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(Dedicated to Professor S.A.Naimpally on his 62th Birthday)

THE INTEGRATION OF CENRTAIN PRODUCTS OF THE MULTIVARIABLE *H*-FUNCTION WITH FOX'S *H*-FUNCTION AND A GENERAL CLASS OF POLYNOMIALS

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ABSTRACT

This paper presents six finite integrals involving the product of Jacobi Polynomials, Fox's *H*-function, the multivariable *H*- function and a general class of polynomials. The results established here are quite general in nature and a large number of (known and new) integrals can be obtained by specializing the parameters suitably of the various function involved in them.

1. INTRODUCTION

Srivastava and Panda [7] defined the multivariable *H*-function as (*see also* [8])

$$H\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} = H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}$$

$$\begin{pmatrix} [(a):e';\ldots;e^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ [(c):\psi';\ldots;\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]; \end{pmatrix}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} V(\xi_1,\ldots,\xi_r)\,U_1(\xi_1)\ldots U_r(\xi_r)\,z_1^{\xi_1}\ldots z_r^{\xi_r}\,d\xi_1\ldots d\xi_r \qquad \ldots (1)$$

where $$v(\xi_1,\ldots,\xi_r) = \frac{\prod_{J=1}^{\lambda}\Gamma\left[1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}\xi_i\right]}{\prod_{j=\lambda+1}^{A}\Gamma\left[a_j-\sum_{i=1}^{r}\theta_j^{(i)}\xi_i\right]\prod_{j=1}^{C}\Gamma\left[1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}\xi_i\right]}$$

and

$$U_i(\xi_i) = \frac{\prod_{j=1}^{u^{(i)}} \Gamma\left[a_j^{(i)} - \delta_j^{(i)} \xi_i\right] \prod_{j=1}^{v^{(i)}} \Gamma\left[1 - b_j^{(i)} + \phi_j^{(i)} \xi_i\right]}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma\left[1 - d_j^{(i)} + \delta_j^{(i)} \xi_i\right] \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma\left[b_j^{(i)} - \phi_j^{(i)} \xi_i\right]}$$

$$i = 1, \ldots, r \qquad \ldots (3)$$

The multiple integral in (1) converges absolutely if

$$\left| arg\,(Z_i) \right| < \frac{1}{2} T_i \pi \quad i = 1, \ldots, r \qquad \ldots (4)$$

where

$$T_i = -\sum_{j=\lambda+1}^{A} \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{C} \psi_j^{(i)}$$

$$+ \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} > 0, \; i = 1, \ldots, r \qquad \ldots (5)$$

We shall require the following definition of the general class of polynomials ([9], p.1, Eq.(1))

$$S_n^m[x] = \sum_{\alpha'}^{[n/m]} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'} x^{\alpha'}, \; \alpha' = 0, 1, 2, \ldots \qquad \ldots (6)$$

where m is arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constants, real or complex.

By suitably specializing the coefficients $A_{n,\alpha'}$, the general class of polynomials $S_n^m[x]$ can be reduced to the well-known classical orthogonal polynomials such as Hermite polynomials, Jacobi polynomials and its various pecial cases, Legendre polynomals, Tchebycheff polynomials, Laguerre polynomials, Bessel polynomials, Gould-Hopper polynomials, Brafman polynomials etc. and their various combinations.

The seies representaion of Fox's H-function (see also [5])

$$H_{P,Q}^{M,N}\left[z \left| \begin{matrix} (e_p), E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right] = \sum_{g=1}^{M} \sum_{G=0}^{\infty} (-1)^G \phi(\eta_G) z^{\eta_G} \{G!\,F_g\}^{-1}, \qquad \ldots (7)$$

where

$$\phi(\eta_G) = \prod_{j=1, j\neq g}^{M} \Gamma(f_j - F_j \eta_G) \prod_{j=1}^{N} \Gamma(1 - e_j + E_j \eta_G)$$

$$\cdot \left\{ \prod_{j=M+1}^{Q} \Gamma(1 - f_j + F_j \eta_G) \prod_{j=N+1}^{P} \Gamma(e_j - E_j \eta_G) \right\}^{-1} \qquad \ldots (8)$$

and $\eta_G = (f_g + G)/F_g$

The following notations and known results will be used throughout this paper :

$$(\alpha)_w = \frac{\Gamma(\alpha+w)}{\Gamma(\alpha)} = \begin{cases} 1, \text{ is } w = 0 \\ \alpha(\alpha+1)\ldots(\alpha+w-1), \forall \quad w \in \{1, 2, 3, \ldots\} \end{cases} \quad \ldots (9)$$

