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MULTIVARIABLE ANALOGUES OF A CLASS OF POLYNOMIALS $T_n^{(a,k)}(x, r, p)$

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ABSTRACT

In the present paper, we introduce a multivariable analogue of the generalized class of polynomials $T_n^{(a,k)}(x, r, p)$ of Chandel through the Rodrigues' formula (1.4). Its further generalization is also discussed by Rodrigues' formula (6.1). Our polynomials may also very well be regarded as multivariable analogues of Lagurre, Hermite, Bessel polynomials and Truesdell polynomials (Erdelyi [9], Toscano [16], generalized Truesdell polynomials of Singh [11]. Chak's polynomials [5], generalized Laguerre polynomials of Singh-Srivastava [12] (see also Chatterjea [8]), Chatterjea's generalized Bessel polynomials [7] and generalized Hermite function of Gould-Hopper [10].

1. Introduction.

Srivastava and Singhal [15] studied a class of polynomials defined by generalized Rodrigues' formula (see also [14,p.401, problem 34]) :

$$(1.1)\quad G_n^{(\alpha)}(x, r, p, k) = \frac{1}{n!} x^{-\alpha - kn} \exp(px^r)\left(x^{k+1}\frac{d}{dx}\right)^n \{x^\alpha \exp(-px^r)\},$$

where the parameters $\alpha, r, p,$ and k are arbitrary in general. Subsiquently, but independently, to introduce generalized Stirling numbers and polynomials Chandel ([1], [2], [3]) studied a slight variation of Srivastava - Singhal polynomials in the form :

$$(1.2)\quad T_n^{(\alpha,k)}(x, r, p) = x^{-\alpha} \exp(px^r)\, \Omega_x^n \{x^\alpha \exp(-px^r)\}$$

where, and in what follows, $\Omega_x = x^k \frac{d}{dx}$.

Recently, Chandel and Tiwari ([4], (1.7)) introduced a multivariable analogue of Hermite polynomials defined by Rodrigues' formula

$$(1.3)\quad H_{n_1,\ldots,n_m}^{(b;\, h_1,\ldots,h_m;\, r_1,\ldots,r_m)}(x_1, \ldots, x_m)$$

$$= (-1)^{n_1 + \dots + n_m} (1 + h_1 x_1^{r_1} + \dots + h_m x_m^{r_m})^b$$

$$\frac{d^{n_1}}{dx_1^{n_1}} \cdots \frac{d^{n_m}}{dx_m^{n_m}} (1 + h_1 x_1^{r_1} + \dots + h_m x_m^{r_m})^{-b},$$

where $n_1, \dots, n_m$ are positive integers while $h_1, \dots, h_m$; $r_1, \dots, r_m$ and b are any numbers real or complex independent of variables $x_1, \dots, x_m$.

In the present paper, first we introduce the multivariable analogue

$T^{(a_1, \dots, a_m; h_1, \dots, k_m; r_1, \dots, r_m; p_1, \dots, p_m; b)}_{n_1, \dots, n_m} (x_1, \dots, x_m)$

of the class of polynomials of Chandel ([1], [2], [3]), defined by

(1.4) $T^{(a_1, \dots, a_m; k_1, \dots, k_m; r_1, \dots, r_m; p_1, \dots, p_m; b)}_{n_1, \dots, n_m} (x_1, \dots, x_m)$

$$= x_1^{-a_1} \dots x_m^{-a_m} (1 + p_1 x_1^{r_1} + \dots + p_m x_m^{r_m})^b$$

$$\Omega_{x_1}^{n_1} \dots \Omega_{x_m}^{n_m} \left\{ x_1^{a_1} \dots x_m^{a_m} \left(1 + p_1 x_1^{r_1} + \dots + p_m x_m^{r_m} \right)^{-b} \right\}, \Omega_{x_i} = x^{k_i} \frac{\partial}{\partial x_i},$$

where $a_1, \dots, a_m, k_1, \dots, k_m, r_1, \dots, r_m, p_1, \dots, p_m, b$ are arbitrary in general independent of variables $x_1, \dots, x_m$ but no $k_i = 1; i = 1, \dots, n$. Finally, we shall discuss further generalization of (1.4) through Rodrigues' formula (6.1).

