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(Dedicated to the memory of Professor K. L. Singh)

ON THE INVERSE LAPLCE TRANSFORM OF CERTAIN FUNCTIONS INVOLVING ONE AND MORE VARIABLES

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ABSTRACT

In this paper, we first establish three interesting and useful formulas giving the inverse Laplace transform of various products of algebraic powers and general classes of polynomials involving one and more variables. The generalizations of these formulas have also been established in § 3. In view of the generality of the polynomials considered here, on specializing the coefficients, our results would reduce to a large number of formulas giving inverse Laplace transforms of simpler functions and known classes of polynomials.

1. RESULTS REQUIRED

$$L^{-1}\left\{(p^2+b^2)^{-(2\mu+1)/2}\, S_N^M\left[z(p^2+b^2)^{-\eta}\right]\,;x\right\}$$

$$=\sqrt{\pi}\left(\frac{x}{2b}\right)^{\mu}\sum_{k=0}^{[N/M]}(-N)_{Mk}\,B_{N,k}\,\frac{z^k}{k!}\,\frac{1}{\Gamma(\mu+\eta k+(1/2))}\left(\frac{x}{2b}\right)^{\eta k}J_{\mu+\eta k}(bx) \qquad \dots (1.1)$$

where

$$S_N^M[x]=\sum_{k=0}^{[N/M]}(-N)_{Mk}\,B_{N,k}\,\frac{x^k}{k!} \qquad \dots (1.2)$$

stands for a generol class of polynomials introduced by Srivastava ([4, p. 1, Eq. (1)]; see also [7]). The result (1.1) is valid under the following conditons:

$\eta \geqslant 0$, $Re(p) > 0$ and $Re(2\mu+1) > 0$.

$$L^{-1}\left\{\{p+(p^2+b^2)^{1/2}\}^{-\nu} S_N^M\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\right]; x\right\}$$

$$= b^{-\nu} \sum_{k=0}^{[N/M]} (-N)_{MK}\, B_{N,K} \frac{z^k}{k!} (\nu+\rho k)\, b^{\rho k} x^{-1} J_{\nu+\rho k}(dx) \qquad \dots (1.3)$$

provided that

$\rho \geqslant 0$, $Re(p) > 0$.

$$L^{-1}\left\{p^{-2\lambda}(p^2+a^2)^{-\nu} S_n^{m_1,\dots,m_s}\left[y_1 p^{-2\rho_1}(p^2+a^2)^{-\sigma_1}, \dots, y_s p^{-2\rho_s}(p^2+a^2)^{-\sigma_s}\right]; x\right\}$$

$$= \sum_{k_1,\dots,k_s=0}^{m_1k_1+\dots+m_sk_s \leqslant n} (-n)_{m_1k_1+\dots+m_sk_s} A(n;k_1,\dots,k_s) \frac{y_1^{k_1}\dots y_s^{k_s}}{k_1!\dots k_s!}$$

$$\cdot \frac{x^{2\lambda'+2\nu'-1}}{\Gamma(2\lambda'+2\nu')}\, {}_1F_2\left[\nu';\lambda'+\nu',\lambda'+\nu'+1/2; -\frac{a^2x^2}{4}\right] \qquad \dots (1.4)$$

where

$$\lambda' = \lambda + \sum_{i=1}^{s} \rho_i k_i \text{ and } \nu' = \nu + \sum_{i=1}^{s} \sigma_i k_i \qquad \dots (1.5)$$

and $\rho_i, \sigma_i \geqslant 0$ $(i=1,\dots,s)$, $Re(p) > 0$, $Re(\lambda) > 0$, $Re(\nu) \geqslant 0$.

