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(Dedicated to the memory of Professor K. L. Singh)

A FINITE DOUBLE INTEGRAL INVOLVING A GENERAL MULTIVARIABLE POLYNOMIAL AND THE MULTIVARIABLE H-FUNCTION

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ABSTRACT

In this paper we evaluate a general finite double integral involving the product of algebraic and exponential functions, a general multi-variable polynomial, and the H function of several variables. Some new and interesiting special cases of our main integral formula have been considered briefly.

1. INTRODUCTION

The general class of polynomials $S_n^m[x]$ introduced by Srivastva [3] has been further extended by Srivastava and Garg [4] to a multi-variable polynomial in the following manner [4]:

$$S_n^{M_1,\ldots,M_r}[x_1,\ldots,x_r]=\sum_{k_1,\ldots,k_r=0}^{M_1k_1+\ldots+M_rk_r\leqslant n}(-n)_{M_1k_1+\ldots+M_rk_r}\cdot A(n;k_1,\ldots,k_r)\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_r^{k_r}}{k_r!},\quad\ldots(1.1)$$

where $M_1,\ldots,M_r$ are arbitrary positive integers and the coefficients $A(n;k_1,\ldots,k_r)$ $(n,k_i\geqslant 0,\ i=1,\ldots,r)$ are arbitrary constants, real or complex.

The parameters of the multivariable H-function introduced and studied by Srivastava and Panda ([6], [7]) will be displayed in the following contracted notation [5, p. 251, Eq. (C.1)].

$$H[z_1,\ldots,z_r] = H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j;\alpha'_j,\ldots,\alpha^{(r)}_j)_{1,p}: \\ (b_j;\beta'_j,\ldots,\beta^{(r)}_j)_{1,q}: \end{matrix} \begin{matrix} (c'_j,\gamma'_j)_{1,p_1};\ldots;(c^{(r)}_j,\gamma^{(r)}_j)_{1,p_r} \\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d^{(r)}_j,\delta^{(r)}_j)_{1,q_r} \end{matrix}\right] \quad \ldots (1.2)$$

Here, all the Greek letters are assumed to positive numbers for standardization purposes; the definition of the multivariable H-function given by (1.2) will, however, be meaningful even if some of these quantities are zero. The details of this function can be found in the papers and book referred to above.

We shall require the following known integrals ([1, p 450] and [2, p. 255]) for the evaluation of our main integral

$$\int_0^{\pi/2} e^{\omega(\alpha+\beta)\theta} (\sin\theta)^{\alpha-1} (\cos\theta)^{\beta-1}\, d\theta = \frac{e^{(\pi/2)\omega\alpha}\,\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad \ldots (1.3)$$

$(Re(\alpha) > 0,\ Re(\beta) > 0,\ \omega = \sqrt{-1}$,

$$\int_0^1 x^{c-1}(1-x)^{d-1}[ax+b(1-x)]^{-c-d} \exp\{-zax/[ax+b(1-x)]\} \cdot {}_2F_1\left[\alpha,\beta;c;\frac{ax}{ax+b(1-x)}\right] dx$$

$$= \frac{epx\{-z\}\,\Gamma(c)\,\Gamma(d)\,\Gamma(c+d-\alpha-\beta)}{a^c\, b^d\, \Gamma(c+d-\alpha)\,\Gamma(c+d-\beta)}\ {}_2F_2\,[d, c+d-\alpha-\beta\ ;\ c+d-\alpha,\ c+d-\beta\ ;\ z] \quad \ldots (1.4)$$

$Re(c) > 0$, $Re(d) > 0$; a and b are non-zero integers such that $ax + b(1-x) \neq 0,\ 0 \leqslant x \leqslant 1)$.

