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(Dedicated to the memory of Professor K. L. Singh)

VIBRATION ANALYSIS OF A NON-UNIFORM CIRCULAR PLATE SUBJECTED TO ONE-DIMENSIONAL STEADY-STATE TEMPERATURE DISTRIBUTION

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ABSTRACT

The effect of temperature on the vibration of solid bodies is significant, especially in mechanical constructions, where certain parts of the body have to operate unde el vated temperatures. The interest in this direction has highly increased because of rapid developments in space technology, high speed atmospheric flights, and nuclear energy applications The present paper aims at discussing the effect of linearly transient temperature fields on the natural freqnencies of a circular plate. The frequencies, deflections and moments corresponding to the first two modes of vibration are computed for clamped and simply supported edge conditions by using Frobenius method.

1. INTRODUCTION

With the advancements of space technology and high sqeed atmospheric flights, it has become necessary to ebtain a greate insight into the dynamic behaviour of plates working under temperature fields. The thermal dependence of frequency of plates of different shapes is of great importance. The effect of temperature on the modulus of elasticity of materials is far from negligible especially in the design of aircrafts and rockets in which certain parts have to operate under elevated temperatures. An up-to-date account of problems involved in the vibrations of plates has been given by Leissa ([5], [6]). The frequencies and normal modes of a circular plate of a circular plate of parabolically varying thickness with free edge have evaluated by Harris [2]. Jian [3] investigated the free axisymmetric vibrations of clamped and simply supported circular plates of lineatly varying thickness. Gupta and Lal [1] have observed the buckling and vibrations of circular plate of variable thickness. The axisymmetric vibrations of circular plates of linearly varying thickness have also been studied by Tomar et al. ([11] to [14]) by taking elastic foundation, shear deformation and nonhomogeneiy into account. Tomar and Tiwari [15] have adopted the linea variation of temperature for the computation of free vibrations of a circular plate of variable thickness by a method based on Rayleigh's quotient. Recently Tomar and Gupta ([8], [9]) considered the effect of harmonic and linear temperature variations of axisymmetric vibrations of orthotropic circular plates of variable thickness respectively.

An attempt is made here to study the effect of linearly varying temperature on the free vibrations of circular plates of linearly varying thickness.

2. EQUATION OF MOTION

For an isotropic circular plate of variable thickness, as derived after expressing the equation of motion [10] in polar coordinates and assuming that the axis of the plate coincides with the radial direction, and that the thickness of the plate h, Young's modulue E, density ρ and flexural rigidity D are functions of r, the differential equation is governed by

$$Eh^2 \frac{\partial^4 w}{\partial r^4} + 2[h^2 \frac{\partial E}{\partial r} + \left\{3h(\frac{\partial h}{\partial r}) + \frac{h^2}{r}\right\} E]\frac{\partial^3 w}{\partial r^3} + [\, h^2 + \left\{ 6h(\frac{\partial h}{\partial r})\right.$$

$$\left. + (2+\nu)\frac{h^2}{r}\right\}\frac{\partial E}{\partial r} + \left\{3h(\frac{\partial^2 h}{\partial r^2}) + 6(\frac{\partial h}{\partial r})^2 - \frac{h^2}{r^2} + (2+\nu)\frac{3h}{r}(\frac{\partial h}{\partial r})E\right]\frac{\partial^2 w}{\partial r^2}$$

$$+ [\, \frac{\nu h^2}{r}\frac{\partial^2 E}{\partial r^2} + \left\{\frac{6\nu h}{r}(\frac{\partial h}{\partial r}) - \frac{h_2}{r^2}\right\}\frac{\partial E}{\partial r} + \left\{3\nu\frac{h}{r}(\frac{\partial^2 h}{\partial r^2}) + \frac{6\nu}{r}(\frac{\partial h}{\partial r})^2\right.$$

$$\left. - \frac{3h}{r^2}(\frac{\partial h}{\partial r}) + \frac{h^2}{r^3}\right\}E]\frac{\partial w}{\partial r} + 12(1-\nu^2)\rho\frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

where w is the transverse displacement and ν the Poisson's ratio.