Also

$$S_n^{m_1,\dots,m_s}[y_1,\dots,y_s] = \sum_{k_1,\dots,k_s=0}^{m_1k_1+\dots+m_sk_s \leqslant n} (-n)_{m_1k_1+\dots m_sk_s}$$

$$\cdot A(n;k_1,\dots,k_s) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_s^{k_s}}{k_s!} \qquad \dots (1.6)$$

stands for a general class of multivariable polynomials (see, e.g., [5]).

$$L^{-1}\left\{(p^2+a^2)^{1/2}\{p+(p^2+a^2)^{1/2}\}^{-\lambda} S_n^{m_1,\dots m_s}\right.$$

$$\left[y_1 \{p+(p^2+a^2)^{1/2}\}^{-\rho_1},\ldots,y_s\{p+(p^2+a^2)^{1/2}\}^{-\rho_s}\right] ; x \Big\}$$

$$= \sum_{k_1,\ldots,k_s=0}^{m_1k_1+\ldots+m_sk_s \leqslant n} (-n)_{m_1k_1+\ldots+m_sk_s} A(n;k_1,\ldots,k_s)$$

$$\cdot \frac{y_1^{k_1}\ldots y_s^{k_s}}{k_1!\ldots k_s!} a^{-\lambda'} \; J_{\lambda'}(ax) \qquad \ldots(1\ 7)$$

where λ' is given by (1.5) and

$\rho_i > 0,\ i = 1,\ldots,s\ ;\ Re(\lambda) > -1,\ Re(p) \geqslant 0.$

$$L^{-1} \left\{p^{-\nu} q^{-\mu} S_N^M\left[z\, p^{-\rho}\, q^{-\sigma}\right] ; x,y \right\}$$

$$= x^{\nu-1}\, y^{\mu-1} \sum_{k=0}^{[N/M]} (-N)_{MK}\, B_{N,K} \frac{z^K\, x^{\rho K}\, y^{\sigma K}}{k!\, \Gamma(\nu+\rho k)\, \Gamma(\mu+\sigma k)} \qquad \ldots(1.8)$$

where

$\rho,\ \sigma \geqslant 0,\ Re(\nu) > 0,\ Re(\mu) > 0,\ Re(p) > 0,\ Re(q) > 0,$

he results (1.1), (1.3), (1.4), (1.7) and (1.8) are not available in the literature. In order to evaluate (1.1) use series representation for $S_N^M[x]$ given by (1.2) and make use of a known result ([2,*p*.239, *Eq*. (18)]). Similarly the results (1.3), (1.4), (1.7) and (1.8) can be established if we use the known results ([2, *p*. 240, *Eq*. (23)]), ([2, *p*. 238, *Eq* (10)]), ([2, *p*. 240, *Eq*. (21)]), ([1, *p*. 137, *Eq*. (287)]) respectively instead of ([2, *p*. 239, *Eq* (18)]).

$$L^{-1} \left\{ p^{ms_2}\, q^{ns_1}\, (p^m q^n + a)^{-s_1-s_2} ; x, y \right\}$$

$$= \frac{x^{ms_1-1}\; y^{ns_2-1}}{\Gamma(s_1+s_2)}\, {}_1\Psi_2 \left[\begin{matrix}(s_1+s_2,\ 1) \\ (ms_1, m);\ (ns_2, n)\end{matrix} ; -ax^m y^n \right] \qquad \ldots(1.9)$$

$Re(s_1) > 0,\ Re(s_2) > 0,\ Re(p) > 0,\ Re(q) > 0,\ m,n > 0.$

The result (1.9) is a special case of a known result ([3,*p*. 118] ; see also [6, *p*. 69, *Eq*. (5. 4. 1)]).

