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(Dedicated to Professro S.A. Naimpally on his 62th Birthday)

SOME INTEGRALS INVOLVING A GENERAL CLASS OF POLYNOMIALS, FOX'S *H*-FUNCTION AND THE MULTIVARIABLE *H*-FUNCTION

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ABSTRACT

In the present paper, we evaluate four finite integrals involving the products of Fox's H-function, the multivariable H-function and a general class of polynomials. On account of the most general nature of the functions involved herein a very large number of known and new integrals involving simpler special functions and orthogonal polynomials follows as particular cases of our main integrals.

1. INTRODUCTION

Srivastava [5] introduced the genral class of polynomials (see also Srivastava and Singh [9]) :

$$S_n^m[x] = \sum_{l=0}^{n/m} \frac{(-n)_{ml}}{l\,!} A_{n,l}\, x^l \;, \; l = 0, 1, 2, \ldots \qquad \ldots (1.1)$$

where m is an arbitrary postive integer and the coefficients $A_{n,l}\,(n, l \geq 0)$ are arbitrary constants real or complex. By suitable specializing the coefficients $A_{n,l}$, the general claass of polynomials can be reduced to a large spectrum of polynomials as cited in the paper's referred to above.

The series representation of Fox's H-function (see [4] and [1])

$$H_{P,Q}^{M,N}\left[z \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right. \right] = \sum_{G=0}^{M} \sum_{g=1}^{\infty} (-1)^G \phi(\eta_G)\, z^{\eta_G} \{G\,!\,F_g\}^{-1}, \qquad \ldots (1.2)$$

where

$$\phi(\eta_G) = \prod_{j=1, j\neq g}^{M} \Gamma(f_j - F_j \eta_G) \prod_{j=1}^{N} \Gamma(1 - e_j + E_j \eta_G)$$

$$\cdot\left\{\prod_{j=M+1}^{Q}\Gamma(1-f_j+F_j\eta_G)\prod_{j=N+1}^{P}\Gamma(e_j-E_j\eta_G)\right\}^{-1} \qquad \ldots (1.3)$$

and

$$\eta_G=(f_g=G)/F_g\,.$$

For the multivariable H-function defined by Srivastava and Panda ([5] and [7]; see also [6]), we establish four finite integrals in the next section.

2. THE MAIN INTEGRALS

The following integrals have been derived in this section :

$$\int_s^t (x-s)^{u-1}(t-x)^{v-1}(x-w)^{-u-v}\, S_n^m\left[z\left(\frac{x-s}{x-w}\right)^h\left(\frac{t-x}{x-w}\right)^k\right]$$

$$\cdot\, H_{P,Q}^{M,N}\left[y\left(\frac{x-s}{x-w}\right)^{h'}\left(\frac{t-x}{x-w}\right)^{k'}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$

$$\cdot\, H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:\\ [(c):\psi',\ldots,\psi^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]\;;\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\;;\end{matrix}\begin{matrix}z_1\left(\frac{x-s}{x-w}\right)^{h_1}\left(\frac{t-x}{x-w}\right)^{k_1}\\ \vdots\\ z_r\left(\frac{x-s}{(x-w)}\right)^{h_r}\left(\frac{t-x}{x-w}\right)^{k_r}\end{matrix}\right)dx$$

$$=\sum_{l=1}^{n/m}\frac{(-n)_{ml}}{l\,!}A_{n,l}\,z^l\sum_{g=1}^{M}\sum_{G=0}^{\infty}\frac{(-1)^G}{G\,!\,F_g}\,\phi(\eta_G)\,y^{\eta_G}$$

$$\cdot\,(t-s)^{u+v-1+(h+k)l+(h'+k')\eta_G}(t-w)^{-u-hl-h'\eta_G}(s-w)^{-v-kl-k'\eta_g}$$

$$H_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda+2\;:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[1-u-hl-h'\eta_G:h_1,\ldots,h_r]\\ [1-u-v-(h+k)\,l-(h'+k')\,\eta_G:h_1+k_1,\ldots,h_r+k_r]\end{matrix}\right.$$