It is clear that

(1.5) $\lim\limits_{b \to \infty} T^{(a_1, \dots, a_m; k_1, \dots, k_m; r_1, \dots, r_m; p_1/b, \dots, p_m/b; b)}_{n_1, \dots, n_m} (x_1, \dots, x_m)$

$$= T^{(a_1, k_1)}_{n_1} (x_1, r_1, p_1) \dots T^{(a_m, k_m)}_{n_m} (x_m, r_m, p_m),$$

where $T^{(a, k)}_n (x, r, p)$ are polynomials of Chandel ([1], [2], [3]) defined by (1.2).

For $k_1 = \dots = k_m = a_1 = \dots a_m = 0$, (1.4) reduces to (1.3), showing

(1.6) $T^{(0, \dots, 0; 0, \dots, 0; r_1, \dots, r_m; p_1, \dots, p_m; b)}_{n_1, \dots, n_m} (x_1, \dots x_m)$

$$= (-1)^{n_1 + \dots + n_m} H^{(b; p_1, \dots, p_m; r_1, \dots, r_m)}_{n_1, \dots, n_m} (x_1, \dots, x_m).$$

For $k_1 = \dots k_m = 0$, (1.4) reduces to the polynomials of Chandel and Tiwari [5, p. 760 (5.1)] defined by

(1.7) $H^{(a_1, \dots, a_m; r_1, \dots, r_m; p_1, \dots, p_m; b)}_{n_1, \dots, n_m} (x_1, \dots, x_m)$

$$= (-1)^{n_1 + \dots + n_m} x_1^{-a_1} \dots x_m^{-a_m} [1 + p_1 x_1^{r_1} + \dots + p_m x_m^{r_m}]^b$$

$$\frac{d^{n_1}}{dx_1^{n_1}} \cdots \frac{d^{n_m}}{dx_m^{n_m}} \left\{x_1^{a_1} \ldots x_m^{a_m} (1 + p_1 x_1^{r_1} + \ldots + p_m x_m^{r_m})^{-b}\right\},$$

as special case of their polynomials [5, p. 757 (1.1)].

Choosing $k_1 = \ldots = k_m = 0$ and replacing p_i by p_i/b $(i = 1, \ldots, m)$ and then taking $b \to \infty$, (1.4) reduces to the polynomials of Chandel and Tiwari [5, p. 760 (5.2)] defined by

$$(1.8)\ E_{n_1, \ldots, n_m}^{(a_1, \ldots, a_m; r_1, \ldots, r_m; p_1, \ldots, p_m)} (x_1, \ldots x_m)$$

$$= (-1)^{n_1 + \ldots + n_m} x_1^{-a_1} \ldots x_m^{-a_m} \exp[-(p_1 x_1^{r_1} + \ldots p_m x_m^{r_m})]$$

$$\frac{d^{n_1}}{dx_1^{n_1}} \cdots \frac{d^{n_m}}{dx_m^{n_m}} \left\{x_1^{a_1} \ldots x_m^{a_m} \exp(p_1 x_1^{r_1} + \ldots + p_m x_m^{r_m})\right\},$$

as special case of their polynomials [5, p.557 (1.1)].

Our polynomials may also very well be regarded as the multivariable analogue of Laguree, Hermite, Bessel polynomials and the Truesdell polynomials (Erdelyi [9], Toscano [16]), generalized Truesdell polynomials of Singh [11], Chak's polynomials [6], Singh and Srivastava generalized Laguerre polynomials [12] (see also Chatterjea [8]), Chatterjea's generalized Bessel polynomials [7] and generalized Hermite function of Gould and Hopper [10].