2. MAIN INTEGRAL

We shall evaluate the following general integral :

$$\int_0^1 \int_0^1 x^{c-1} y^{\rho-1} (1-x)^{d-1} (1-y)^{\sigma-1} \left\{(1-y^2)^{1/2}+\omega y\right\}^{\rho+2\sigma} [ax+b(1-x)]^{-c-d}$$

$$. \exp\left\{-zax/[ax+b(1-x)]\right\} {}_2F_1[\alpha,\beta;c; \frac{ax}{ax+b(1-x)}]$$

$$. S_n^{M_1,\ldots M_r}\left[t_1 y^{u_1}(1-y^2)^{v_1}\left\{(1-y^2)^{1/2}+\omega y\right\}^{u_1+2v_1}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\delta_1},\ldots,\right.$$

$$\left. t_r y^{u_r}(1-y^2)^{v_r}\left\{(1-y^2)^{1/2}+\omega y\right\}^{u_1+2v_r}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\delta_r}\right]$$

$$. H\left[z_1 y^{\rho_1}(1-y^2)^{\sigma_1}\left\{(1-y^2)^{1/2}+\omega y\right\}^{\rho_1+2\sigma_1}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_1},\ldots,\right.$$

$$\left. z_s y^{\rho_s}(1-y^2)^{\sigma_s}\left\{(1-y^2)^{1/2}+\omega y\right\}^{\rho_s+2\sigma_s}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_s}\right]dx\,dy$$

$$= \Gamma(c) \exp\left\{-z+\pi\omega\rho/2\right\} a^{-c} b^{-d} \sum_{p=0}^{\infty} \sum_{k_1,\ldots,k_r=0}^{M_1k_1+\ldots+M_rk_r \leqslant n}$$

$$. (-n)_{M_1k_1+\ldots+M_rk_r} A(n;k_1,\ldots,k_r) \frac{z^p}{p!} \prod_{i=1}^{r}\left\{\frac{t_i^{k_i}(b)^{-\delta_i k_i}\exp\{\omega u_i k_i \pi/2\}}{k_i!}\right\}$$

$$. H^{0,\ N+4:m_1,n_1;\ldots;m_s,n_s}_{P+4,\ Q+3:p_1,q_1;\ldots;p_s,q_s}\left\{\begin{matrix} z_1 & b^{-\theta_1}\exp(\omega\pi\rho_1/2) \\ \vdots & \\ z_s & b^{-\theta_s}\exp(\omega\pi\rho_s/2)\end{matrix}\right|$$

$$(1-p-d-\sum_{i=1}^{r}\delta_i k_i;\theta_1,\ldots,\theta_s),\ (1-p-d-c+\alpha+\beta-\sum_{i=1}^{r}\delta_i k_i;\theta_1,\ldots,\theta_s)$$

$$(b_j;\beta_j',\ldots,\beta_j^{(s)})_{1,Q},\ (1-c-d-p+\alpha-\sum_{i=1}^{r}\delta_i k_i;\theta_1,\ldots,\theta_s),$$

$$(1-\rho-\sum_{i=1}^{r}u_i k_i;\rho_1,\ldots,\rho_s),\ (1-2\sigma-2\sum_{i=1}^{r}v_i k_i;2\sigma_1,\ldots,2\sigma_s),$$

$$(1-c-d-p+\beta-\sum_{i=1}^{r}\delta_i k_i;\theta_1,\ldots,\theta_s),$$

$$(a_j;\alpha'_j,\ldots,\alpha^{(s)}_j)_{1,p} \qquad : (c'_j,\gamma'_j)_{1,p_1};\ldots;$$

$$(1-\rho-2\sigma-\sum_{i=1}^{r}(u_i+2v_i)k_i;\rho_1+2\sigma_1,\ldots,\rho_s+2\sigma_s): (d'_j,\delta'_j)_{1,q_1};\ldots;$$

$$\left.\begin{matrix}(c^{(s)}_j,\gamma^{(s)}_j)_{1,p_s}\\ (d^{(s)}_j,\delta^{(s)}_j)_{1,q_s}\end{matrix}\right] \qquad \ldots (2.1)$$

The integral (2.1) is valid if the following sets of (sufficient) conditions are satisfied.

(i) $M_1,\ldots,M_r$ are arbitrary positive integers and the coefficients $A(n;k_1,\ldots,k_r)$ $(n;k_1,\ldots,k_r \geqslant 0)$ are arbitrary constants, real or complex.

(ii) a and b are non-zero integers such that $ax+b(1-x)\neq 0, 0\leqslant x\leqslant 1$.

