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(Decided to Professor S.A. Naimpally on his 62th Birthday)

TAYLOR INSTABILITY IN THE PRESENCE OF VARIABLE MAGNETIC FIELD AND SUSPENDED PARTICLES IN POROUS MEDIA

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ABSTRACT

Rayleigh-Taylor instability in the presence of vairable magnetic field and suspended particles through porous medium has been studied. The system is stable for stable density stratification. The magnetic field can stabilize a system which was unstable in its absence, for unstable density stratification. The medium permeability and suspended particles number density have decreasing as well as increasing effect on the growth rates.

1. INTRODUCTION

A comprehensive account of Rayleigh-Taylor instability under varying assumptions of hydromagnetics has been given by Chandrasekhar [2]. A contradiction was observed by Chandra [1], when he compared the theory for onset of convection in fluid heated from below and his experiment. He performed the experiement in an air layer and found that the instability depended on the depth of the layer. Scanlon and Segal [4] have studied the effect of suspended particles on the onset of Benard convection and found that critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The thermal instability of fluids in a porous medium in the presence of suspended particles has been studied by Sharma and Sharma [6]. The effects of suspended particles and medium permeability were found to destabilize the layer. Sharma et al. [5] have considered the effect of suspended particles on the onset of bénard convection in hydromagnetics and found that suspended particles destabilize the layer whereas the effect of magnetic field was stabilizing.

Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest importance of porosity in astrophysical context) McDonnel [3]). The problem of the hydromagnetic stability of conducting fluid of variable density in the presence of suspended particles through porous medium plays an important role in stability of stellar atmospheres, in ground water hydrology or in oil recovery etc. In the present paper, a study has been made of the Rayleigh-Taylor instability of conducting fluid in the presence of suspended particles and variable magnetic field through porous medium with continuous stratificaions in fluid density, fluid viscosity, suspended particles number density and magnetic field.

2. PERTURBATION EQUATIONS

Consider an incompressible infinitely conducting fluid mixed with suspended particles through porous medium in which fluid density ρ, fluid viscosity μ and suspended particles number density N are functions of vertical component z only and layers of fluid are arranged in horizontal strata. A vaiable magnetic field $\vec{H}(H_0(z), 0, 0)$ and $\vec{g}(0, 0, -g)$ pervade the system. Initially the system is in a static state and the character of equilibrium of this static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let $\rho, \mu, p, \vec{v}(u, v, w)$ denote the density, the viscosity, the pressure and the veloctiy of pure fluid respectively. $\vec{v_p}(l, r, s)$ and $N(\bar{x}, t)$ denote the velocity and the number density of suspended particles. $K = 6\pi/\mu\in'$ is Stokes' drag, where $\in'$ is the particle radius, μ_e is magnetic permeability and $\bar{x} = (x, y, z)$. Then the equations of motion and continuity for the fluid are

$$\frac{\rho}{\in}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\in}(\vec{v}\,.\,\nabla)\,\vec{v}\right] = -\nabla p + \rho\vec{g} + \frac{KN}{\in}(\vec{v_p} - \vec{v}) + \frac{\mu_e}{4\pi}(\nabla \times \vec{H}) \times \vec{H} - \frac{\mu}{k_1}\vec{v}, \tag{1}$$

$$\nabla\,.\,\vec{v} = 0, \tag{2}$$

where $\in$ is porosity and k_1 is peremeability of the medium.

Since int he presence of suspended particles, the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude and opposite in sign in the equations of motion for suspended paticles. Assuming distances between paricles quite large compared with their diameter and so neglecting the buoyancy force exerted on particles and inter- particle reactions, the equaitons of motion and continuity for particles become

$$\frac{mN}{\in}\left[\frac{\partial \vec{v_p}}{\partial t} + \frac{1}{\in}\left(\vec{v_p}\,.\,\nabla\right)\vec{v_p}\right] = mN\vec{g} + \frac{KN}{\in}(\vec{v} - \vec{v_p}). \tag{3}$$

$$\in \frac{\partial N}{\partial t} + \nabla\,.\,(N\vec{v_p}) = 0, \tag{4}$$

where mN is the mass of particles per unit volume.