1. MAIN RESULTS

$$L^{-1}\left\{p^{-2\lambda}\,(p^2+a^2)^{-\nu}\,(p^2+b^2)^{-(2\mu+1)/2}\;S_N^M\left[z(p^2+b^2)^{-\eta}\right]\right.$$

$$\left.\cdot S^{m_1,\ldots,m_s}\left[y_1\;p^{-2\rho_1}\,(p^2+a^2)^{-\sigma_1},\ldots,y_s\;p^{-2\rho_s}\,(p^2+a^2)^{-\sigma_s}\right];x\right\}$$

$$=\sum_{K=0}^{[N/M]}\;\sum_{k_1,\ldots,k_s=0}^{m_1+k_1+\ldots+m_sk_s\leqslant n}(-n)_{m_1k_1+\ldots+m_sk_s}$$

$$(-N)_{MK}\,B_{N,K}\,A(n;k_1,\ldots,k_s)\;\cdot\frac{y_1^{k_1}\cdots y_s^{k_s}\,z^K(x)^{2\lambda'+2\nu'+2\mu+2\eta k}}{k_1!\ldots k_s!\,k!\,\Gamma(1+2\lambda'+2\nu'+2\mu+2\eta K)}$$

$$\cdot F^{0:1;1}_{2:0;0}\left[\begin{matrix}-\!- & : \\ 1/2+\lambda'+\nu'+\mu+\eta K,\;1+\lambda'+\nu'+\mu+\eta K & :\end{matrix}\right.$$

$$\left.\begin{matrix}\nu\;;\;\mu+\eta K+(1/2)\;; \\ -\;;\qquad -\qquad ;\end{matrix}\;\frac{-a^2x^2}{4},\frac{-b^2x^2}{4}\right]\qquad\ldots(2.1)$$

where λ', ν' are given by (1.5), the conditions of validity of (2.1) are

(i) $\rho_i,\sigma_i,\eta,\geqslant 0$, where $i=1,\ldots s$,

$Re(p)\vartriangleright 0$, $Re(\lambda+\nu)>0$, $Re(2\mu+1)>0$

(ii) M, $m_1,\ldots,m_s$ are arbitrary positive integers, the coefficients $B_{N,K}$, $A(n;k_1,\ldots,k_s)$, $(N,n,K,k_1,\ldots,k_s\geqslant 0)$ are arbitrary constants, real or complex.

$$L^{-1}\{(p^2+b^2)^{1/2}\{p+(p^2+a^2)^{1/2}\}^{-\lambda}\{p+(p^2+b^2)^{1/2}\}^{-\nu}$$

$$S_N^M\left[z\{p+(p^2+b^2)^{1/2}\}^{-\rho}\right]$$

$$,S_n^{m_1,\ldots,m_s}\left[y_1\{p+(p^2+a^2)^{1/2}\}^{-\rho_1},\ldots,y_s\;\{p+(p^2+a^2)^{1/2}\}^{-\rho_s}\right];x\Big\}$$

$$=\sum_{K=0}^{[M/N]}\;\sum_{k_1,\ldots,k_s=0}^{m_1k_1+\ldots+m_sk_s\leqslant n}(-n)_{m_1k_1+\ldots+m_sk_s}\,(-N)_{MK}\,B_{N,K}$$

$$A(n;k_1,\ldots,k_s)\ \frac{z^K}{K!}\ \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_1^{k_s}}{k_s!}\ \frac{(x/2)^{\lambda'+\nu+\rho K}}{\Gamma(\lambda'+\nu+\rho K)}$$

$$F^{0:2;2}_{2:1;1}\left[\begin{matrix} \text{——} & : \frac{1}{2}(\lambda'+2),\ \frac{1}{2}(\lambda'+1)\ ; \\ \frac{1}{2}(\lambda'+\nu+\rho K+2),\ \frac{1}{2}(\lambda'+\nu+\rho K+1) & : \lambda'+1\ ; \end{matrix}\right.$$

$$\left.\begin{matrix} \frac{1}{2}(\nu+\rho K),\ \frac{1}{2}(\nu+\rho K+1)\ ; \\ \nu+\rho K+1\ ; \end{matrix}\ -\frac{a^2x^2}{4},\ -\frac{b^2x^2}{4}\right] \qquad \ldots(2.2)$$

provided that

(i) $\rho,\ \rho_i \geqslant 0\ (i=1,\ldots,s)$, $Re(\lambda)$, $Re(\nu)$ and $Re(p) > 0$

(ii) $M,\ m_1,\ldots,m_s$ are arbitrary positive integers. $B_{N,K}$. $A(n;k_1,\ldots,ks)$ are arbitrary constants, real or complex.