Introducing the non-dimensional variables $H = \frac{h}{a}$, $R = \frac{r}{a}$ and $W = \frac{w}{a}$, equation (1) reduces to

$$\bar{E} H^2 \frac{\partial^4 W}{\partial R^4} + 2[H^2 \frac{\partial \bar{E}}{\partial R} + \left\{3H(\frac{\partial H}{\partial R}) + \frac{H^2}{R}\right\}\bar{E}]\frac{\partial^3 W}{\partial R^3} + H^2\frac{\partial^2 \bar{E}}{\partial R^2} + \left\{6H(\frac{\partial H}{\partial R})\right.$$

$$\left. + (2+\nu)\frac{H^2}{R}\right\}\frac{\partial \bar{E}}{\partial R} + \left\{3H(\frac{\partial^2 H}{\partial R^2}) + 6(\frac{\partial H}{\partial R})^2 - \frac{H^2}{R^2} + (2-\nu)\frac{3H}{R}(\frac{\partial H}{\partial R})\right\}$$

$$\bar{E}]\frac{\partial^2 W}{\partial R^2} + [\, \frac{\nu H^2}{R}\frac{\partial^2 \bar{E}}{\partial R^2} + \left\{6\frac{\nu H}{R}(\frac{\partial H}{\partial R}) - \frac{H^2}{R^2}\right\}\frac{\partial \bar{E}}{\partial R} + \left\{3\nu\frac{H}{R}(\frac{\partial^2 H}{\partial R^2})\right.$$

$$\left. + \frac{6\nu}{R}(\frac{\partial H}{\partial R})^2 - \frac{3H}{R^2}(\frac{\partial H}{\partial R}) + \frac{H^2}{R^3}\right\}\bar{E}\,]\frac{\partial W}{\partial R} + 12(1-\nu^2)a^2\bar{\rho}\frac{\partial^2 W}{\partial t^2} = 0, \quad (2)$$

where 'a' is the radius of the plate, $\bar{E} = \frac{E}{a}$ and $\bar{\rho} = \frac{\rho}{a}$.

Here, the plate material is assumed to be subjected to a linear temperature distribution in radial direction in the following form

$$T = T_0\,(I-R), \tag{3}$$

where T denotes the temperature excess above the reference temperature at any point at a distance $R = \frac{r}{a}$ from the centre and T_0 denotes the temperature excess above the reference temperature at the end $R = 0$.

Furthermore, most of the engineering materials are found to have a linear relationship between the modulus of elasticity and temperature [7], therefore one can have

$$\overline{E}\,(T) = \overline{E}_0(1-\xi\, T), \tag{4}$$

where $\overline{E}_0$ is the modulus of elasticity of the material at the reference temperature and ξ is a constant.

Assuming the temperature at $R = 1$ as the reference temperature, the modulus variation becomes

$$\overline{E}(R) = \overline{E}_0\,[1-\eta(1-R)], \tag{5}$$

where $\eta = \xi\, T_0$. $0 \leqslant \eta \leqslant 1$, a parameter known as thermal gradient.

Thus by taking eqation (5) with the following type of variation in the thickness of the plate

$$H = H_0\,(1-\beta R), \tag{6}$$

where $H_0 = H\,|_{R=0}$ and β is a taper constant. Equation (2) becomes

$$\left\{1-\eta\,(1-R)\right\}(1-\beta R)^2\,\frac{\partial^4 W}{\partial R^4} + 2\eta\Big[\,(1-\beta R)^2 + \Big\{-3\beta(1-\beta R)$$

$$+\frac{(1-\beta R)^2}{R}\Big\}\left\{1-\eta(1-R)\right\}\Big]\,\frac{\partial^3 W}{\partial R^3} + -\Big[6\eta\beta(1-\beta R)+(2+\nu)\eta\,\frac{(1-\beta R)^2}{R}$$

$$+\left\{6\beta^2-\frac{(1-\beta R)^2}{R^2}-3\beta(2+\nu)\,\frac{(1-\beta R)}{R}\right\}\left\{1-\eta\,(1-R)\right\}\Big]\,\frac{\partial^2 W}{\partial R^2}$$

$$+\Big[-6\nu\eta\beta\,\frac{(1-\beta R)}{R}-\frac{\eta(1-\beta R)^2}{R^2}+\Big\{\frac{6\nu\beta^2}{R}+\frac{3\beta(1-\beta R)}{R^2}$$

$$+ \frac{(1-\beta R)^2}{R^3}\Big\} \Big\{ 1 - \eta(1-R) \Big\} \Big] \frac{\partial W}{\partial R} + \frac{(1-\nu^2)\, a^2 \bar{\rho}}{E_0\, I^*} \frac{\partial^2 W}{\partial t^2} = 0, \qquad (7)$$

where $I^* = \frac{H^3{}_0}{12}$.