Let $\delta\rho$, δp, $\vec{v}(u, v, w)$, $\vec{V}_p(l, r, s)$ and $\vec{h}(h_x, h_y, h_z)$ denote the perturbations in density ρ, pressure p, fluid velocity (0,0,0), particles velocity (0,0,0) and magnetic field $\vec{H}(H_0(z), 0, 0)$ respectively. Then the linearized perturbation equations governing the motion are

$$\frac{\rho}{\in}\frac{\partial \vec{v}}{\partial t} = -\nabla\,\delta p + \vec{g}\,\delta\rho - \frac{\mu}{k_1}\vec{v} + \frac{KN}{\in}(\vec{v_p} - \vec{v}) + \frac{\mu_e}{4\pi}[(\nabla\times\vec{h})\times\vec{H} + (\nabla\times\vec{H})\times\vec{h}] \tag{5}$$

$$\nabla\,.\,\vec{v} = 0\,, \tag{6}$$

$$\nabla\,.\,\vec{h} = 0\,, \tag{7}$$

$$\in\frac{\partial \vec{h}}{\partial t} = (\vec{H}\,.\,\nabla)\,\vec{v} - (\vec{v} - (\vec{v}\,.\,\nabla)\,\vec{H}\,, \tag{8}$$

$$m\frac{\partial \vec{v}_p}{\partial t} = k\,(\vec{v} - \vec{v}_p)\,, \tag{9}$$

$$\in\frac{\partial}{\partial t}\delta\rho = -w\frac{d\rho}{dz}\,. \tag{10}$$

3. DISPERSION RELATION AND DISCUSSION

Analyzing the disturbances into normal modes, we seek the solutions whose dependence on space-time coordinates is of the form

$$f(z)\exp\,(ik_x x + ik_y y + nt)\,, \tag{11}$$

where $f(z)$ is some function of z, n is the growth rate and k_x, k_y $\left[k = (k_x^2 + Sk_y^2)^{1/2}\right]$ are horizontal wave numbers.

Eliminating $\vec{v}_p$ between eqs. (5), (9) and using (11), we get

$$\left[\frac{\rho n}{\in} + \frac{\mu}{k_1} + \frac{m\,N\,n}{\in\,(1 + mn/K)}\right]\vec{v} = -\nabla\,\delta p + \frac{\mu_e}{4\pi}\left[\hat{i}\,h_z\,DH_0 + \hat{j}\,H_0\,(i\,k_x\,h_y - ik_y\,h_x)\right.$$

$$\left. + \hat{k}\,H_0\,(i\,k_x h_z - Dh_x - h_x)\frac{DH_0}{H_0}\right] + \vec{g}\,\delta\rho\,. \tag{12}$$

Writing eqs. (12) and (8) in the component form, and using (10) (11); we obtain

$$\left[\frac{\rho n}{\in} + \frac{\mu}{k_1} + \frac{mNn}{\in\,(1 + mn/K)}\right]u = -i\,k_x\,\delta p + \frac{\mu_e}{4\pi}h_z\,DH_0\,, \tag{13}$$

$$\left[\frac{\rho n}{\epsilon}+\frac{\mu}{k_1}+\frac{mNn}{\epsilon(1+mn/K)}\right] v = -ik_y\,\delta p+\frac{\mu_e H_0}{4\pi}\left(ik_x h_y - ik_y h_x\right), \quad (14)$$

$$\left[\frac{\rho n}{\epsilon}+\frac{\mu}{k_1}+\frac{mNn}{\epsilon\,(1+mn/K)}\right] w = -D\,\delta p+\frac{\mu_e H_0}{4\pi}\left(ik_x h_z - Dh_x - h_x\frac{DH_0}{H_0}\right)+\frac{g(D_\rho)\,w}{\epsilon n},$$

(15)

$$\epsilon\, n\, h_x = ik_x H_0\, u - w\, DH_0, \quad (16)$$

$$\epsilon\, n\, h_y = ik_x H_0\, v, \quad (17)$$

$$\epsilon\, n\, h_z = ik_x H_0\, w. \quad (18)$$

Eliminating h_x, h_y, h_z from eqs. (13)-(15) with the help of eqs. (16)- (18), we get

$$\left(\frac{n'\rho}{\epsilon}+\frac{\mu}{k_1}\right) u = -ik_x\,\delta p+\frac{ik_x\mu_e H_0}{4\pi\epsilon n}\left(DH_0\right) w, \quad (19)$$

$$\left(\frac{n'\rho}{\epsilon}+\frac{\mu}{k_1}\right) v = -ik_y\,\delta p+\frac{ik_x\mu_e}{4\pi\epsilon n}\left(i\,k_x v - i\,k_y u\right)+\frac{\mu_e H_0}{4\pi\epsilon n}\, ik_y\left(D\,H_0\right) w,$$