$$[1-v-kl-k'\eta_G;k_1,\ldots,k_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];$$

$$[(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];$$

$$\left.\begin{matrix}z_1\left(\frac{t-s}{t-w}\right)^{h_1}\left(\frac{t-s}{s-w}\right)^{k_1}\\ \vdots\\ z_r\left(\frac{t-s}{t-w}\right)^{h_r}\left(\frac{t-s}{s-w}\right)^{k_r}\end{matrix}\right) \qquad \ldots (2.1)$$

where $h > 0, k > 0, h' > 0, k' > 0, h_i > 0, k_i > 0$,

$Re(u + h' f_L/F_L + \sum_{i=1}^{r} h_i d_j^{(i)}/\delta_j^{(i)}) > 0$,

$Re(v + k' f_L/F_L + \sum_{i=1}^{r} k_i d_j^{(i)}/\delta_j^{(i)}) > 0, w < s < t$,

$|arg\, y| < T\pi/2, \; |arg\,(z_i)| < T_i \pi/2, T > 0, T_i > 0$,

$i = 1, ..., r, \; j = 1, ..., u^{(i)}, L = 1, ..., M$,

m is an arbitrary positive integrer and the coefficients $A_{n,l}\,(n, l \geq 0)$ are arbitrary cosntants real or complex.

$$\left(T = \sum_{1}^{M} E_i - \sum_{N+1}^{P} E_i + \sum_{1}^{M} F_i - \sum_{m+1}^{Q} F_i\right).$$

$$\int_0^t x^{u-1}(t-x)^{v-1}\, S_n^m\left[yx^h(t-x)^k\right] H_{P,Q}^{M,N}\left[z\, x^{h'}(t-x)^{k'} \middle| \begin{matrix}(e_p, E_p)\\(f_Q, F_q)\end{matrix}\right]$$

$$\cdot H_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');...;(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):e',...,e^{(r)}]:\\ [(c):\psi',...,\psi^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi^1];...;[(b^{(r)}):\phi^{(r)}];z_1 x^{h_1}(t-x)^{k_1}\\ \vdots \\ [(d'):\delta^1];...;[(d^{(r)}:\delta^{(r)}];z_r x^{h_r}(t-x)^{k_r}\end{matrix}\right) dx$$

$$= \sum_{l=0}^{n/m} \frac{(-n)_{ml}}{l!} A_{n,l}\, y^l \sum_{g=1}^{M} \sum_{G=0}^{\infty} \frac{(-1)^G}{G!\, F_g} \phi(\eta_G) z^{\eta_G} t^{u+v+(h+k)l(h'+k')\eta_G - 1}$$

$$\cdot H_{A+2,C+1:[B',D'];...;[B^{(r)},D^{(r)}]}^{0,\;\lambda+2:(u',v');...;(u^{(r)},v^{(r)})}\left(\begin{matrix}[1-u-hl-h'\eta_G : h_1,...,h_r],\\ [l-u-v-(h+k)l-(h'+k')\eta_G :\end{matrix}\right.$$

$$[1-v-kl-k'\eta_G : k_1;...;k_r], [(a):e';...e^{(r)}]:[(b'):\phi'];...;[b^{(r)}):\phi^{(r)}]:$$

$$h_1+k_1;...;h_r+k_r], \qquad [(c):\psi';...;\psi^{(r)}]:[(d'):\delta'];...;[(a^{(r)}):\delta^{(r)}]:$$

$$\left.\begin{matrix}z_1\, t^{h_1+k_1}\\ \\ z_r\, t^{h_r+k_r}\end{matrix}\right). \qquad \text{... (2.2)}$$

where

$h > 0, k > 0, h' > 0, k' > 0, h_i > 0, k_i > 0, \; |arg\, z| < T\pi/2$,

$$| \arg (z_i) | < T_i \pi/2 , Re\left(u + h' f_L/F_L + \sum_{i=1}^{r} h_i \, d_j^{(i)}/\delta_j^{(i)}\right) > 0,$$

$$Re\left(v + k' f_L/F_L + \sum_{i=1}^{r} k_i \, d_j^{(i)}/\delta_j^{(i)}\right) > 0, i = 1, ..., r, \ j = 1, ..., u^{(i)},$$