3. SOLUTION

For harmonic vibrations, the substitution

$$W(R,t) = \overline{W}(R)\, e^{i\omega t}$$

is made tnto equation (7), which reduces to

$$\sum_{\lambda=0}^{\infty} a_\lambda [B_1(\lambda) R^{c+\lambda-4} + B_2(\lambda) R^{c+\lambda-3} + B_3(\lambda) R^{c+\lambda-2}$$

$$+ B_4(\lambda) R^{c+\lambda+1} + G_{05}\, R^{c+\lambda}] = 0, \qquad (8)$$

where

$$\overline{W}(R) = \sum_{\lambda=0}^{\infty} a_\lambda\, R^{c+\lambda},\ a_0 \neq 0, \qquad (9)$$

is the assumed series solution for $\overline{W}$ and c is the exponent of singularity.

$$B_1(\lambda) = G_{01} b_\lambda^{(3)} + G_{02} b_\lambda^{(2)} + G_{03} b_\lambda^{(1)} + G_{04} b_\lambda^{(0)}.$$

$$B_2(\lambda) = G_{11} b_\lambda^{(3)} + G_{12} b_\lambda^{(2)} + G_{13} b_\lambda^{(1)} + G_{14} b_\lambda^{(0)}.$$

$$B_3(\lambda) = G_{21} b_\lambda^{(3)} + G_{22} b_\lambda^{(2)} + G_{23} b_\lambda^{(1)} + G_{24} b_\lambda^{(0)}.$$

$$B_4(\lambda) = G_{31} b_\lambda^{(3)} + G_{32} b_\lambda^{(2)} + G_{33} b_\lambda^{(1)} + G_{34} b_\lambda^{(0)}.$$

$G_{01} = 1 - \eta$, $G_{11} = \eta - 2(1-\eta)\,\beta$, $G_{21} = -2\eta\beta + (1-\eta)\beta^2$,

$G_{31} = \eta\beta^2$, $G_{02} = 2(1-\eta)$, $G_{12} = 4\eta - 10(1-\eta)\beta$, $G_{22} = -14\eta\beta + 8(1-\eta)\beta^2$,

$G_{32} = 10\eta\beta^2$, $G_{03} = -(1-\eta)$, $G_{13} = (1+\nu)\,\eta - (4+3\nu)(1-\eta)\beta$,

$G_{23} = -(14+5\nu)\ \eta\beta + (11+3\nu)\ (1-\eta)\ \beta^2,\quad G_{33} = (19+4\nu)\ \eta\beta^2,$

$G_{04} = 1-\eta,\quad G_{14} = (1-\eta)\ \beta,\quad G_{24} = 3\ (1-2\nu)\ \eta\beta - 2(1-3\nu)\ (1-\eta)\ \beta^2,$

$G_{34} = -3(1-4\nu)\eta\beta^2,\ G_{05} = -\dfrac{\Omega^2}{1^*},\ \Omega^2 = \dfrac{(1-\nu^2)}{E_0}\ a^2\omega^2,$

ω is the radian frequency and Ω-frequency parameter,

$b_\lambda^{(0)} = c+\lambda,\ b_\lambda^{(1)} = (c+\lambda)(c+\lambda-1),\ b_\lambda^{(2)} = (c+\lambda)(c+\lambda-1)\ (c+\lambda-2),$

For (9) to be the solution, the coefficients of various powers of R in the expression obtained substituting (9) in (8), must be identically zero. Thus by equating to zero the coefficient of the lowest power of R, yields the indicial equation

$c^2(c-2)^2 = 0$

whch gives $c = 0, 0, 2, 2.$

Further, equating to zero the coefficients of the next subsequent powers of R, it is found that $a_1 = 0$, a_2 is indeterminate for $c = 0$. Also, the remaining a_λ's ($\lambda = 3,4,\ldots$) can be written in terms of a_0 and a_2. Hence, assuming

$$a_\lambda = F_\lambda^{(0)}\ a_0 + F_\lambda^{(2)}\ a_2, \tag{10}$$

where

$F_0^{(0)} = F_2^{(2)} = 1,\ F_1^{(0)} = F_2^{(0)} = F_0^{(2)} = F_1^{(2)} = 0,$

and rest $F_\lambda^{(i)}$'s ($i=0,2;\ \lambda=3,4,\ldots$) can be calculated from the recurrence relation

$$F_\lambda^{(i)} = -[B_2(\lambda-1)\ F_{\lambda-1}^{(i)} + B_3(\lambda-2)\ F_{\lambda-2}^{(i)} + B_4(\lambda-3)\ F_{\lambda-3}^{(i)} + G_{05}\ F_{\lambda-4}^{(i)}]/B_1(\lambda), \tag{11}$$

with the consideration that

$$B_j(\lambda-s) = \begin{cases} B_j(\lambda-s) & \text{if } \lambda-s \geqslant 0, \\ 0 & \text{if } \lambda-s < 0. \end{cases}$$

and

$$F^{(i)}_{\lambda-s} = \begin{cases} F^{(i)}_{\lambda-s} & \text{if } \lambda-s \geqslant 0, \\ 0 & \text{if } \lambda-s < 0 \end{cases}$$