0)

$$\left(\frac{n'\rho}{\epsilon}+\frac{\mu}{k_1}\right) w = -D\,\delta p+\frac{\mu_e H_0}{4\pi\epsilon n}\Big[-k_x^2 H_0\, w - D\,(ik_x H_0\, u - w\, D\, H_0)$$

$$-(i\,k_x H_0\, u - w\, DH_0)\,\frac{DH_0}{H_0}\Big]+\frac{g\,(D\rho)\,w}{\epsilon\, n}, \quad (21)$$

where
$$n' = n\left[1+\frac{mN}{\rho\,(1+mn/K)}\right]. \quad (22)$$

Multiplying eq. (19) by $-ik_x$, (20) by $-ik_y$; adding and using (6), we get

$$\left(\frac{n'\rho}{\epsilon}+\frac{\mu}{k_1}\right) Dw = -k^2\,\delta p+\frac{\mu_e H_0^2 k_x k_y}{4\,\pi\,\epsilon\, n}\,\zeta+\frac{\mu_e H_0 k^2 (DH_0)}{4\,\pi\,\epsilon\, n}\, w, \quad (23)$$

where
$$\zeta = ik_x v - ik_y u. \quad (24)$$

Multiplying (19) by $-ik_y$ (20) by ik_x and adding, we obtain

$$\zeta = 0, \quad (25)$$

which together with (6) yields

$$u = \frac{ik_x}{k^2}\, Dw. \quad (26)$$

Eliminating δp between eqs. (21), m (23) and using (25), (26), after a little algebra, we obtain

$$\frac{1}{\epsilon}\left[D\,(n'\,\rho\,D\,w) - n'\,K^2\,\rho\,w\right] + \frac{gK^2\,(D\rho)\,w}{n\,\epsilon} + \frac{\mu}{k_1}(D^2 - K^2)\,w + \frac{(D\mu)\,(Dw)}{k_1}$$

$$+\frac{\mu_e\,K_x^2}{4\,\pi\,\epsilon\,n}\left[H_0^2\,(D^2 - k^2)\,w + D.(H_0^2)\,Dw\right] = 0\,. \quad (27)$$

Assume the stratifications in density, viscosity, number density and magnetic field of the form

$$\rho = \rho_0\,e^{\beta z}\,,\ \ \mu = \mu_0\,e^{\beta z}\,, N = N_0\,e^{\beta z}\,, H_0^2 = H_1^2\,e^{\beta z}\,, \quad (28)$$

where ρ_0, μ_0, N_0, H_1 and β are constants. Here we consider the case of two free boundaries at z=0 and z=d. Assuming $\beta\,d << 1$, i.e. the variation in density at two neighbouring points in velocity field is much less than the average density, has a negligible effect on the inertia of the fluid. The appropriate boundary conditions are

$$w = D^2 w = 0 \ (29) \qquad \text{at } z = 0 \ \text{ and } z = d\,. \quad (29)$$

The solution of (27) staisfying (29) is given by

$$w = A\,\sin\frac{m'\,\pi\,z}{d}\,, \quad (30)$$

where m' is an integer. Substituting (30) into (27), we obtain the dispersion relation

$$n^3 + \left[\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0\,\epsilon}{k_1}\right]n^2 + \left[k_x^2\,V_A^2 + \frac{\nu_0\,\epsilon\,K}{m\,K_1} - \frac{gK^2\,\beta}{L}\right]n$$