Next, for the Jacobi polynomials $P_w^{(\alpha,\beta)}(x)$ [6, p. 25 4, E_q. (1)], we have

$$P_w^{(\alpha,\beta)}(t+\rho)\, P_w^{(\alpha,\beta)}(t-\rho)\, \frac{(-1)^w (1+\alpha)_w (1+\beta)_w}{(w!)^2}$$

$$= \sum_{R=0}^{w} (-w)_R \frac{(1+\alpha+\beta+w)_R}{(1+\alpha)_R (1+\beta)_R} P_R^{(\alpha,\alpha)}(x)\, t^R \quad \ldots (10)$$

$$\rho^w P_w^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right) = \frac{(1+\alpha)_W}{w!} \sum_{R=0}^{w} \frac{(-w)_R}{(1+\alpha)_R} P_R^{(\alpha,\alpha)}(x)\, t^R \quad \ldots (11)$$

$$\frac{1}{\rho}(1-t-\rho)^{-\alpha}(1+t+\rho)^{\beta} = 2^{-\alpha-\beta} \sum_{R=0}^{\infty} P_R^{(\alpha,\beta)}(x)\, t^R \quad \ldots (12)$$

In each of the formulas (10), (11) and (12), and throughout this paper, $\rho = (1-2xt+t^2)^{1/2}$. Formulas (10), (11) and (12) can be found, for example in [1,p.945] and [3,p.172], respectively.

$$\int_{-1}^{1} (1-x)^{\alpha}(1+x)^{v} P_w^{(\alpha,\beta)}(x)\, S_n^m[(1+x)^h]\, H_{P,Q}^{M,N}\left[z(1+x)^{h'} \middle| \begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right]$$

$$. H\begin{bmatrix} z_1(1+x)^{h_1} \\ \vdots \\ z_r(1+x)^{h_r} \end{bmatrix} dx$$

$$= 2^{\alpha+v+h'\eta_G+h\alpha'+1}\,\Gamma(\alpha+w+1) \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'}$$

$$. \frac{(-1)^G\, \phi(\eta_G)}{G!\, G_g} z^{\eta_G}$$

$$. H_{A+2,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[-v-h'\eta_G-h\alpha':h_1;\ldots;h_r]',\\ [(c):\psi';\ldots;\psi^{(r)}].\end{matrix}\right.$$

$$[\beta-v-h'\eta_G-h\alpha':h_1;\ldots;h_r],[(a):\theta';\ldots;\theta^{(r)}]:$$
$$[\beta-v-h'\eta_G+w:h_1;\ldots;h_r],[-\alpha-v-h'\eta_G-h\alpha'-w-1:h_1;\ldots;h_r]:$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];2^{h_1}z_1\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];2^{h_r}z_r\end{matrix}\right) \quad \ldots (13)$$

where

$Re(\alpha) > -1$, $Re(v) > -1$, $h > 0$, $h' > 0$, $h_i > 0$, $|\arg z| < \frac{T\pi}{2}$, $|\arg|(z_i)| < \frac{T_i\pi}{2}$,

$T > 0$, $T_i > 0$, $\text{Re}\left(v + h'f_L/F_L + \sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}\right) > -1$, $i = 1, \ldots, r$,

$j = 1, \ldots, u^{(i)}$, $L = 1, \ldots, M$, and m is an arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constants, real or complex.

$$\int_{-1}^{1} (1-x)^{u} (1+x)^{v} P_{w}^{(\alpha,\beta)}(x)\; S_{n}^{m}\left[(1-x)^{h}(1+x)^{k}\right]$$

$$H_{P,Q}^{M,N}\left[z(1-x)^{h'}(1+x)^{k'}\left|\begin{matrix}(e_P, E_P)\\(f_Q, F_Q)\end{matrix}\right.\right].\, H\begin{pmatrix} z_1(1-x)^{h_1}(1+x)^{k_1}\\ \vdots \\ z_r(1-x)^{h_r}(1+)^{k_r}\end{pmatrix} dx$$

$$= 2^{u+v+(h'+k')\eta_G+(h+k)\alpha'+1} \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{l=0}^{w} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'}$$