For brevity, we shall write

$$T_{n_1, \ldots, n_m}^{(a_1, \ldots, a_m; k_1, \ldots, k_m; r_1, \ldots, r_m; p_1, \ldots, p_m; b)} (x_1, \ldots, x_m)$$

as

$$T_{n_1, \ldots, n_m}^{(a_1, \ldots, a_m; [k_m]; [r_m]; [p_m]; b)} (x_1, \ldots, x_m)$$

2. **Generating Relation.** Starting with the Rodrigues' formula (1.4) and making an appeal to the result due to Chandel [1, p.43, (2.5)]

$$(2.1)\ e^{t\Omega_x} \{f(x)\} = f(x\{1 - (k-1)t\, x^{k-1}\}^{-1/(k-1)}),$$

we derive the following generating relation :

$$(2.2)\quad \sum_{n_1, \ldots, n_m = 0}^{\infty} T_{n_1, \ldots, n_m}^{(a_1, \ldots, a_m; [k_m]; [r_m]; [p_m]; b)} (x_1, \ldots, x_m) \frac{t_1^{n_1}}{n_1!} \cdots \frac{t_m^{n_m}}{n_m!}$$

$$= (1 + p x_1^{r_1} + \ldots + p_m x_m^{r_m})^b\, [1 - (k_1 - 1) t_1 x_1^{k_1 - 1}]^{-a_1/(k_1 - 1)}$$

$$\ldots [1 - (k_m - 1) t_m x_m^{k_m - 1}]^{-a_m/(k_m - 1)}$$

$$\left[1 + \frac{p_1 x_1^{r_1}}{[1 - (k_1 - 1) t_1 x_1^{k_1 - 1}]^{r_1/(k_1 - 1)}} + \right.$$

$$\left. \dots + \frac{p_m x_m^{r_m}}{[1-(k_m-1)t_m x_m^{k_m-1}]^{r_m/(k_m-1)}} \right]^{-b}.$$

3. Explicit form. Starting with the generating relation (2.2), we derive

$$(3.1) \quad T_{n_1, \dots, n_m}^{(a_1, \dots, a_m; [k_m]; [r_m]; [p_m]; b)}(x_1, \dots, x_m)$$

$$= (1 + p_1 x_1^{r_1} + \dots p_m x_m^{r_m})^b (k_1-1)^{n_1} \dots (k_m-1)^{n_m} x_1^{n_1(k_1-1)} \dots$$

$$x_m^{n_m(k_m-1)} \left(\frac{a_1}{k_1-1}\right)_{n_1} \dots \left(\frac{a_m}{k_m-1}\right)_{n_m}$$

$$\sum_{s_1, \dots, s_m=0}^{\infty} \frac{(b)_{s_1 + \dots + s_m} \left(\frac{a_1}{k_1-1}+n_1\right)_{r_1 s_1/(k_1-1)} \dots \left(\frac{a_m}{k_{m-1}}+n_m\right)_{r_m s_m/(k_m-1)}}{\left(\frac{a_1}{k_1-1}\right)_{r_1 s_1/(k_1-1)} \dots \left(\frac{a_m}{k_m-1}\right)_{r_m s_m/(k_m-1)}}$$

$$\frac{(-p_1 x_1^{r_1})^{s_1}}{s_1!} \dots \frac{(-p_m x_m^{r_m})^{s_m}}{s_m!},$$

which can be written in the following form of multiple hypergeometric function of Srivastava and Daoust [13] :

$$(3.2) \quad T_{n_1, \dots, n_m}^{(a_1, \dots, a_m; [k_m]; [r_m]; [p_m]; b)}(x_1, \dots, x_m)$$

$$= (k_1-1)^{n_1} \dots (k_m-)^{n_m} x_1^{n_1(k_1-1)} \dots x_m^{n_m(k_m-1)} \left(\frac{a_1}{k_1-1}\right)_{n_1} \dots \left(\frac{a_m}{k_m-1}\right)_{n_m}$$

$$(1 + p_1 x_1^{r_1} + \dots + p_m x_m^{r_m})^b$$

$$F_{0:1;\dots;1}^{1:1;\dots;1}\left(\begin{array}{l} [b:1,\dots,1] : \left[\frac{a_1}{k_1-1}+n_1 : \frac{r_1}{k_1-1}\right]; \dots; \left[\frac{a_m}{k_m-1}+n_m : \frac{r_m}{k_m-1}\right]; \\ - : \left[\frac{a_1}{k_1-1} : \frac{r_1}{k_1-1}\right]; \dots; \left[\frac{a_m}{k_m-1} : \frac{r_m}{k_m-1}\right]; \end{array} \right.$$