$(j = 2,3,4, ;\ \lambda = 3,4,5,\dots ;\ s = 1,2,3,4 ;\ i = 0,2)$,

the following solution corresponding to $c = 0$, is obtained

$$\overline{W} = \left[1 + \sum_{\lambda=3}^{\infty} a_\lambda R^\lambda\right] a_0 + \left[R^2 + \sum_{\lambda=3}^{\infty} a_\lambda R^\lambda\right] a_2. \tag{12}$$

It is also evident that the solution corresponding to other values of c is included in equation (12).

4. CONVERGENCE OF THE SOLUTION

The convergence of the solution (12) is tested by applying the technique used by Lamb [4]. Thus, equating to zero the coefficient of the general term occurring in (8) and taking the limit as $\lambda \to \infty$, one has

$$(1-\eta)\mu^4 + \{\eta - 2(1-\eta)\beta\}\mu^3 - \{2\eta\beta - (1-\eta)\beta^2\}\mu^2 + \eta\beta^2\mu = 0, \tag{13}$$

where

$$\mu = \lim_{\lambda \to \infty} \frac{a_{\lambda+1}}{a_\lambda}.$$

The roots of the equation (13) are $\mu = 0, \beta, \beta$ and $-\frac{\eta}{1-\eta}$. Thus in the interval $0 \leqslant R < 1$, when $|\mu| < 1$ and hence for all $|\beta| < 1$ and $\eta < 0.5$, the solution (12) is uniformly convergent.

5. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

The frequency equations for clamped (C) and simply supported (SS) circular plates are obtained by using the following boundary conditions:

C-PLATES. For a circular plate clamped at the edge $r = a$, the deflection and slope of toe plate element at the edge snould be zero i. e.

$$w(r,t)\,|_{r=a} = \frac{\partial w(r,t)}{\partial t}\,|_{r=a} = 0$$

$$\text{or } \overline{W}\,|_{R=1} = \frac{\partial \overline{W}}{\partial R}\,|_{R=1} = 0. \qquad (14)$$

When the boundary conditions (14) are enforced, equation (11) yields the following frequency equation:

$$\begin{vmatrix} Q_1(\Omega) & Q_2(\Omega) \\ Q_3(\Omega) & Q_4(\Omega) \end{vmatrix} = 0\,, \qquad (15)$$

where

$$Q_1(\Omega) = 1 + \sum_{\lambda=3}^{\infty} F_\lambda^{(0)},\ Q_2(\Omega) = 1 + \sum_{\lambda=3}^{\infty} F_\lambda^{(2)},\ Q_3(\Omega) = \sum_{\lambda=3}^{\infty} \lambda\, F_\lambda^{(0)}$$

and

$$Q_4(\Omega) = 2 + \sum_{\lambda=3}^{\infty} \lambda\, F_\lambda^{(2)}.$$

SS-PLATES. For a circular plate simply-supported at the edge $r = a$, the boundary conditions are

$$w(r,t)\,|_{r=a} = M_r(r,t)\,|_{r=a} = 0,$$

or

$$\overline{W}\,|_{R=1} = \left[\frac{\partial^2 \overline{W}}{\partial R^2} + \frac{\nu}{R}\frac{\partial \overline{W}}{\partial R}\right]|_{R=1} = 0. \qquad (16)$$

Applying the boundary conditions (16) on equation (12) the characteristics equation for the determination of frequencies, is obtained as

$$\begin{vmatrix} Q_1(\Omega) & Q_2(\Omega) \\ Q_5(\Omega) & Q_6(\Omega) \end{vmatrix} = 0\,, \qquad (17)$$

where

$$Q_5(\Omega) = \sum_{\lambda=3}^{\infty} \lambda(\lambda+\nu-1)\ \overset{(0)}{F_\lambda} \quad \text{and}$$

$$Q_6(\Omega) = 2(1+\nu) + \sum_{\lambda=3}^{\infty} \lambda(\lambda+\nu-1)\ \overset{(2)}{F_\lambda}\ .$$