$$\cdot \frac{(-1)^G \phi(\eta_G)}{G!\,F_g}\, z^{\eta_G} \frac{(-w)_l (\alpha+\beta+w+1)_l}{(\alpha+1)_l\, l!}$$

$$\cdot H_{A+2,C+1:[B',D'];[B^{(r)},D^{(r)}]}^{[0,\lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[-v-h'\eta_G-k\alpha':k_1;\ldots;k_r],\\ [(c):\psi';\ldots;\psi^{(r)}],\end{matrix}\right.$$

$$[-u-h\alpha'-l:h_1;\ldots;h_r],\ [(a):\theta';\ldots;\theta^{(r)}]:$$

$$[-u-v-(h'+k')\eta_G-(h+k)\alpha'-l-1:\ h_1+k_1;\ldots;h_r+k_r];$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\ 2^{h_1+k_r}z_1\\ \vdots\\ [(d'):\delta'];\ldots;[(d)^{(r)}):\delta^{(r)}];\ 2^{h_r+k_r}z_r\end{matrix}\right), \qquad \ldots (14)$$

where

$$Re(u) > -1, Re(v) > -1, h > 0, k > 0, h' > 0, k' > 0, h_i > 0,$$

$$k_i > 0,\ |\,arg\,(z)\,| < \frac{T\pi}{2},\ |\,arg\,(z_i)\,| < \frac{T_i\pi}{2},\ T > 0, T_i > 0,$$

$$Re(u + h' f_L/F_L + \sum_{i=1}^{r} h_i\; d_j^{(i)}/\delta_j^{(i)} > -1,$$

$$Re(v + k' f_L/F_L + \sum_{i=1}^{r} k_i\; d_j^{(i)}/\delta_j^{(i)}) > -1, i = 1, \ldots, r,$$

$j = 1, \ldots, u^{(i)}$, $L = 1, \ldots, M$, and

m is an arbitrary positive integer and the coefficients $A_{n.\alpha'}$, $(n, \alpha' >= 0)$ are arbitrary constants, real or complex. The integral (14) can be established by making use of a known result recently obtained by Chaurasia and Girdhari Lal in [2,*p*.100,*Eqn*.(1.12)].

2. MAIN INTEGRALS

The following integrals have been evaluated in this section

$$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{v} P_{w}^{(\alpha,\beta)} (t+\rho) P_{w}^{(\alpha,\beta)} (t-\rho) S_{n}^{m} [(1+x)^{h}]$$

$$. H_{P,Q}^{M,N}\left[z (1+x)^{h'} \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right.\right] H \begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r} \end{pmatrix} dx$$

$$= 2^{\alpha+v+h'\eta_G+h\alpha'+1} (-1)^{w} \frac{\Gamma(1+\alpha+w)\,\Gamma(1+\beta+w)}{(w\,!)^2}$$

$$. \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{R=0}^{w} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'} \frac{(-1)^G \phi(\eta_G) z^{\eta_G}}{G!\,F_g}$$

$$. \frac{(-w)_R (1+\alpha+\beta+w)_R}{\Gamma(1+\beta+R)}$$

$$. H_{A+2,\,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\,\lambda+2\;:(u',v');\ldots;(u^{(r)},v^{(r)})} \left(\begin{matrix} [-v-h'\eta_G-h\alpha' : h_1;\ldots;h_r], \\ [(c):\psi';\ldots;\psi^{(r)}], \end{matrix} \right.$$

$$[\alpha-v-h'\eta_G-h\alpha' : h_1;\ldots;h_r],\ [(a):e';\ldots;e^{(r)}]:$$
$$[\alpha-v-h'\eta_G-h\alpha'+R : h_1;\ldots;h_r],\ [-\alpha-v-h'\eta_G-h\alpha'-R-1 : h_1;\ldots;h_r]:$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b)^{(r)}):\phi^{(r)}];\ 2^{h_1} z_1 \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\ 2^{h_r} z_r \end{matrix} \right), \qquad \ldots (15)$$

where

$\mathrm{Re}(\alpha) > -1$, $\mathrm{Re}(v) > -1$, $h > 0$, $h' > 0$, $h_i > 0$,

$\mathrm{Re}(v + h' f_L/F_L + \sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}) > -1$, $|\arg z| < \frac{T\pi}{2}$,

