.28540	− 0.87623	− 1.46660
0.5	− 0.56901	− 1.28600	− 2.00020
0.6	− 0.81289	− 1.62230	− 2.43090
0.7	− 0.97224	− 1.82110	− 2.66920
0.8	− 0.97184	− 1.76220	− 2.54700
0.9	− 0.69042	− 1.21770	− 1.74450
1.0	− 1.14060	− 1.90170	− 2.66200

4. Discussion

The numerical values of the velocity of the lower liquid for different values of Hartmann number M_1 and Reynolds number R_1 with y have been listed in Table-I and Table II, respectively. A study of these two tables shows that the velocity of the lower liquid decreases as the intensity of magnetic field M_1 increases and similar in the case when Reynolds number R_1 increases. Besides, we observe that on increasing y the velocity of the lower liquid decreases in both the tables.

The numerical values of the velocity of the upper liquid for different values of Hartmann number M_2 and Reynolds number R_2 with y at $U = 2.0$ have been listed in Table-III and Table IV, respectively. It is observed from these tables that the velocity of the upper liquid decreases as the intensity of the magnetic field M_2 increases. Again when Reynolds number R_2 increases the velocity also decreases for given value of parameter. It is interesting to note that when y increases the velocity of the upper liquid decreases while it increases again as y increases from $y = 0.5$ to $y = 1.0$.

The numerical values for the temperature of the lower liquid for different values of Eckert number E_{c_1} at $P_r = .71$ are shown in Table-V. We observe that the temperature of the lower liquid increases as the Eckert number E_{c_1} increases.Besides on increasing y temperature of the lower liquid decreases.

The numerical values for the temperature of the upper liquid for different values of the Eckert number E_{c_2} at $P_r = 0.71$ and $U = 2.0$ are shown in Table-VI. Here we observe that the temperature of the upper liquid decreases as the Eckert number E_{c_2} increases.Besides on increasing y temperature of the upper liquid decreases.

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