6. DEFLECTION AND MOMENTS

Usiing the boundary conditions at $R = 1$ with $a_0 = 1$ on the deflection function $\overline{W}$ (12), one finds that the value of a_2 i. e.

$$a_2 = -\frac{1 + \sum_{\lambda=3}^{\infty} \overset{(0)}{F_\lambda}}{1 + \sum_{\lambda=3}^{\infty} \overset{(2)}{F_\lambda}},$$

Thus after substituting for a_0 and a_2, equation (12) becomes

$$\overline{W} = \left[1 + \sum_{\lambda=3}^{\infty} \overset{(0)}{F_\lambda} R^\lambda\right] - \frac{1 + \sum_{\lambda=3}^{\infty} \overset{(0)}{F_\lambda}}{1 + \sum_{\lambda=3}^{\infty} \overset{(2)}{F_\lambda}} \left[R^2 + \sum_{\lambda=3}^{\infty} \overset{(2)}{F_\lambda} R^\lambda\right] \qquad (18)$$

Applying the boundary conditions at $R = 1$ with same values of a_0 and a_2 on the non-dimensional moment parameter i. e. on $\overline{M} = \frac{M_R}{D_0}$

one gets

$$\overline{M} = -\{1-\gamma(1-R)\}\ (1-\beta R)^3\ \Big[\sum_{\lambda=3}^{\infty} \lambda(\lambda-1+\nu)\ \overset{(0)}{F_\lambda}\ R^{\lambda-2}$$

$$- \frac{1 + \sum_{\lambda=3}^{\infty} \overset{(0)}{F_\lambda}}{1 + \sum_{\lambda=3}^{\infty} \overset{(2)}{F_\lambda}} \{2(1+\nu) + \sum_{\lambda=3}^{\infty} \lambda(\lambda-1+\nu)\ \overset{(2)}{F_\lambda}\ R^{\lambda-2}\}\Big] \qquad (19)$$

where

$$D_0 = \frac{E_0 H_0^3 a_0}{12(1-\nu^2)} .$$

6. DISCUSSION OF RESULTS

In both the cases of edge conditions i.e. clamped and simplysupported of circular plates, the frequeucy parameter $\Omega (= a\omega\sqrt{(1-\nu^2)} / (\bar{E}_0/\bar{\rho}))$, for the first two modes of vibration have been computed for different values of taper constant β and thermal gradient η with fixed values of Poisson's ratio and initial thickness taken as 0.3 and 0.1 respectively. In computing the the frequency parameter, terims upto an accuracy of 10^{-8} on their absolute values in the seres have been retained.

In Figure-1, the effect of linear thickness on the frequency parameter Ω of heated and unheated circular plates, is shown. The values of Ω for $\eta = 0.0$ have been found to decrease 39.81% and 31.37% for a clamped circular plate, 27.61% and 28.56 for SS-plate for the first and second modes of vibration, respectively. On the other hand, other hand, for heated plate, these decrements are 40.31, 31.32 and 28 74 29 12 percent respectively. Thus, one can conclude from the above percentage the shapes of the curves, for heated and unheated plates for both the edge conditions and modes of vibration considered here, are almost linear.

The effect of thermal gradient η on the frequency parameter Ω, for clamped and simply supported edge conditions for the first two modes of vibration of the uniform and non-uniform circular plates, has been shown in Fig 2 It is found for $\beta = 0.0$, the Ω decreases 7.72% and 9 95% for clamped plate and 10.09% and 10.43 for SS-plate for the first two modes of vibration respectively. When the value of β is taken 0.3, the decrement with linearly varying temperature gradient η, are noted 8.02, 10.07 and 10.66, 9.85 percent for the same boundary conditions and modes of vibration considered above. The effect of linear temperature, on the frequencies of uniform and non-uniform clamped circular plates is higher than simply-supported plate, is also noted.

The transverse displacement $\overline{W}$ and moment parameter $\overline{M}$, for the first two modes for both edge conditions, are shown in Figures 3 and 4 resyectively.

To make a comparision with the results in references [3] and [14], the value of Ω was replaced by $\frac{a\omega}{H_0}\sqrt{12(1-\nu^2)\ \bar{\rho}\ /\ \bar{E}_0}$ and the frequencies were computed for the first two modes of vibration for both clamped and simply supported edge conditions, for three values of the paper constant. These results are shown in Table 1. It can be seen from this table that the results are in good agreement.