$$\left. -p_1 x_1^{r_1}, \dots, -p_m x_m^{r_m} \right).$$

4. Applications of Generating Relation. Making an appeal to the generating relation (2.2), we derive

(4.1) $T^{(a_1+c_1,\ldots,a_m+c_m;[k_m];[r_m];[p_m];b+d)}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$

$$=\sum_{s_1=0}^{n_1}\cdots\sum_{s_m=0}^{n_m}\binom{n_1}{s_1}\cdots\binom{n_m}{s_m}T^{(a_1,\ldots a_m;[k_m];[r_m];[p_m];b)}_{n_1-s_1,\ldots,n_m-s_m}(x_1,\ldots,x_m)$$

$T^{(c_1,\ldots,c_m;[k_m];[r_m];[p_m];d)}_{s_1,\ldots,s_m}(x_1,\ldots,x_m).$

(4.2) $T^{(a_1+c_1,\ldots,a_m+c_m;[k_m];[r_m];;[p_1;b)}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$

$$=\sum_{s_1=0}^{n_1}\cdots\sum_{s_m=0}^{n_m}\cdots\binom{n_1}{s_1}\cdots\binom{n_m}{s_m}(k_1-1)^{s_1}\cdots(k_m-1)^{s_m}\left(\frac{a_1}{k_1-1}\right)_{s_1}\cdots\left(\frac{a_m}{k_m-1}\right)_{s_m}$$

$x_1^{s_1(k_1-1)}\cdots x_m^{s_m(k_m-1)}\,T^{(c_1,\ldots,c_m;[k_m];[r_m];[p_m];b)}_{n_1-s_1,\ldots,n_m-s_m}(x_1,\ldots x_m),$

and

(4.3) $T^{(a_1,\ldots,a_m;[k_m];[r_m];[p_m];b+d)}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$

$$=(1+p_1x_1^{r_1}+\ldots+p_mx_m^{r_m})^b\sum_{s_1,\ldots,s_m=0}^{\infty}(b)_{s_m+\ldots+s_1}\frac{(-p_1x_1^{r_1})^{s_1}}{s_1!}\cdots\frac{(-p_mx_m^{r_m})^{s_m}}{s_m!}$$

$T^{(a_1+r_1s_1,\ldots,a_m+r_ms_m;[k_m];[r_m];[p_m];d)}_{n_1,\ldots,n_m}(x_1,\ldots x_m)\,.$

5. Recurrence Relations. Starting with generating relation (2.2). we derive

(5.1) $T^{(a_1+k_1-1,a_2,\ldots,a_m;[k_m];[r_m];[p_m];b)}_{n_1,\ldots,n_m}(x_1,\ldots x_m)$

$$+(k_1-1)x_1^{k_1-1}\,T^{(a_1+k_1-1,a_2,\ldots a_m;[k_m];[r_m];[p_m];b)}_{n_1-1,n_2,\ldots,n_m}(x_1,\ldots,x_m)$$

$$=T^{(a_1,\ldots,a_m;[k_m];[r_m];[p_m];b)}_{n_1,\ldots n_m}(x_1,\ldots x_m),$$

which suggests to unify m- recurrence relations in the foolowing form :

(5.2) $T^{(a_1,\ldots,a_{i-1},a_i+k_i-1,a_{i+1},\ldots,a_m;[k_m];[r_m];[p_m];b)}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$

$$+(k_i-1)x_i^{k_i-1}\,n_i\,T^{(a_1,\ldots,a_{i-1},a_i+k_i-1,a_{i+1},\ldots,a_m;[k_m];[r_m];[p_m];b)}_{n_1,\ldots,n_{i-1},n_i-1,n_{i+1},\ldots,n_m}(x_1,\ldots,x_m)$$

$$=T^{(a_1,\ldots,a_m;[k_m];[r_m];[p_m];b)}_{n_1,\ldots,n_m}(x_1,\ldots,x_m),$$

$i=1,\ldots,m$

An appeal to generating relation (2.2) also gives

(5.3) $(1+p_1 x_1^{r_1}+\ldots+p_m x_m^{r_m})\, T_{n_1,\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b-1)}(x_1,\ldots,x_m)$