$L = 1,...,M$, m is an arbitrary postive integers and the coefficients $A_{n,l}$ $(n, l \geq 0)$ arbitrary constants are real or complex.

$$\int_0^1 x^{u-1}(1-x)^{v-1} \, {}_2F_1(\alpha, \beta : u : x) S_n^m[y(1-x)^h] \, H_{P,Q}^{M,N}\left[z(1-x)\, h' \left| \begin{matrix} (e_P, E_P) \\ (f_Q, F_Q) \end{matrix} \right.\right]$$

$$. \, H_{A, C: (B', D'); ...; (B^{(r)}, D^{(r)})}^{0, \lambda : (u', v'); ...; (u^{(r)}, v^{(r)})} \begin{bmatrix} [(a) : e', ..., e^{(r)}] : \\ [(c) : \psi', ..., \psi^{(r)}] : \end{bmatrix} \left. \begin{matrix} [(b') : \phi'] ; ...; [(b^{(r)}) : \phi^{(r)}] ; z_1(l-x)^{h_1} \\ \vdots \\ [(d') : \delta'] ; ...; [(d^{(r)}) : \delta^{(r)}] ; z_r(1-x)^{h_r} \end{matrix} \right) dx$$

$$= \sum_{l=0}^{n/m} \frac{(-n)_{ml}}{l!} A_{n,l} Y^l \sum_{g=1}^{M} \sum_{G=0}^{\infty} \frac{(-1)^G}{G! \, F_g} \phi(\eta_G) \, z^{\eta_G}$$

$$. \, \Gamma(u) \, H_{A+2, C+1: [B', D']; ...; [B^{(r)}, D^{(r)}, D^{(r)}]}^{0, \lambda+1 : (u', v'); ...; (u^{(r)}, v^{(r)})} \left(\begin{matrix} [1 - v - hl - h'\eta_G : h_1, ..., h_r] \\ [1 + \alpha - u - v - hl - h'\eta_G : h_1, ..., h_r] \end{matrix} \right.$$

$$\begin{matrix} [1 - u - v - hl - h'\eta_G + \alpha + \beta : h_1 ; ... ; h_r], [(a) : e', ..., e^{(r)}] : \\ [1 + \beta - u - v - hl - h'\eta_G : h_1, ..., h_r], [(c) : \psi', ... \psi^{(r)}] : \end{matrix}$$

$$\left. \begin{matrix} [(b') : \phi'] ; ...; [(b^{(r)} : \phi^{(r)}] : z_1 \\ \vdots \\ [(d') : \delta'] ; ...; [(d^{(r)} : \delta^{(r)}] : z_r \end{matrix} \right), \qquad ... (2.3)$$

where

$h > 0, h' > 0, h_i > 0, T > 0, T_i > 0, \ | \arg z | < T\pi/2, \ | \arg (z_i) | < T_i \pi/2,$

$$Re\,(u + v - \alpha - \beta) > 0, Re\,(u) > 0, Re\left(v + h' f_L/F_L + \sum_{i=1}^{r} h_i \, d_j^i/\delta_j^{(i)}\right) > 0,$$

$i = 1,...,r, j = 1,..., u^{(i)}, l = 1,...,M$, m is an arbitrary poistive integer and the coefficients $A_{n,l}$ $(n, l \geq 0)$ are arbitrary constants real or complex.