TABLE 1

A comparison of the natural frequencies of circular plate of linearly varying thickness, $H_0 = 0.1$, $\nu = 0.3$, $\eta = 0.0$

β	Clamped plate		Simply-supported plate	
	First Mode	Second Mode	First Mode	Second Mode
0.1	9.40578	37.37706	4.66706	28.07945
	(9.40578)*	(37.37705)	(4.66707)	(28.07944)
	(9.4058)**	(37.3771)	(4.6671)	(28.0795)
0.3	7.80704	32.47526	4.14675	24.74402
	(7.80705)	(32.47527)	(4.14676)	(24.74401)
	(7.8071)	(32.4753)	(4.1468)	(24.7440)
0.5	6.23385	27.34074	3.63707	21.28774
	(6.23384)	(27.34073)	(3.63707)	(21.28773)
	(6.2339)	(27.3408)	(3.6371)	(21.2878)

* Reference [3]

**Reference [14]

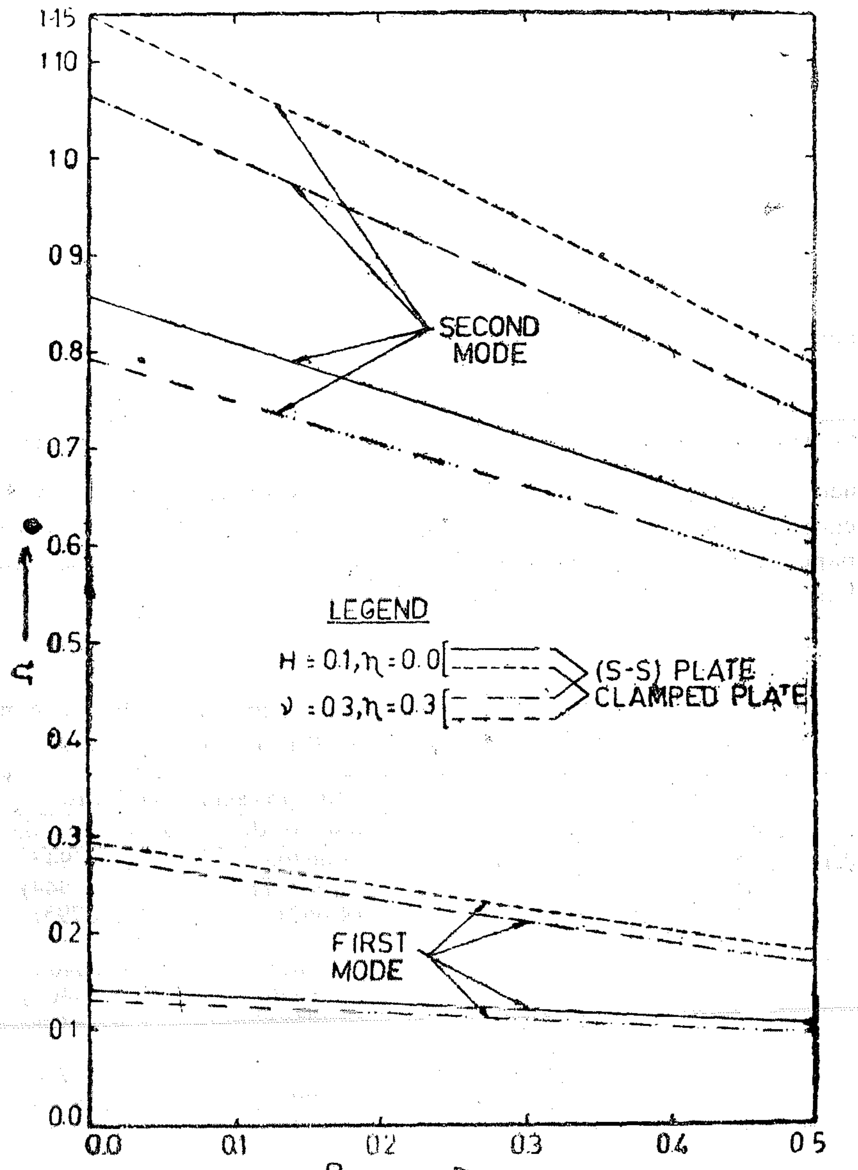

FIG 11 EFFECT OF TAPER CONSTANT β ON THE FREQUENCY PARAMETER 'Ω'