(iii) $\min\limits_{1\leqslant i\leqslant r} \{Re(u_i), Re(v_i), Re(\delta_i)\} > 0$,

$\min\limits_{1\leqslant j\leqslant s} \{Re(\rho_j), Re(\sigma_j), Re(\theta_j)\} \geqslant 0$.

(iv) $Re(c) > 0$, $Re(d) + \sum_{i=1}^{s} \theta_i\xi_i > 0$, and

$Re(\rho) + \sum_{i=1}^{s} \rho_i\xi_i > 0$, $Re(\sigma) + \sum_{i=1}^{s} \sigma_i\xi_i > 0$,

where $\xi_i = \min\limits_{1\leqslant j\leqslant m_i} \left\{Re(d^{(i)}_j/\delta^{(i)}_j)\right\}$, $i=1,\ldots,s$

(v) $U_i > 0$, $|\arg z_i| < (1/2)\, U_i\pi$ $(i=1,\ldots,s)$.

where

$$U_i = -\sum_{j=N+1}^{P}\alpha^{(i)}_j - \sum_{j=1}^{Q}\beta^{(i)}_j + \sum_{j=1}^{n_i}\gamma^{(i)}_j - \sum_{j=n_i+1}^{p_i}\gamma^{(i)}_j + \sum_{j=1}^{m_i}\delta^{(i)}_j - \sum_{j=m_i+1}^{q_i}\delta^{(i)}_j$$

(vi) The series on the right-hand side of (2.1) is absolutely convergent.

Proof. To establish the integral formula (2.1), we first use the series representation of the multivariable polynomial given by (1.1), and express the multivariable H-function in terms of its Mellin-Barnes contour integral. Then interchanging the order of integration and summation (which is justified due to absolute convergence of the various integrals involved in the proces under the conditions stated with result) and evaluating the x- and y-integrals with the help of (1.3) (1.4), respectively, we find that

$$L.\ H.\ S.\ \text{of (2.1)} = \sum_{k_k,\ldots,k_r=0}^{M_1k_1+\ldots+M_rk_r\leqslant n} (-n)_{M_1k_1+\ldots+M_rk_r}$$

$$\cdot\ A\,(n;k_1,\ldots,k_r)\prod_{i=1}^{r}\left\{\frac{t_i^{k_i}}{k_i!}\right\}\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi\,(\xi_1,\ldots,\xi_s)$$

$$\prod_{j=1}^{s}\left\{\theta_j\,(\xi_j)\,z_j^{\xi_j}\right\}\left(\frac{exp\{-z\}\ \Gamma\,c)\,\Gamma(d+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\xi_j)}{(a)^C\,(b)_{d+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\xi_j}}\right.$$

$$\frac{\Gamma\,(c+d\ \alpha-\beta+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j)}{\Gamma\,(c+d-\alpha+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j)\,\Gamma\,(c+d-\beta+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j,\,\xi_j)}$$

$$\cdot\ {}_2F_2\left[d+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j,\ c+d\ \alpha-\beta+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j\ ;\right.$$

$$\left.\left.c+d-a+\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j,\ c+d-\beta-\sum_{i=1}^{r}\delta_ik_i+\sum_{j=1}^{s}\theta_j\,\xi_j\ ;\ z\right]\right)$$

$$\cdot\left(exp\left\{\rho+\sum_{i=1}^{r}u_ik_i+\sum_{j=1}^{s}\rho_j\xi_j\right\}\Gamma(\rho+\sum_{i=1}^{r}u_ik_i+\sum_{j=1}^{s}\rho_j\xi_j).\right.$$

$$\cdot \frac{\Gamma(2\sigma + 2 \sum_{i=1}^{r} v_i k_i + 2 \sum_{j=1}^{s} \sigma_j \xi_j)}{\Gamma(\rho + 2\sigma + \sum_{i=1}^{r} (u_i + 2v_i)k_i + \sum_{j=1}^{s} (\rho_j + 2\sigma_j) \xi_j)} \Bigg) d\xi_1, \ldots, d\xi_s. \quad \ldots(2.2)$$

On expressing ${}_2F_2[x]$ as a hypergeometric series and changing the oteder of integration and summation, if we interpret the resulting contour integral in terms of the multivariable H-function, we get at once the integral (2.1).