$|\arg(z_i)| < \frac{T_i \pi}{2}$, $T > 0$, $T_1 > 0$, $i = 1, \ldots, r$, $j = 1, \ldots, u^{(i)}$,

$L = 1, \ldots, M$

and m ia an arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constant, real or complex.

$$\int_{-1}^{1} \rho^{w} (1-x)^{\alpha} (1+x)^{v} P_{w}^{(\alpha,\alpha)} \left(\frac{1-xt}{\rho}\right) S_{n}^{m}\left[(1+x)^{h}\right]$$

$$.\, H_{P,Q}^{M,N}\left[z\,(1+x)\,h' \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right] H \begin{pmatrix} z_1\,(1+x)^{h_1} \\ z_r\,(1+x)^{h_r} \end{pmatrix} dx$$

$$= 2^{\alpha + v + h'\eta_G + h\alpha' + 1}\, \frac{\Gamma(1+\alpha+w)}{w\,!} \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{R=0}^{w} \frac{(-n)_m \alpha'}{\alpha'\,!} A_{n,\alpha'}$$

$$.\, \frac{(-1)^G\,\phi(\eta_G)}{G\,!\,F_g}\, z^{\eta_G}\,(-w)_R\, t^R$$

$$.\, H_{A+2,\,C+2:[B',D']:\ldots;[B^{(r)},D^{(r)}]}^{0,\ \lambda+2\ :(u',v');\ldots;(u^{(r)},v^{(r)})} \left(\begin{matrix} [-v\,h'\eta_G - h\alpha' : h_1 ; \ldots; h_r], \\ [(c) : \psi' ; \ldots; \psi^{(\rho)}], \end{matrix} \right.$$

$$[\alpha - v - h'\eta_G - h\alpha' : h_1 ; \ldots; h_r],\ [(a) : e' ; \ldots; e^{(r)}] :$$
$$[\alpha - v - h'\eta_G - h\alpha' + R : h_1; \ldots; h_r],\ [-\alpha - v - h'\eta_G - h\alpha' - R - 1 : h_1; \ldots; h_r] :$$

$$\left. \begin{matrix} [(b') : \phi'] ; \ldots; [(b^{(r)}) : \phi^{(r)})] ;\ 2^{h_1} z_1 \\ [(d') : \delta'] ; \ldots; [(d^{(r)}) : \delta^{(r)}] ;\ 2^{h_r} z_r \end{matrix} \right) \qquad \ldots (16)$$

where

$\mathrm{Re}(\alpha) > -1$, $\mathrm{Re}(v) > -1$, $h > 0$, $h' > 0$, $h_i > 0$,

$\mathrm{Re}(v + h' f_L / F_L + \sum_{i=1}^{r} h_i\, d_j^{(i)} / \delta_j^{(i)}) > -1$,

$|\arg z| < \frac{T\pi}{2}$, $|\arg(z_i)| < \frac{T_i \pi}{2}$, $T > 0$, $T_i > 0$,

$i = 1, \ldots, r, j = 1, \ldots, u^{(i)}$, $L = 1, \ldots, M$, and

m is an arbitrary positive integer5 and the coefficients $A_{n,\alpha'}$ $(n, \alpha' >= 0)$ are arbitrary constant, real or complex.

$$\int_{-1}^{1} \frac{1}{\rho}(1+t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}(1-x)^{\alpha}(1+x)^{v} S_n^m\left[(1+x)^h\right]$$

$$.\, H_{P,Q}^{M,N}\left[z\,(1+x)^{h'} \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right] H \begin{pmatrix} z_1\,(1+x)^{h_1} \\ z_r\,(1+x)^{h_r} \end{pmatrix} dx$$

$$= 2^{v + h'\eta_G + h\alpha' - \beta + 1} \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{R=0}^{\infty} \frac{(-n)_{m\alpha'}}{\alpha'\,!} A_{n,\alpha'}$$

$$\cdot \frac{(-1)^G \phi(\eta_G)}{G!\,F_g} z^{\eta_G} \Gamma(1+\alpha+R)\, t^R$$

$$\cdot H^{0,\ \lambda+2\ :\ (u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+2:[B',D'];\ldots;[B^{(r)},D^{(r)}]} \left(\begin{matrix} [-v-h'\eta_G-h\alpha';h_1;\ldots;h_r], \\ [(c):\psi',\ldots,\psi^{(r)}], \end{matrix} \right.$$