Also λ' is defined by (1.5)

$$L^{-1}\left\{ p^{ms_2-\nu}\ q^{ns_1-\mu}\ (p^m q^n + a)^{-s_1-s_2}\ S_N^M\left[z\,p^{-\rho}\,q^{-\sigma}\right];\ x,\ y \right\}$$

$$= \frac{x^{ms_1+\nu-1}\ y^{ns_2+\mu-1}}{\Gamma(s_1+s_2)} \sum_{K=0}^{[N/M]} (-N)_{MK}\, B_{N,K}\, \frac{z^K}{K!}\, x^{-\rho K}\, y^{-\sigma K}$$

$$\cdot\ {}_1\Psi_2\left[\begin{matrix} (s_1+s_2,\ 1) \\ (\nu+\rho K+ms_1, m),\ (\mu+\sigma K+ns_2, n) \end{matrix}\ ;\ -a x^m y^n\right] \qquad \ldots(2.3)$$

provided that

(i) $\rho,\ \sigma,\ Re(\nu),\ Re(\mu),\ Re(p),\ Re(q) > 0$, $m\ n$ are real numbers.

(ii) M is arbitrary positive integer and $B_{N,K}$ $(N,\ K \geqslant 0)$ is an arbitrary constant, real or complex.

Proof of (2.1). Using the well-known convolution theorem for Laplace transforms to the pair of equations (1.1) and (1.4), we find that

$$L.H.S.\ \text{of (2.1)} = \sum_{K=0}^{[N/M]}\ \sum_{k_1,\ldots,k_s=0}^{m_1k_1+\cdots+m_sk_s \leqslant n} (-n)_{m_1k_1+\cdots+m_sk_s}$$

$$(-N)_{MK}\, A(n;k_1,\ldots,k_s)\, B_{N\, K}\, \frac{\sqrt{\pi}\ (2b)^{-\mu-\eta K}}{(\Gamma(\mu+\eta K+1/2))\,\Gamma(\lambda'+2\nu')}\, \frac{z^K}{K!}\, \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}$$

$$\int_0^x (x-u)^{\mu+\eta K}\, J_{\mu+\eta K}\,\{b(x-u)\}\, u^{2\lambda'+2\nu'-1}$$

$${}_1F_2\left[\nu';\ \lambda'+\nu',\ \lambda'+\nu'+\tfrac{1}{2};\ \frac{-a^2u^2}{4}\right] du \qquad \ldots(2.4)$$

Now, using the known result ([6, *p.* 61, *Eq.* (5.2.1)]) to evaluate u-integral and the results ([6, *p.* 15. *Eq.* (2. 3. 6); *p.* 19, *Eq.* (2.6.11)]) and the Gamma duplication formula to simplify the expression so obtained, we easily arrive at the required result (2.1).

Proof of (2.2). To prove the result (2.2), we proceed on the lines similar to those in the proof of (2.1). Here we use the transform pairs (1.3) and (1.7), instead of (1.1) and (1.4).

Poorf of (2 3) To prove (2.3). we apply the convolution theorem of Laplace transforms for two variables for the pair (1.8) and (1.9), and we get

$$L.H.S.\text{ of }(2.3) = \frac{1}{\Gamma(s_1+s_2)} \sum_{K=0}^{[N/M]} (-N)_{MK}\, B_{N,K}\, \frac{z^K}{K!}\, \frac{1}{\Gamma(\nu+\rho K)\Gamma(\mu+\sigma K)}$$

$$\cdot \int_0^x \int_0^y (x-u)^{\nu+\rho K-1}\, (y-u)^{\mu+\sigma K-1}\, u^{ms_1-1}\, v^{ns_2-1}$$

$$\cdot\ {}_1\Psi_2\left[\begin{matrix}(s_1+s_2,\ 1)\\ (ms_1,\ m),\ (ns_2,\ n)\end{matrix};\ -au^m v^n\right] du\, dv \qquad \ldots(2.5)$$

Writing ${}_1\Psi_2$ in terms of its bilateral series, changing the order of integration and summation, and evaluating the u, v-integrals, we easily arrive at the desired result (2.3).