$$= T_{n_1,\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x_1,\ldots,x_m)$$

$$+\sum_{i=1}^{m} p_i x_i^{r_i}\, T_{n_1,\ldots,n_m}^{(a_1,\ldots,a_{i-1},a_i+r_i,a_{i+1},a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x_1,\ldots,x_m)$$

and

(5.4) $T_{n_1+1,n_2,\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x_1,\ldots,x_m)$

$$= a_1 x_1^{k_1-1}\, T_{n_1,\ldots,n_m}^{(a_1+k_1-1,a_2,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x_1,\ldots,x_m)$$

$$- b p_1 r_1 x_1^{r_1+k_1-1}\, T_{n_1,\ldots,n_m}^{(a_1+r_1+k_1-1,a_2,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b+1)}(x_1,\ldots,x_m),$$

which further suggests to unify m- similar results in the form :

(5.5) $T_{n_1,\ldots,n_{i-1},n_i+1,n_{i+1},\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x_1,\ldots,x_m)$

$$= a_i x_i^{k_i-1}\, T_{n_1,\ldots,n_m}^{(a_1,\ldots a_{i-1},a_i+k_i-1,a_{i+1},\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b)}(x,\ldots,x_m)$$

$$- b p_i r_i x_i^{r_i+k_i-1}\, T_{n_1,\ldots,n_m}^{(a_1,\ldots,a_{i-1},a_i+r_i+k_i-1,a_{i+1},\ldots,a_m;\,[k_m];\,[r_m];\,[p_m];\,b+1)}(x_1,\ldots,x_m),$$

$i=1,\ldots,m.$

6. Generalization. Consider

(6.1) $G_{n_1,\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m])}(x_1,\ldots,x_m)$

$$= x_1^{-a_1}\ldots x_m^{-a_m}\,[G(p_1x_1^{r_1}+\ldots+p_m x_m^{r_m})]^{-1}$$

$$\Omega_{x_1}^{n_1}\ldots\Omega_{x_m}^{n_m}\{x_1^{a_1}\ldots x_m^{a_m}\,G(p_1x_1^{r_1}+p_m x_m^{r_m})\},$$

where

(6.2) $G(z)=\sum_{n=0}^{\infty}\gamma_n x^n,\ r_0\neq 0.$

It is clear that for $\gamma_n=\dfrac{(-1)^n(b)_n}{n!}$, (6.1) reduces to (1.4), while for $\gamma_n=\dfrac{(-1)^n}{n!}$, (6.1) gives

(6.3) $E_{n_1,\ldots,n_m}^{(a_1,\ldots,a_m;\,[k_m];\,[r_m];\,[p_m])}(x_1,\ldots,x_m)$

$$= T_{n_1}^{(a_1,k_1)}(x_1,r_1,p_1)\ldots T_{n_m}^{(a_m,k_m)}(x_m,r_m,p_m).$$

7. Generating Relation. Starting with the Rodrigues' formula (6.1) and making a appeal to (2.1) , we drive the following generating relation :

$$(7.1)\quad \sum_{n_1,\dots,n_m=0}^{\infty} G_{n_1,\dots,n_m}^{(a_1,\dots,a_m;[k_m];[r_m];[p_m])}(x_1,\dots,x_m)\frac{t_1^{n_1}}{n_1!}\cdots\frac{t_m^{n_m}}{n_m!}$$

$$=\frac{[1-(k_1-1)t_1x_1^{k_1-1}]^{-a_1/(k_1-1)}\dots[1-(k_m-1)t_m x_m^{k_m-1}]^{-a_m/(k_m-1)}}{G(p_1x_1^{r_1}+\dots p_m x_m^{r_m})}$$

$$G\left(\frac{p_1x_1^{r_1}}{[1-(k_1-1)t_1x_1^{k_1-1}]^{r_1/(k_1-1)}}+\dots+\frac{p_m x_m^{r_m}}{[1-(k_m-1)t_m x_m^{k_m-1}]^{r_m/k_m-1)}}\right).$$