$$\left. \begin{matrix} [\beta-v-h'\eta_G-h\alpha':\ h_1;\ldots;h_r],[(a):e';\ldots;e^{(r)}]: & [(b'):\phi' \\ [\beta-h'\eta_G-h\alpha'+R:h_1;\ldots;h_r],[-\alpha-v-h'\eta_G-h\alpha'-R-1:h_1;\ldots;h_r]:[(d'):\delta' \end{matrix} \right. \quad \ldots (17)$$

where

$Re(\alpha) > -1$, $Re(v) > -1$, $h > 0$, $h' > 0$, $h_i > 0$

$Re(v + h' f_L/F_L + \sum_{i=1}^{r} h_i\ d_j^{(i)}/\delta_j^{(i)}) > -1$.

$|arg\, z| < \frac{T\pi}{2}$, $|arg(z_i)| < \frac{T_i\pi}{2}$, $T > 0$, $T_i > 0$,

$i = 1, \ldots, r, j = 1, \ldots, u^{(i)}$, $L = 1, \ldots, M$

and m is an arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constant, real or complex.

$$\int_{-1}^{1} (1-x)^u (1+x)^v P_w^{(\alpha,\beta)}(t+\rho)\, P_w^{(\alpha,\beta)}(t-\rho)\ S_n^m\left[(1-x)^h (1+x)^k\right].$$

$$\cdot H^{M,N}_{P,Q}\left[z(1+x)^{h'}(1+x)^{k'} \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right.\right] H\left(\begin{matrix} z_1 (1-x)^{h_1}(1+x)^{k_1} \\ z_r (1-x)^{h_r}(1+x)^{k_r} \end{matrix} \right) dx$$

$$= 2^{u+v+(h'+k')\eta_G+(h+k)\alpha'+1} \frac{(-1)^w \Gamma(1+\alpha+w)\Gamma(1+\beta+w)}{(w!)^2}$$

$$\cdot \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{l=0}^{R} \sum_{R=0}^{\infty} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'} \frac{(-1)^G \phi(\eta_G)}{G!\,F_g} z^{\eta_G}$$

$$\frac{(-R)_l (1+\alpha+\beta+R)_l}{(\alpha+1)_l\, l!} \frac{(-W)_R (1+\alpha+\beta+w)_R}{\Gamma(1+\alpha+R)\,\Gamma(1+\beta+R)} t^R$$

$$\cdot H^{0,\ \lambda+2\ :\ (u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]} \left(\begin{matrix} [-v-h'\eta_G-k\alpha':k_1;\ldots;k_r], \\ [(c):\psi';\ldots;\psi^{(r)}], \end{matrix} \right.$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)});2^{h_1+k_1}z_1 \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\ 2^{h_r+k_r}z_r \end{matrix} \right), \quad \ldots (18)$$

where

$\mathrm{Re}(u) > -1, \mathrm{Re}(v) > -1, h > 0, k > 0, h' > 0, k' > 0, h_i > 0, k_i > 0,$

$\mathrm{Re}(u + h' f_L/F_L + \sum_{i=1}^{r} h_i \; d_j^{(i)}/\delta_j^{(i)}) > -1,$

$\mathrm{Re}(u + h' f_L/F_L + \sum_{i=1}^{r} k_i \, d_j^{(i)}/\delta_j^{(i)}) > -1,$

$|\arg z| < \frac{T\pi}{2}, \; |\arg(z_i)| < T_i \pi/2, T > 0, T_i > 0,$

$i = 1, \ldots, r, \; j = 1, \ldots, u^{(i)}, L = 1, \ldots, M,$ and
m is an arbitrary positive integer and the coefficients A_n, $(n, \alpha' \geq 0)$ are arbitrary constants, real or complex.