$$+\frac{k_x^2\,V_A^2\,K}{m} - \frac{gk^2\,\beta\,K}{m\,L} = 0, \quad (31)$$

where $\nu_0 = \dfrac{\mu_0}{\rho_0}\,, V_A = \left(\dfrac{\mu_e\,H_0^2}{4\pi\rho}\right)^{1/2} = \left(\dfrac{\mu_e\,H_1^2}{4\,\pi\,\rho_0}\right)^{1/2}$ and $\dfrac{N}{\rho}$ are constant everywhere and $L = \left(\dfrac{m'\,\pi}{d}\right)^2 + k^2$. For $\beta < 0$ (stable stratification), eq. (31) does not allow any positive root of n and so the system is stable. For $\beta > 0$ (unstable stratification), the system is stable or unstable according as

$$k_x^2 V_A^2 \gtrless \frac{gk^2\beta}{L}\,. \quad (32)$$

In the absence of magnetic field, the system is unstable for $\beta > 0$. However, the system can be stabilized by magnetic field, as can be seen from eq. (32), if

$$V_A^2 > \frac{g\,\beta\,K^2}{k_x^2\,L}\,.$$

If $\beta > 0$ and $V_A^2 < \frac{gK^2\beta}{k_x^2 L}$, eq. (31) has at least one positive root. Let this positive root be n_0, then

$$n_0^3 + \left[\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0 \in}{k_1}\right] n_0^2 + \left[k_x^2 V_A^2 + \frac{\nu_0 \in K}{mk_1} - \frac{gK^2\beta}{L}\right] n_0$$

$$+\frac{k^2{}_x V_A^2 K}{m} - \frac{gk^2 \beta\, K}{m\,L} = 0. \qquad (33)$$

To find the role of permeability and suspended particles number density on the growth rate unstable modes, we examine the natures of dn_0/dk_1 and dn_0/dN_0. Equation (33) yields

$$\frac{dn_0}{dk_1} = \frac{n_0 \nu_0 \in \left(n_0 + \frac{K}{m}\right)}{k_1^2\left[3n_0^2 + 2\left(\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0 \in}{k_1}\right) n_0 + \left(k_x^2 V_A^2 + \frac{\nu_0 \in K}{mk_1} - \frac{gK^2\beta}{L}\right)\right]}. \qquad (34)$$

This shows that growth rates decrease of increase with permeability according as the denominator is negative or positive.

Equation (33) also yields

$$\frac{dn_0}{dN_0} - \frac{\left(\frac{K}{\rho_0}\right) n_0^2}{3\,n_0^2 + 2\left(\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0 \in}{k_1}\right) n_0 + \left(k_x^2 V_A^2 + \frac{\gamma_0 \in K}{mk_1} - \frac{gK^2\beta}{L}\right)}.$$

Thus, if in addition to $K_x^2 V_A^2 < \frac{gk^2\,\beta}{L}$ we have the condition

$$\left|K_x^2\, V_A^2 - \frac{gk^2\beta}{L}\right| \;<\; 3\,n_0^2 + 2\,n_0\left(\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0 \in}{k_1}\right) + \frac{\nu_0 \in K}{m\,k_1}\,,$$

then dn_0/dN_0 is always negative and so the growth rate decreases with the increase in the suspended particles number density. However the growth rate increases with the number density for the region

$$\left|k_x^2 V_A^2 - \frac{gK^2\beta}{L}\right| > 3\,n_0^2 + 2n_0\left(\frac{K}{m} + \frac{KN_0}{\rho_0} + \frac{\nu_0 \in}{k_1}\right) + \frac{\nu_0 \in K}{m\,k_1}.$$

REFERENCES

[1] K. Chandra, Instability of fluid heated from below, *Proc. Roy. Soc. London* Sect.A 164 (1938), 231.

[2] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability,* Dover Publication, New York, 1981.

[3] J.W. McDonnel, *Cosmic Dust,* John Wiley and Sons, Toronto, p. 330.

[4] J.W. Scanlon and L.A. Segel, Some effects of suspended particles on the onset of Bénard convection, *Phys. Fluids* **16** (1973), 1573-1578.

[5] R.C. Sharma, K. Prakash and S.N. Dube, Effect of suspended particles on the onset of bénard convections in hydromagnetics *Acta Phys. Hungar.* **40** (1976), 3-10.

[6] R.C. Sharma and K.N. Sharma, Thermal instability of fluids through a porous medium in the presence of suspended particles, rotation and solute gradient, *J. Math. Phys. Sci.* **16** (1982), 167-181.