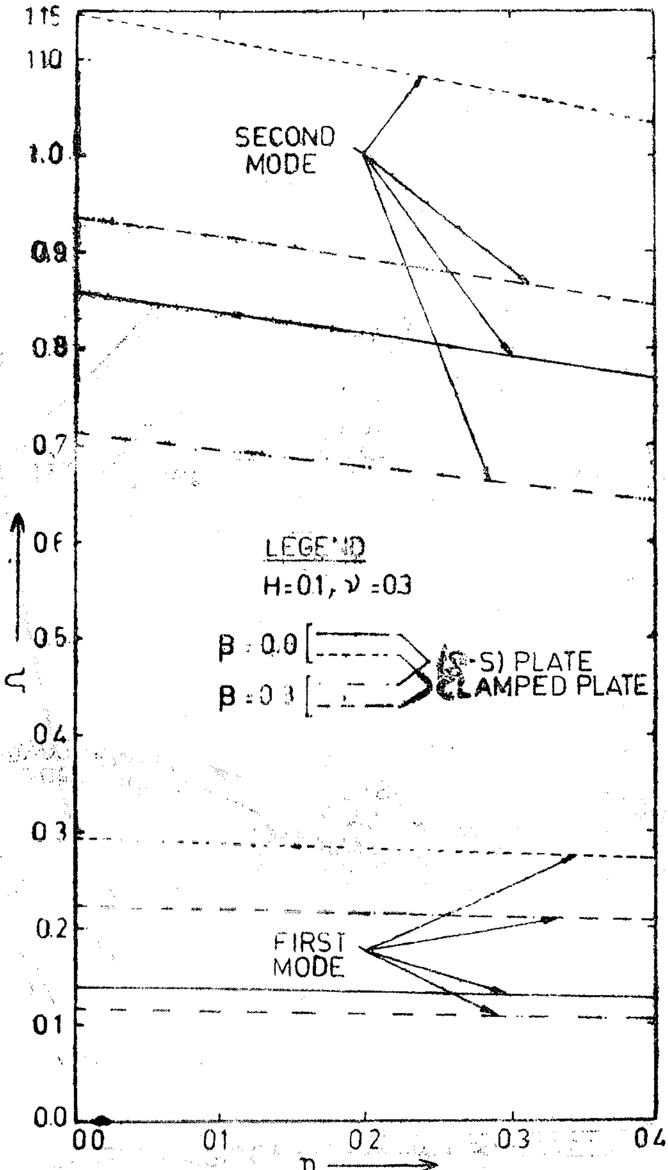

FIG 2 EFFECT OF THERMAL GRADIENT 'η' ON THE FREQUENCY PARAMETER 'Ω'