3. GENERALIZATIONS OF THE FORMULAS (2.1) TO (2.3)

Now we establish three more formulas (given below) which are generalizations of the formulas (2.1) to (2.3).

$$L^{-1}\left\{p^{-2\lambda}(p^2+a^2)^{-\nu}S_n^{m_1,\dots,m_s}\left[y_1p^{-2\rho_1}(p^2+a^2)^{-\sigma_1},\dots,y_sp^{-2\rho_s}(p^2+a^2)^{-\sigma_s}\right]\right.$$

$$\left.\cdot\prod_{i=1}^{r}\left\{(p^2+b_i^2)^{-(2\mu_i+1)/2}\,S_{N_i}^{M_i}\left[z_i(p^2+b_i^2)^{-\eta_i}\right]\right\};x\right\}$$

$$=x^{2\lambda+2\nu+2\sum_{i=1}^{r}(\mu_i)+r-1}$$

$$\sum_{k_1,\dots,k_s=0}^{m_1k_1+\dots+m_sk_s\leqslant n}\ \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_r=0}^{[N_r/M_r]}(-n)_{m_1k_1+\dots+m_sk_s}$$

$$A(n;k_1,\dots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}\sum_{i=1}^{r}\left\{(-N_i)_{M_iK_i}B_{N_i,K_i}\frac{(z_i)^{K_i}}{K_i!}\right\}$$

$$\cdot\frac{(x)^{2\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+\sum_{i=1}^{r}(\eta_iK_i)}}{\Gamma\left(2\lambda+2\nu+r+2\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+2\sum_{i=1}^{r}(\mu_i+\eta_iK_i)\right)}$$

$$F_{2:0;\ ;0}^{0:1;\dots;1}\left[\begin{matrix}\text{---}\\ \lambda+\nu+\sum_{j=1}^{s}(\rho_j+\sigma)k_j+\sum_{i=1}^{r}\mu_i+\eta_iK_i)+\frac{r}{2}\end{matrix}\right.$$

$$\begin{matrix}\text{---} & :\nu+\sum_{j=1}^{s}\rho_jk_j:\mu_1+\eta_1K_1+;\dots;\\ \lambda+\nu+\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+\sum_{i=1}^{r}(\mu_i+\eta_iK_i)+\frac{r+1}{2}; & \text{--}\quad:\quad\text{--}\end{matrix}$$

$$\left.\begin{matrix}\mu_r+\eta_rK_r+\frac{1}{2}; \\ \text{--}\quad;\end{matrix}\ \frac{-a^2x^2}{4},\frac{-b_1^2x^2}{4},\ ,\frac{-b_r^2x^2}{4}\right]. \qquad \dots(3.1)$$

provided that

(i) $Re(p)>0$, $Re(\lambda+\nu)>0$, $Re(2\mu+1)>0$. ρ_j, σ_j, $\geqslant 0$, and

$\eta_i, \mu_i > 0 \; (i=1,\ldots,r,\; j=1,\ldots,s)$

(ii) $m_1,\ldots,m_s, M_1, \ldots, M_r$ are arbitrary positive integers, the coefficients $A(n;k_1,\ldots,k_s)$, $B_{N_1, K_1},\ldots,B_{N_r, K_r}$ $(n_j, k_j, N_j, K_i \geqslant 0,$ $i=1,\ldots,r\;;\; j=1,\ldots,s)$ are arbitrary constants, real or complex.