8. Explicit Form. Starting with the generating relation (7.1), we derive the following explicit form :

$$(8.1)\quad G_{n_1,\dots,n_m}^{(a_1,\dots a_m;[k_m];[r_m];[p_m])}(x_1,\dots,x_m)$$

$$=\frac{\left(\frac{a_1}{k_1-1}\right)_{n_1}\cdots\left(\frac{a_m}{k_m-1}\right)_{n_m}(k_1-1)^{n_1}x_1^{(k_1-1)n_1}\dots(k_m-1)^{n_m}x_m^{(k_m-1)n_m}}{G(p_1x_1^{r_1}+\dots+p_m x_m x_m^{r_m})}.$$

$$\sum_{s_1,\dots,s_m=0}^{\infty}\frac{\gamma_{s_1+\dots+s_m}\left(\frac{a_1+(k_1-1)n_1}{k_1-1}\right)_{r_1s_1/(k_1-1)}\cdots\left(\frac{a_m+(k_m-1)n_m}{k_m-1}\right)_{r_ms_m/(k_m-1)}}{\left(\frac{a_1}{k_1-1}\right)_{r_1s_1/(k_1-1)}\cdots\left(\frac{a_m}{k_m-1}\right)_{r_ms_m/(k_m-1)}}$$

$$\cdot\frac{(p_1x_1^{r_1})^{s_1}}{s_1!}\cdots\frac{(p_m x_m^{r_m})^{s_m}}{s_m!}.$$

9. Recurrence Relations. Making an appeal to generating relation (7.1), we derive

$$(9.1)\quad G_{n_1,\dots,n_m}^{(a_1,\dots,a_m;[k_m];[r_m];[p_m])}(x_1,\dots,x_m)$$

$$=G_{n_1,\dots,n_m}^{(a_1+k_1-1,a_2,\dots,a_m;[k_m];[r_m];[p_m])}(x_1,\dots,x_m)$$

$$-(k_1-1)n_1x_1^{k_1-1}G_{n_1-1,n_2,\dots,n_m}^{(a_1+k_1-1,a_2,\dots,a_m;[k_m];[r_m];[p_m])}(x_1,\dots,x_m),$$

which suggests that m-results similar to (9.1) can be unified in the form :

$$(9.2)\quad G^{(a_1,\ldots,a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$$

$$= G^{(a_1,\ldots,a_{i-1},a_i+k_i-1,a_{i+1},\ldots,a_m;[k_m];(r_m];[p_m])}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$$

$$-(k_i-1)\,n_i\,x_i^{k_i-1}$$

$$G^{(a_1,\ldots,a_{i-1},a_i+k_i-1,a_{i+1},\ldots,a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_{i-1},n_i-1,n_{i+1},\ldots,n_m}(x_1,\ldots,x_m),$$

$$i=1,\ldots,m.$$

Also

(9.3)

$$p_j r_j x_j^{r_j+k_j} G^{(a_1,\ldots,a_{j-1},a_j+r_j+k_j-1,a_{J+1},\ldots,a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_{i-1},n_i+1,n_{i+1},\ldots,n_m}(x_1,\ldots,x_m)$$

$$-a_i p_j r_j x_i^{k_i-1} x_j^{r_j+k_j}\times$$

$$G^{(a_1,\ldots,a_{i-1},a_i+k_i-k,a_{i+1},\ldots,a_{j-1},a_j+r_j+k_j-1,a_{j+1},\ldots,a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_m}(x_1,\ldots,x_m)$$

$$=p_i r_i x_i^{r_i+k_i} G^{(a_1,\ldots,a_{i-1},a_i+r_i+k_i-1,a_{i+1},\ldots a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_{j-1},n_j+1,n_{j+1},\ldots,n_m}(x_1,\ldots,x_m)$$

$$-p_i r_i a_j x_j^{k_j-1} x_i^{r_i+k_i}\,.$$

$$G^{(a_1,\ldots,a_{i-1},a_i+r_i+k_i-1,a_{i+1},\ldots,a_{j-1},a_j+k_j-1,a_{j+1},\ldots,a_m;[k_m];[r_m];[p_m])}_{n_1,\ldots,n_m}(x_1,\ldots,x_m),$$

where $i,j\in\{1,\ldots,m\}$, $i\neq i$.

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