$$\int_{-1}^{1} (1-x)^u (1+x)^v \rho^w P_w'(\alpha, \alpha)\left(\frac{1-xt}{\rho}\right) S_n^m\left[(1-x)^h (1+x)^k\right]$$

$$. H_{P,Q}^{M,N}\left[z(1-x)^{h'} (1+x)^{k'} \left| \begin{matrix} (e_l, E_P) \\ (f_Q, F_Q) \end{matrix} \right.\right] H \left(\begin{matrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{matrix} \right) dx$$

$$= 2^{u+v+(h'+k')\eta_G + (h+k)\alpha' + 1} \frac{\Gamma(1+\alpha+w)}{w!}$$

$$. \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{l=0}^{R} \sum_{R=0}^{w} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'} \frac{(-1)^G \phi(\eta_G)}{G! F_g} z^{\eta_G}$$

$$. \frac{(-R)_l (2\alpha+R+1)_l}{(\alpha+1)_l \, l!} \frac{(-w)_R}{\Gamma(1+\alpha+R)} t^R$$

$$. H_{A+2, C+1 : [B', D']; \ldots; [B^{(r)}, D^{(r)}]}^{0, \; \lambda+1 \; : (u'+v'); \ldots; (u^{(r)}, v^{(r)})} \left(\begin{matrix} [-v - h'\eta_G - k\alpha' : k_1; \ldots; k_r], \\ [(c) : \psi'; \ldots; \psi^{(r)}] \end{matrix} \right.$$

$$[-u - h\alpha' - l : h_1; \ldots; h_r], [(a) : e'; \ldots; e^{(r)}] :$$
$$[-u - v - (h'+k')\eta_G - (h+k)\alpha' - l - 1 : h_1 + k_1; \ldots; h_r + k_r] :$$

$$\left. \begin{matrix} [(b') : \phi']; \ldots; [(b)^{(r)}) : \phi^{(r)}] \; ; \; 2^{h_1+k_1} z_1 \\ [(d') : \delta']; \ldots; [(d^{(r)}) : \delta^{(r)}] \; ; \; 2^{h_r+k_r} z_r \end{matrix} \right) , \qquad \ldots (19)$$

where

$Re(u) > -1, Re(v) > -1, h > 0, k > 0, h' > 0, k' > 0, h_i > 0, k_i > 0,$

$Re\left(u + h' f_L/F_L + \sum_{i=1}^{r} h_i \; d_j^{(i)} / \delta_j^{(i)}\right) > -1,$

$Re\left(v + k' \; f_L/F_L + \sum_{i=1}^{r} k_i \, d_j^{(i)}/\delta_j^{(i)}\right) > -1,$

$|\arg z| < \frac{T\pi}{2}, \; |\arg(z_i)| < \frac{T_i \pi}{2}, T > 0, T_i > 0,$

$i = 1, ..., r,\ j = 1, ..., u^{(i)},\ L = 1, ..., M$
and m is an arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constants, real or complex.

$$\int_{-1}^{1} (1-x)^{u} (1+x)^{v} \frac{1}{\rho} (1-t+\rho)^{-\alpha} (1+t+\rho)^{-\beta}$$

$$. S_{n}^{m} \left[1-x)^{h} (1+x)^{k}\right] H_{P,Q}^{M,N} \left[z (1-x)^{h'} (1+x)^{k'} \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right]$$

$$. H \begin{pmatrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{pmatrix} dx$$

$$= 2^{-\alpha-\beta+u+v+(h'+k')\eta_G+(h+k)\alpha'+1}$$

$$. \sum_{\alpha'=0}^{[n/m]} \sum_{g=1}^{M} \sum_{G=0}^{\infty} \sum_{R=0}^{R} \sum_{R=0}^{\infty} \frac{(-n)_{m\alpha'}}{\alpha'!} A_{n,\alpha'} \frac{(-1)^{G} \phi(\eta_G)}{G!\,F_g} z^{\eta_G}$$

$$. \frac{(-R)_l\ (1+\alpha+\beta+R)_l}{(\alpha+1)_l\ l!} t^{R}$$

$$. H_{A+2,\, C+1 : [B', D'] ; ... ; [B^{(r)}, D^{(r)}]}^{0,\ \lambda+2\ : (u', v') ; ... ; (u^{(r)}, v^{(r)})} \left(\begin{matrix} [-v-h'\eta_G - k\alpha' : k_1 ; ... ; k_r] , \\ [(c) : \psi' ; ... ; \psi^{(r)}] , \end{matrix} \right.$$

$$[-u-h\alpha'-l : h_1 ; ... ; h_r] , [(a) : \theta' ; ... ; \theta^{(r)}] :$$
$$[-u-v (h'+k') \eta_G - (h+k) \alpha' - l - 1 : h_1 + k_1 ; ... ; h_r + k_r] :$$