$$\int_{-1}^{1} (1+x)^{u-1} (1-x)^{v-1} P_w^{(\alpha, \beta)} (1 - \frac{st}{2}(1-x)) \, S_n^m \left[y(1-x)^h (1+x)^k\right]$$

$$. H_{P,Q}^{M,N}\left[z\,(1+x)^{h'}(1-x)^{k'}\,\middle|\,\begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right]$$

$$. H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):e',\ldots,e^{(r)}]:\\ [(c):\psi',\ldots,\psi^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];z_1(1+x)^{h_1}(1-x)^{k_1}\\ \vdots\\ [(d'):\delta'];\ldots;[(d)^{(r)}):\delta^{(r)}];z_r(1+x)^{h_r}(1-x)^{k_r}\end{matrix}\right)dx$$

$$= \sum_{l=0}^{n/m}\frac{(-n)_{ml}}{l!}A_{n,l}\,y^l\sum_{g=1}^{M}\sum_{G=0}^{\infty}\frac{(-1)^G}{G!\,F_g}\,\phi(\eta_G)\,z^{\eta_G}$$

$$\cdot\frac{2^{u+v+(h+k)l+(h'+k')\eta_G-1},(\alpha+1:w)}{w!}\sum_{R=0}^{w}\frac{(-w:R)(1+\alpha+\beta+w:R)}{R!\,(\alpha+1:R)}\left(\frac{st}{2}\right)^R$$

$$. H_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\ \lambda+2\ :(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[1-u-hl-h'\eta_G:h_1,\ldots,h_r],\ \eta_G-R\\ [1-u-v-(h+k)l-(h'+k')\end{matrix}\right.$$

$$\begin{matrix}[1-v-kl-k'\eta_G-R:k_1;\ldots;k_r],\ [(a):\theta',\ldots,\theta^{(r)}]:\\ :h_1+k_1;\ldots;h_r+k_r], \qquad [(c):\psi',\ldots,\psi^{(r)}]:\end{matrix}$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]\ ;\ z_1^2h_1+k_1\\ \vdots\\ [(d'):\delta'];\ldots;[(d^{(r)}):\ \delta^{(r)}]\ ;\ 2^{h_r+k_r}\end{matrix}\right). \qquad \ldots (2.4)$$

where

$$h>0, k>0, h'>0, k'>0, h_i>0, k_i>0, T>0, T_i>0,$$

$$|\arg z|<T\pi/2,\ |\arg(z_i)|<T_i\pi/2,\ Re(u+h'f_l/F_L+\sum_{i=1}^{r}h_i\,d_j^{(i)}/\delta_j^{(i)})>0,$$

$$Re\ (v+k'f_L/F_L+\sum_{i=1}^{r}k_i\,d_j^{(i)}/\delta_j^{(i)}>0, i=1,\ldots,r, j=1,\ldots,u^{(i)},$$

$L = 1,\ldots,M$, m is an arbitrary positive integer and the coefficients $A_{n,l}\,(n, l \geq 0)$ an arbitrary constants real or complex and the series on the right converages.

Proof : To establish (2.1), express the general class of polynomials occuring in integrand (2.1) in series form given by (1.1), and express the Fox's H-function in series with the help of (1.2), and then interchanged the order of summation and integration (which is permissible under the conditions stated above), evaluate the inner integral with the help of a result of Chaurasia ([3],p.210,eq.(2.1)), we arrive at the desired result.

The proofs of the integrals (2.2) to (2.4) can be developed by proceeding on similar lines with the help of known integrals ([2],p.85,eq.(2.6),eq.(2.4) and p.84,eq.(2.2)).

3 SPECIAL CASES

(i) When $n = 0$, the integrals in (2.1) to (2.4) reduce to the following integrals

$$\int_s^t (x-s)^{u-1} (t-x)^{v-1} (x-w)^{-u-v} H_{P,Q}^{M,N}\left[y\left(\frac{x-s}{x-w}\right)^{h'} \left(\frac{t-x}{x-w}\right)^{k'} \middle| \begin{matrix} (e_P, E_P) \\ (f_Q, F_Q) \end{matrix} \right]$$