$$L^{-1}\Big\{(p^2+a^2)^{1/2}\{p+(p^2+a^2)^{1/2}\}^{-\lambda}$$

$$.S_n^{m_1,\ldots,m_s}\Big[y_1\{p+(p^2+a^2)^{1/2}\}^{-\rho_1},\ldots,y_s\{p+(p^2+a^2)^{1/2}\}^{-\rho_s}\Big]$$

$$\sum_{i=1}^{r}\Big\{\{p+(p^2+b_i^2)^{1/2}\}^{-\nu_i}\;.S_{N_i}^{m_i}\Big[z_i\{p+(p^2+b_i^2)^{1/2}\}^{-\sigma_i}\Big]\Big\};\;x\Big\}$$

$$=\left(\frac{x}{2}\right)^{\lambda+\sum_{i=1}^{r}\nu_i}\sum_{k_1,\ldots,k_s=0}^{m_1k_1+\ldots+m_sk_s\leqslant n}\;\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_r=0}^{[N_r/M_r]}$$

$$.\,(-n)_{m_1k_1+\ldots+m_sk_s}A(n;k_1,\ldots,k_s)\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_s^{k_s}}{k_s!}\sum_{i=1}^{r}\Big\{(-N_i)_{M_iK_i}B_{N_i,K_i}\frac{z_i^{K_i}}{K_i!}\Big\}$$

$$.\,\frac{\left(\frac{x}{2}\right)^{\sum_{j=1}^{s}\rho_j+k_j+\sum_{i=0}^{r}\sigma_iK_i}}{\Gamma(1+\lambda+\sum_{j=1}^{s}\rho_jk_j+\sum_{i=1}^{r}(\nu_i+\sigma_iK_i))}\;F^{\,0:2;\ldots;2}_{\,2:1;\ldots;1}\Bigg[\frac{\text{———}}{\tfrac{1}{2}(\lambda+2+\sum_{j=1}^{s}\rho_jk_j+\sum_{i=1}^{r}(\nu_i+\sigma_iK_i)),\;\tfrac{1}{2}(\lambda+1+\sum_{j=1}^{s}\rho_jk_j+\sum_{i=1}^{r}(\nu_i+\sigma_iK_i))}\;:$$

$$\tfrac{1}{2}(\lambda+2+\sum_{j=1}^{s}\rho_jk_j),\;\tfrac{1}{2}(\lambda+1+\sum_{j=1}^{s}\rho_jk_j):\;\tfrac{1}{2}(\nu_1+\sigma_1K_1),\;\tfrac{1}{2}(\nu_1+\sigma_1K_1+1);\ldots;$$

$$\lambda+1+\sum_{j=1}^{s}\rho_jk_j\qquad:\;\nu_1+\sigma_1k_1+1\qquad;\ldots;$$

$$\left.\begin{matrix}\frac{1}{2}(\nu_r+\sigma_r K_r), \frac{1}{2}(\nu_r+\sigma_r K_r+1) ; \\ \nu_r+\sigma_r K_r+1 \quad ;\end{matrix} \quad \frac{-a^2x^2}{4}, \frac{-b_1^2x^2}{4}, \ldots, \frac{-b_r^2x^2}{4}\right]. \quad \ldots(3.2)$$

provided that

(i) $Re(p) > 0, \lambda, \rho_j, \nu_i, \sigma_i \geqslant 0, i=1, \ldots, r; j, \ldots, s$

(ii) $m_1, \ldots, m_s, M_1, \ldots, M_r$ are arbitrary positive integers, $A(n; k_1, \ldots, k_s), B_{N_1, \ldots, i}$ $(n, k_1, \ldots, k_s, N_i K_i \geqslant 0, i, \ldots, r)$ are arbitrary constants, real or complex.