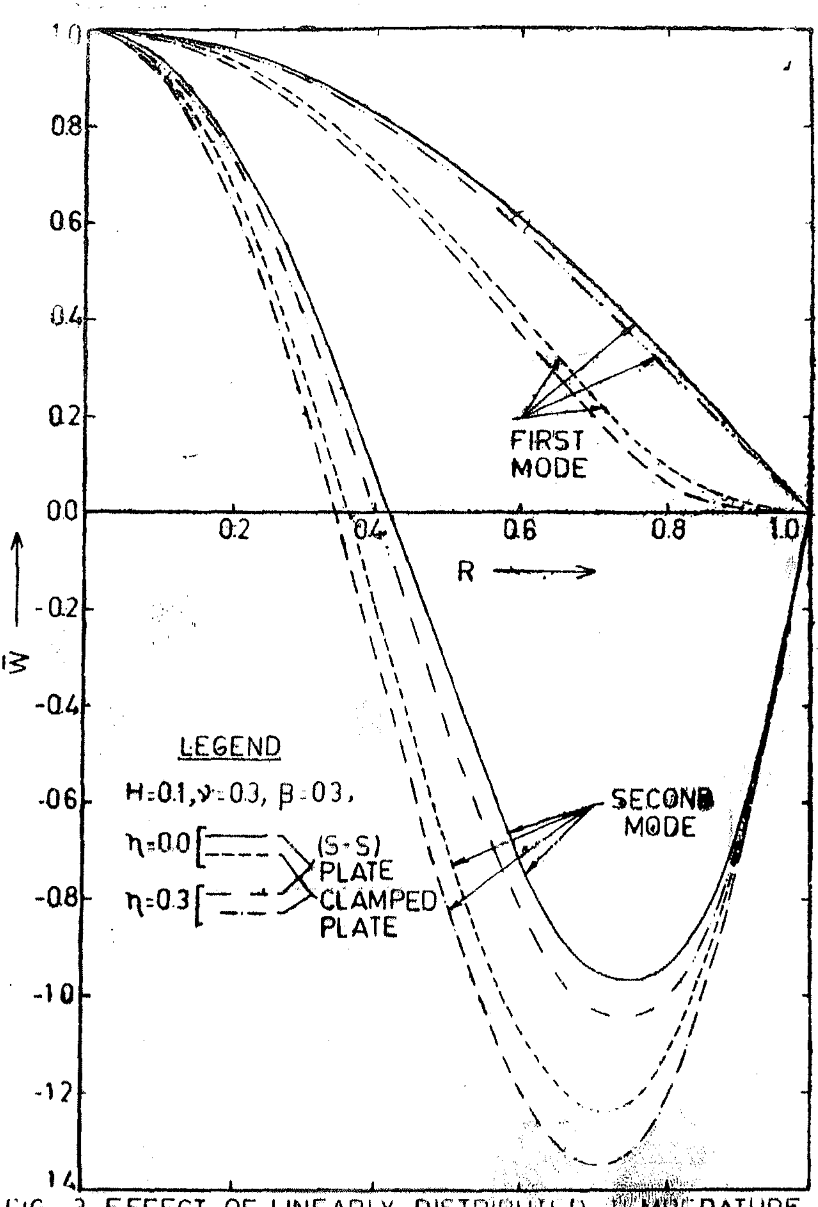

FIG. 3 EFFECT OF LINEARLY DISTRIBUTED TEMPERATURE ON THE DEFLECTION PARAMETER 'W̄' OF CIRCULAR PLATE OF LINEARLY VARYING THICKNESS.

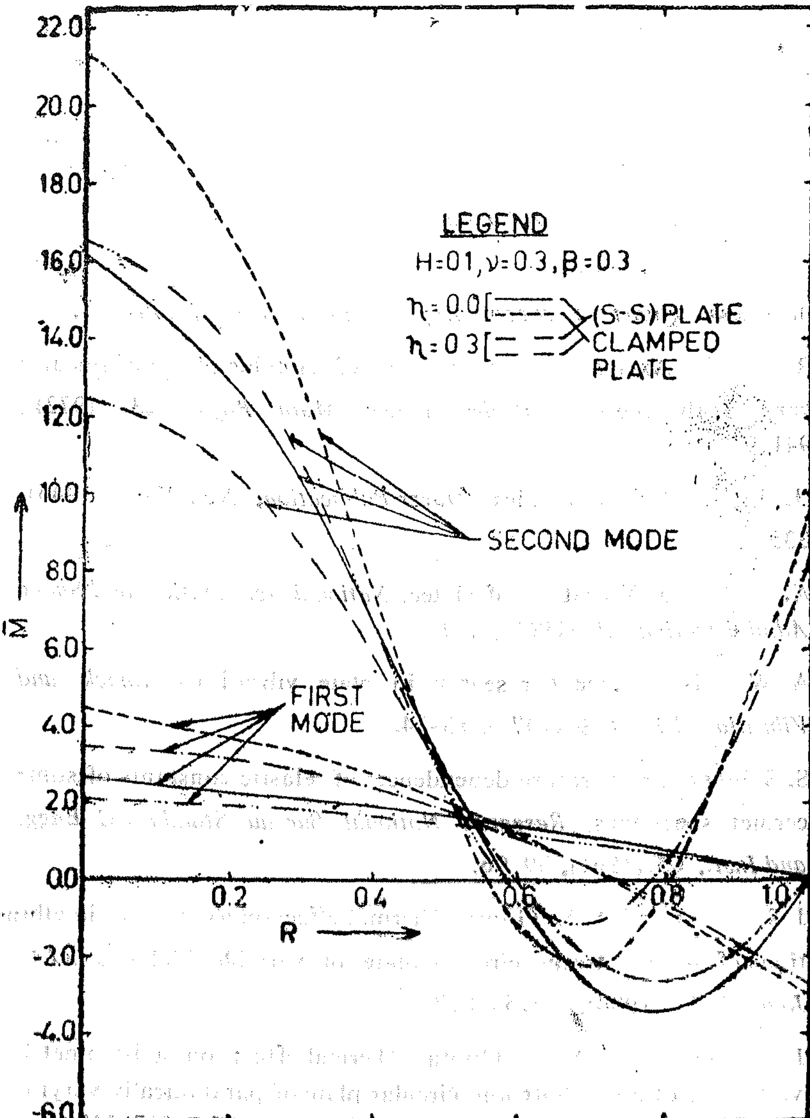

FIG. 4 MOMENT PARAMETERS 'M̄' OF A CIRCULAR PLATE OF LINEARLY VARYING THICKNESS UNDER THE INFLUENCE OF LINEARLY VARYING TEMPERATURE FIELD.

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