$$\cdot H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})} \left(\begin{matrix} [(a):\theta',\ldots,\theta^{(r)}] : \\ [(c):\psi',\ldots,\psi^{(r)}] : \end{matrix} \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}] \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}] \end{matrix} ; \begin{matrix} z_1\left(\frac{x-s}{x-w}\right)^{h_1}\left(\frac{t-x}{x-w}\right)^{k_1} \\ \vdots \\ z_r\left(\frac{x-s}{x-w}\right)^{h_r}\left(\frac{t-x}{x-w}\right)^{k_r} \end{matrix} \right) dx$$

$$= \sum_{g=1}^{M} \sum_{G=0}^{\infty} \frac{(-1)^G}{G!\,F_g} \phi(\eta_G)\, Y^{\eta_G} (t-S)^{u+v-1+(h'+k')\eta_G} (t-w)^{-u-h'\eta_G}$$

$$\cdot (s-w)^{-v-k'\eta_G} H_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\ \lambda+2:(u',v');\ldots;(u^{(r)},v^{(r)})}$$

$$[1-u-h'\eta_G : h_1,\ldots,h_r], [1-v-k'\eta_G : k_1,\ldots,k_r], [(a):\theta',\ldots,\theta^{(r)}] :$$

$$[1-u-v-(h'+k')\eta_G : h_1+k_1;\ldots;h_r+k_r], \qquad [(c):\psi',\ldots,\psi^{(r)}] :$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}] ; z_1\left(\frac{t-s}{t-w}\right)^{h_1}\left(\frac{t-s}{s-w}\right)^{k_1} \\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}) ; z_r\left(\frac{t-s}{t-w}\right)^{h_r}\left(\frac{t-s}{s-w}\right)^{k_r} \end{matrix} \right), \qquad \ldots (3.1)$$

valid under the conditions obtainable from (2.1)

$$\int_0^t x^{u-1}(t-s)^{v-1}\; H^{M,N}_{P,Q}\left[z\,x^{h'}(t-x)^{k'}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right]$$

$$.\; H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]\;:\\ [(c)\;:\;\psi',\ldots,\psi^{(r)}]\;:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)},\phi^{(r)}]\;;\;z_1 x^{h_1}\;(t-x)^{k_1}\\ \vdots\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\;;\;z_r x^{h_r}(t-x)^{k_r}\end{matrix}\right)dx$$

$$=\sum_{g=1}^{M}\sum_{G=0}^{\infty}\frac{(-1)^G}{G!\;F_g}\;\phi(\eta_G)\;z^{\eta_G}t^{u+v+(h'+k')\eta_G-1}$$

$$H^{0,\;\lambda+2\;:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left(\begin{matrix}[1-u-h'\eta_G:h_1,\ldots,h_r]\;:\\ [1-u-v-(h'+k')\eta_G:\end{matrix}\right.$$

$$[1-v-k'\eta_G:k_1,\ldots,k_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];$$

$$h_1+k_1,\quad\ldots,h_r+k_r]\quad,[(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots,[(d^{(r)}):\delta^{(r)}];$$

$$\left.\begin{matrix}z_1\,t^{h_1+k_1}\\ \vdots\\ z_r\,t^{h_r+k_r}\end{matrix}\right),\qquad\ldots(3.2)$$

valid under the conditions obtainable from (2.2)

$$\int_0^1 x^{u-1}(1-x)^{v-1}\;{}_2F_1(\alpha,\beta;u:x)\,H^{M,N}_{P,Q}\left[z(1-x)^{h'}\left|\begin{matrix}(e_p,E_p)\\(f_Q,E_Q)\end{matrix}\right.\right]$$

$$H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{r}]:[(d'):\delta'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}];z_1(1-x)^{h_1}\\ [(d^{(r)}):\delta^{(r)}];z_r(1-x)^{h_r}\end{matrix}\right)dx$$

$$=\sum_{g=1}^{M}\sum_{G=0}^{\infty}\frac{-1)^G}{G!\,F_g}\phi(\eta_G)\;z^{\eta_G}\;\Gamma(u)\,H^{0,\lambda+1\;:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A+2,C+1:(B',D');\ldots;[B^{(r)},D^{(r)}]}$$