$$L^{-1}\left\{(p)^{\sum_{i=1}^{r}(m_i t_i)-\nu}(q)^{\sum_{i=1}^{r}(n_i s_i)-\mu}\sum_{i=1}^{r}\left\{(p^{m_i}q^{n_i}+a)^{s_i+t_i}\right\} S_N^M\left[z\, p^{-\rho} q^{-\sigma}\right]; x, y\right\}$$

$$= (x)^{\sum_{i=1}^{r}(m_i t_i)+\nu-1}(x)^{\sum_{i=1}^{r}(n_i s_i)+\mu-1}\sum_{K=0}^{[N/M]}(-N)_{MK}\, B_{N,K}\,\frac{z^K}{K!}\,.$$

$$F^{0:1;\ldots;1}_{2:0;\ldots;0}\left[\begin{matrix} \text{———} \\ \left(\sum_{i=1}^{r}(m_i s_i)+\nu+\rho K; m_1, \ldots, m_r\right), \sum_{i=1}^{r}(n_i t_i)+\mu+\sigma K; n_1, \ldots, n_r\right)\end{matrix}:\right.$$

$$\left.\begin{matrix}(s_1+t_1, 1); \ldots; (s_r+t_r, 1) ; \\ \text{——} \quad ; \ldots ; \quad \text{——}\end{matrix} \quad -ax^{m_1}y^{n_1}, \ldots, ax^{m_r}y^{n_r}\right]. \quad \ldots(3.3)$$

where $S[x_1, \ldots, x_r]$ stands for the generalized Lauricella function.

The conditions of validity of (3.3) are

(i) $\nu, \mu, \rho, \sigma > 0$, m_i, n_i, s_i, t_i are non-negative integers $(i = 1, \ldots, r)$ and $Re(p), Re(q), Re(a) > 0$.

(ii) M is an arbitrary positive integer and the coefficients $B_{N,K}$ $(N, K \geqslant 0)$ are arbitrary constants, real or complex.

Proof of (3.1). On using the convolution theorem for the transform pair (1.1) and (2.1), and evaluating the integral so obtained, we find that

$$L^{-1}\Big\{ p^{-2\lambda}(p^2+a^2)^{-\nu}(p^2+b_1^2)^{-(2\mu_1+1)/2}(p^2+b_1^2)^{-(2\mu_2+1)/2}$$

$$\cdot S^{M_1}_{N_1}\left[z_1(p^2+b_1^2)^{-\eta_1}\right] S^{M_2}_{N_2}\left[z_2(p^2+b_2^2)^{-\eta_2}\right]$$

$$\cdot S^{m_1,\ldots,m_s}_{n}\left[y_1 p^{-2\rho_1}(p^2+a^2)^{-\sigma_1},\ldots,y_s p^{-2\rho_s}(p^2+a^2)^{-\sigma_s}\right];x\Big\}$$

$$= x^{2\lambda+2\nu+2\mu_1+2\mu_2+1}\sum_{K_1=0}^{[N_1/M_1]}\sum_{K_2=0}^{[N_2/M_2]}\sum_{k_1,\ldots,k_s=0}^{m_1k_1+\ldots+m_sk_s\leqslant n}$$

$$(-n)_{m_1k_1+\ldots+m_sk_s}$$

$$\cdot(-n_1)_{M_1K_1}(-N_2)_{M_2K_2}A(n;k_1,\ldots,k_s)\,B_{N_1,K_1}\,,B_{N_2,K_2}$$

$$\frac{y_1^{k_1}\ldots y_s^{k_s}}{k!\ldots k_s!}\ \frac{z_1^{K_1}z_2^{K_2}}{K_1!\ K_2!}\,.$$

$$\frac{(x)^{\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+2\eta_1K_1+2\eta_2K_2}}{\Gamma(2\lambda+2\nu+2\mu_1+2\mu_2+2\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+2\eta_1K_1+2\eta_2K_2+2)}$$

$$\cdot F^{0:1;1;1}_{2:0;0;0}\left[\begin{matrix}\text{———} \\ \lambda+\nu+\mu_1+\mu_2+\sum_{j=1}^{s}(\rho_j+\sigma_j)k_1+\eta_1K_1+\eta_2K_2+1,\end{matrix}\right.$$

$$\text{———}\quad :\nu+\sum_{j=1}^{s}\sigma_jk_j\ ;$$

$$\lambda+\nu+\mu_1+\mu_2+\sum_{j=1}^{s}(\rho_j+\sigma_j)k_j+\eta_1K_1+\eta_2K_2+3/2\ :\quad\text{———}\quad ;$$

