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(Dedicated to Professor S.A. Naimpally on his 62th Birthday)

ALMOST PERIODIC FUNCTIONS ON FUZZY METRIC SPACE

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1. The notion of fuzzy set is introduced by Zadeh *[7]*. Here all the properties of an interval is not required. Let $M : X \to I := [0, 1]$. $M(x)$ the degree of membership of a point x in X is a fuzzy set M. It is a generalization of characteristic function. General topology has been applied systematically first by Chang [1]. There are mainly two ways of defining fuzzy metric. In one the numerical distances are measured between fuzzy objects. This type was studied by Erceg [3], Deng [2] etc. Second approach deals with the distance between objects called fuzzy. The objects are also fuzzy. We refer to Kuleva-Seikkalu [5], Eklund and Gahler [4].

We need some definitions due to Sharma, Singh and Mishra [6]:

Definition 1. A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is continuous $\in$ - norm if $([0, 1], *)$ is an abelian (topological) monoid with unit 1 such that $a * b \le c * d$ whenever $a \le c$ and $b \le d$ (a, b, c, d are in $[0, 1]$).

Definition 2. The 3-tuple $(X, M, *)$ is a fuzzy metric space if X be arbitrary set, $*$ is continuous $\in$ - norm, and M is fuzzy set $X^2 \times (0, \infty)$ satisfying :

(1.1) $m(x, y, 0) = 0$,

(1.2) $M(x, y, \in) = 1$ for all $t > 0$ iff $x = y$.

(1.3) $M(x, y, \in) = M(y, x, \in)$,

(1.4) $M(x, y, \in_1)^* M(y, z, \in_2) \le M(x, z, \in_1 + \in_2)$

(1.5) $M(x, y, .) : (0, \infty) \to [0, 1]$

is left continous for all x, y, z in X and $C_1, C_2 > 0$.

Now for the first time we define almost periodic functions with values in fuzzy metric space.

Difinition 3. A continuous function on R (real). $T : R \to (X, M, *)$ is called almost periodic if for any member $\in > 0$, one can find $l(\in) > 0$ such that any interval of length $l(\in)$ contains at least one point τ_Γ with property:

$$M\left(T(t+\tau), T(t), \in\right) = 1, t \in R$$

2. Now we shall prove:

Theorem 1. An almost periodic function on fuzzy metric space is uniformly contionuous.

Proof. Let $l = l(\in/3) > 0$ which corresponds to $\in/3, \in > 0$ in the definition of almost periodic function. On the interval $[-1, 1+l]$ the function is continuous and hence it is also uniformly continuous. Let $\delta = \delta(\in/3) > 0, \delta < 1$ such that $t_1, t_2 \in [-1, 1+l]$ with $|t_1 - t_2| < \delta$, we have

$$M(Tt_1, Tt_2, \tfrac{\in}{3}) = 1.$$

Finally, let $t_1 > t_2$ be such that $|t_1 - t_2| < \delta$ and τ be any $(\in/3)$ translation number of T contiained in the interval $[-t_1, -t_1 + 1]$. Since $|t_2 - t_1| < \delta, 0 \le t_1 + \tau \le l$ it follows that $t_2 + \tau$ is situated on the interval $[-1, 1+l]$. Thus

$$\begin{aligned} M(Tt_2, Tt_1, \in) &\ge M\,[Tt_2, T(t_2+\tau), \in/3]^* \\ &\quad M\,[T(t_2+\tau), T(t_1+\tau, \in/3] \\ &\quad * M\,[(Tt_1+\tau), T(t_1), \in/3] \\ &= 1 * 1 * 1 = 1 \end{aligned}$$

This proves the result.

Theorem 2. If T_n is a sequence of almost periodic function with values in $(X, M, *)$ and if

$$\lim_{n \to \infty} T_n(t) = T(t)$$

Uniformly in the sense of convergence of $\in$ —norm, then T is almost periodic.

Proof. Continuity of T is evident. If $\in > 0$, then there exists a natural number $N(\in)$ with

(2.1) $M(T_n t, Tt, \in/3) = 1,\ t \in R,\ n \ge N(\in)$.

We fix n_o for which (2.1) is true and consider $l(\in/3)$ determined from the almost periodicity of T_{n_o} and T an $(\in/3)$ translation number of f_{no}. For any $t \in R$, We have

$$M(T(t+\tau), T(t), \in) \ge M(T(t+\tau), (T_{n_o}(t+\tau), \in/3)$$

$$* M(T_n(t+\tau), T_n t, (\in/3) * M(T_n t, Tt, \in/3)$$

$$= 1 * 1 * 1 = 1$$

which proves the almost periodicity of T.

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