$$\left. \begin{matrix} [(b') : \phi'] ; ... ; [(b^{(r)}) : \phi^{(r)}] ; 2^{h_1+k_1} z_1 \\ \vdots \\ [(d') : \delta'] ; ... ; [(d^{(r)}) : \delta^{(r)}] ; 2^{h_r+k_r} z_r \end{matrix} \right) , \qquad ... (20)$$

where

$Re(u) > -1, Re(v) > -1, h > 0, k > 0, h' > 0, k' > 0, h_i > 0, k_i > 0,$

$$Re \left\{ u + h' f_L/F_L + \sum_{i=1}^{r} h_i\ d_j^{(i)}/\delta_j^{(i)} \right\} > -1,$$

$$Re \left\{ v + k' f_L/F_L + \sum_{i=1}^{r} k_i\ d_j^{(i)}/\delta_j^{(i)} \right\} > -1,$$

$|\arg z| < T\pi/2,\ |\arg (z_i)| < T_i\pi/2,\ T > 0,.\ T_i > 0,$

$i = 1,...,r, j = 1,..., u^{(i)}, L = 1,..., M,$ and
m is an arbitrary positive integer and the coefficients $A_{n,\alpha'}$ $(n, \alpha' \geq 0)$ are arbitrary constants, real or complex.

Proof : To establish (15), multiply both the sides of (10) by

$$(1-x)^{\alpha}(1+x)^{v} S_n^m [(1+x)^h] H_{P,Q}^{M,N}\left[z(1+x)^{h'} \left| \begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right.\right] . H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r}\end{pmatrix}$$

and integration both sides with respect to x between the limits - 1 to 1, we get

$$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{v} P_w^{(\alpha,\beta)}(t+\rho) P_w^{(\alpha,\beta)}(t-\rho) S_n^m\left[(1+x)^h\right]$$

$$. H_{P,Q}^{M,N}\left[z(1+x)^{h'} \left| \begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right.\right] H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r}\end{pmatrix} dx$$

$$= \int_{-1}^{1} \frac{(-1)^w (1+\alpha)_w (1+\beta)_w}{(w!)^2} \sum_{R=0}^{w} \frac{(-w)_R (1+\alpha+\beta+w)_R}{(1+\alpha)_R (1+\beta)_R}$$

$$. P_R^{(\alpha,\alpha)}(x)\, t^R (1-x)^{\alpha} (1+x)^{v} S_n^m\left[(1+x)^h\right] H_{P,Q}^{M,N}\left[z(1+x)^{h'} \left| \begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right.\right]$$

$$. H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r}\end{pmatrix} dx, \qquad \text{.... (21)}$$

Now, express the $H_{P,Q}^{M,N}\left[z(1+x)^{h'} \left| \begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right.\right]$ functions on the right hand side of (21) by its series (7), then interchange the order of integration and summation in th right hand side of (21), which is justified due to the sbsolute convergent of the integral involved in the process and then evaluate the inner integral with the help of (13).

Finally, interpreting by virture of (1), we get the desired result.

The proofs of the formulas (16) to (20) can be developed by prceeding on similar lines with the help of the result (11) through (14) respectively.

3. SPECIAL CASES

Letting $h' \to 0$, the results in (13) to (20) reduces to known results recently obtained by Chaurasia and Girdhari Lal ([2], *eqn.*(1.11), *p.*99, *eqn.* (1.12), *p.*100 and *eqn.* (2.1) to (2.6), *pp.* 101-104) respectively.

The importance of our resluts lies in its manifold generality. In view of the generality of the polynomials $S_n^m[x]$ on suitbly specializing the coefficients $A_{n,\alpha}$, and making a free use of the special cases of $S_n^m[x]$ listed by Srivastava and Singh [10], our results can be reduced to a large number of integrals involving generalized Hermite polynomials, Hermite polynomials, Jacobi polynomials, and its various special cases, Laguerre polynomials, Bessel polynomials, Gould-Hopper polynomials, Brafman polynomials and their various combinations.

Secondly, by specializing the various parameters and variables in Fox's H-function and in the multivaribale H-function from our results, several integrals involving a remarkably wide variety of useful functions (or products of several such functions), which are expressible in terms of *E, F, G* and *H* functions of one and several variables. Thus the results presented in this paper would at once yield a very large number of integrals, involving a large variety of polynomials and various special functions occurring in the problems, of mathematical analysis.

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