$$\begin{bmatrix} \mu_1+\eta_1 K_1+\tfrac{1}{2}\ ;\ \mu_2+\eta_2 K_2+\tfrac{1}{2}\ ; & \\ \text{——} & ;\ \dfrac{-a^2x^2}{4},\ \dfrac{-b_1{}^2x^2}{4},\ \dfrac{-b_2{}^2x^2}{4} \end{bmatrix} \quad \ldots(3.4)$$

Now repeating the above process r times, we finally arrive at the required result (3.1)

Proof of (3.2). The proof of (3.2) is similar to that of ths result (3.1). Here we use the (1.3) aed (2.2), instead of (1.1) and (2.1).

Proof of (3.3). To prove (3.2), we use the convolution theorem of Laplace transforms for the pair (1.8) and (2.3), and evaluating the u, v-double integral so obtained, we get

$$L^{-1}\Big\{(p)^{m_1t_1+m_2t_2-\nu}\,(q)^{m_1s_1+n_2s_2-\mu}\,(p^{m_1}\,q^{n_1}+a)^{-s_1-t_1}$$

$$\cdot\,(p^{m_2}\,q^{n_2}+a)^{-s_2-t_2}\,S_N^M\big[z\,p^{-\rho}\,q^{-\sigma}\big]\,;\,x,\,y\Big\}$$

$$=(x)^{m_1s_1+m_2s_2+\nu-1}\,(y)^{n_1t_1+n_2t_2+\mu-1}\sum_{K=0}^{[N/M]}(-N)_{MK}\,A_{K,N}\,\frac{z^K}{K!}$$

$$\cdot\,F^{0:1;1}_{2:0;0}\begin{bmatrix} \text{————} & : \\ (m_1s_1+m_2s_2+\nu+\rho K;m_1,m_2),\ (n_1y\cdot+n_2t_2+\mu+\sigma K;n_1,n_2) & : \end{bmatrix}$$

$$\begin{bmatrix} (s_1+t_1,1)\ ;\ (s_2+t_2,1)\ ; & \\ \text{———}\ ;\ \text{———}\ ; & -ax^{m_1}y^{n_1},\ -ax^{m_2}y^{n_2} \end{bmatrix} \quad \ldots(3.5)$$

Now repeating the above process r times, we immediately get the required result (3.3).

4. SPECIAL CASES

The formulas established in § 2 and § 3 are quite general in character; these involve a product of general classes of polynomials of one and several variables, which can be reduced to a number of known polynomials as mentioned in the papers [4], [5] and [6].

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REFERENCES

[1] V. A. Ditkin and A. P. Prudnikov, *Operational Calculus in Two Variables and Its Applications* (Translated from the Russian by D. M. G. Wishart), International Series of Monographs on Pure and Applied Mathematics, Vol. 24, Pergamon Press, Oxford 1962.

[2] A. Erdelvi *et al.*, *Tables of Integral Transforms.* Vol. I, McGraw-Hill, New York, 1954.

[3] S. P. Goyal. On double integrals involving the H-functions, *Indian J. Math.* **19** (1977), 119-123.

[4] H. M. Srivastva, A contour integral involving Fox's H-function, *Indian J. Math.* **14** (1972), 1-6.

[5] H. M. Srivastava and M. Garg, Some integrals involving a general class of polynomials and the multivariable H-function, *Rev. Roumaine Phys.* 32 (1987), 685-692.

[6] H. M. Srivastava, K. C. Gupta and S.P. Goyal, *The H-Functions of One and Two Variables with Applications*, South Asian Publishers, New-Delhi and Madras, 1982,

[7] H. M. Srivastava and N. P. Singh, The integration of certain products of the multivariable H-function with a general class of polynomials, *Rend. Circ. Mat. Palermo* (2) 32 (1983), 157-187.