$$[1-v-h'\eta_G:h_1,\ldots,h_r],[1-u-v-h'\eta_G+\alpha+\beta:h_1,\ldots,[(a):e',\ldots,e^{(r)}]:$$

$$[1+\alpha-u-v-h'\eta_G:h_1,\ldots,h_r],[1+\beta-u-v-h'\eta_G:h_1,\ldots,h_r],[(c):\psi',\ldots,\psi^{(r)}]:$$

$$[(b'):\psi^{2}];\ldots;[(b)^{(r)}:\phi^{(r)}]:z_1$$
$$[(d'):\delta'];\ldots;[(a^{(r)}):\delta^{(r)}];z_r \qquad \ldots (3.3)$$

valid under the conditions obtainable from (2.3).

$$\int_{-1}^{1}(1+x)^{u-1}(1-x)^{v-1}P_w^{(\alpha,\beta)}\left(1-\frac{st}{2}(1-x)\right) H_{P,Q}^{M,N}\left[z(1+x)^{h'}(1-x)^{k'}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right]$$

$$H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}];[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{(r)}];[(d'):\delta'];\ldots;\end{matrix}\right.$$
$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}];z_1(1+x)^{h_1}(1-x)^{k_1}\\ \vdots \\ [(d^{(r)}):\delta^{(r)}];z_r(1+x)^{h_r}(1-x)^{k_r}\end{matrix}\right)dx$$

$$=\sum_{g=1}^{M}\sum_{G=0}^{\infty}\frac{(-1)^G}{G!\,F_g}\phi(\eta_G)\frac{2^{u+v+(h'+k')\eta_G-1},(\alpha+1:w)}{w!}$$

$$\sum_{R=0}^{w}\frac{(-w:R)(1+\alpha+\beta+w:R)}{R!\;(\alpha+1:R)}\left(\frac{st}{2}\right)^R$$

$$H_{A+2,C+1:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda+2\;:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(\begin{matrix}[1-u-h'\eta_G:h_1,\ldots,h_r],\\ [1-u-v-(h'+k')\eta_G-R:\end{matrix}\right.$$

$$\begin{matrix}[1-v-k'\eta_G-R:k_1,\ldots,k_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;\\ h_1+k_1;\ldots;h_r+k_r] \qquad ,[(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;\end{matrix}$$

$$\left.\begin{matrix}[(b^{(r)}):\phi^{(r)}]\;;z_1 2^{h_1+k_1}\\ \vdots\\ [(d^{(r)}):\delta^{(r)}]\;;z_r 2^{h_r+k_r}\end{matrix}\right), \qquad \ldots (3.4)$$

valid under the conditions obtainable form (2.4).

(*ii*) Taking n=0 and leting $h'\to 0, k'\to 0$, the integrals in (2.1), (2.2) and (2.4) reduce to the integrals obtained by Chaurasia ([3],p.210,eq.(2.1); [2],p.85,eq.(2.6), p.84,eq.(2.1)).

(*iii*) Leting $h'\to 0$ and n = 0, the integral in (2.3) reduces to an integral obtained by Chaurasia ([2],p.85,eq.(2.4)).

The importance of our results lies in its manifold generality. In view of the generality of the polynomials $S_n^m[x]$, on suitably specializing the coefficients $A_{n,l}$, and making a free use of the special cases of $S_n^m[x]$ listed by Srivastava and Singh [9], our resluts can be reduced to a large number of integrals (and also its particular cases) involving generalized Hermite polynomials, Hermite polynomials, Jacobi polynomials and its various special cases, Laguerre polynomials, Bessel polynomials, Gould-Hopper polynomials, Brafman polynomials and their various combinations.

Secondly, by specializing the various parameters and variables in the multivariables H-funcion, we can obtain from our integrals, several integrals, involving a remarkably wide variety of useful functions (or products of several such functions), which are expressible in terms of E,F,G and H-functions of one and several variables. Thus the integrals presented in this paper would at once yield a very large number of results involving a large variety of polynomials and various special functions occurring in the literature.

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