3. SPECIAL GASES

I. Taking $u_i = v_i = 0$ $(i=1,\ldots,r)$, $\rho_j = \sigma_j = 0$ $(j=1,\ldots,s)$ in (2.1), and evaluating the y-integral, we arrive at the following integral which is believed to be new.

$$\int_0^1 x^{\sigma-1} (1-x)^{d-1} [ax+b(1-x)]^{-\sigma-d} \exp\left\{-zax/[ax+b(1-x)]\right\}$$

$$\cdot\ {}_2F_1\left[\alpha,\beta,c;\frac{ax}{ax+b(1-x)}\right] S_n^{M_1,\ldots,M_r}\left[t_1\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\delta_1},\ldots,\right.$$

$$\left. t_r\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\delta_r}\right] H\left[z_1\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_1},\ldots,z_s\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_s}\right] dx$$

$$= \Gamma(c)\, e^{-z} a^{-\sigma} b^{-d} \sum_{p=0}^{\infty} \sum_{k_1,\ldots,k_r=0}^{M_1k_1+\ldots+M_rk_r \leqslant n} (-n)_{M_1k_1+\ldots+M_rk_r}$$

$$\cdot\ A(n;k_1,\ldots k_r) \frac{z^p}{p!} \prod_{i=1}^{r} \left\{\frac{t_i^{k_i} (b)^{\delta_i k_i}}{k_i!}\right\}$$

$$\cdot\ H^{0,N+2:m_1,n_1;\ldots;m_s,n_s}_{P+2,Q+2:p_1,q_1;\ldots;p_s,q_s}\left[\begin{matrix} z_1 b^{-\theta_1} \\ \vdots \\ z_s b^{-\theta_s}\end{matrix}\ \middle|\ \begin{matrix}(1-p-d-\sum_{i=1}^{r}\delta_i k_i;\theta_1,\ldots,\theta_s), \\ (b_j;\beta_j',\ldots,\beta_j^{(s)})_{1,Q}\end{matrix}\right],$$

$$(1-p-c-d+\alpha+\beta-\sum_{i=1}^{r}\delta_i k_i;\ \theta_1,\ldots,\theta_s),\ (a_j;a'_j,\ldots,a_j^{(s)})_{1,p} \quad :$$

$$(1-p-c-d+\alpha-\sum_{i=1}^{r}\delta_i k_i;\ \theta_1,\ldots,\theta_s),(1-p-c-d+\beta\ \sum_{i=1}^{r}\delta_i k_i;\ \theta_1,\ldots,\theta_s) :$$

$$\left.\begin{matrix}(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(s)},\gamma_j^{(s)})_{1,p_s}\\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(s)},\delta_j^{(s)})_{1,q}\end{matrix}\right], \qquad \ldots(3.1)$$

provided that

(i) $Re(\delta_i) \geqslant 0\ (i=1,\ldots,r),\ Re(\theta_j) \geqslant 0,\ j=1,\ldots,s;$

(ii) $Re(c) > 0,\ Re(d)+\sum_{j=1}^{s}\theta_j\xi_j > 0,$

where $\xi_j = \min_{j\leqslant i\leqslant m_j}\left\{Re(d_i^{(j)}/\delta_i^{(j)})\right\},\ j=1,\ldots,s;$

(iii) conditions (i), (ii), (v) and (vi) stated with (2.1) are also satisfied.

II. Similarly, if we put $\delta_i = 0\ (i=1,\ldots,r)$ and $\theta_j = (\ j=1,\ldots,s)$ in (2.1), we get the following result:

$$\int_0^1 y^{\rho-1}(1-y^2)^{\sigma-1}((1-y^2)^{1/2}+\omega y)^{\rho+2\sigma}\, S_n^{M_1,\ldots M_r}\Big[t_1\, y^{u_1}(1-y^2)^{v_1}$$

$$\{(1-y^2)^{1/2}+\omega y\}^{u_1+2v_1},\ldots,t_r\, y^{u_r}(1-y^2)^{v_r}\{(1-y^2)^{1/2}+\omega y\}^{u_r+2v_r}\Big]$$

$$.\ H\Big[z_1\, y^{\rho_1}(1-y^2)^{\sigma_1}\{(1-y^2)+\omega y\}^{\rho_1+2\sigma_1},\ldots,z_s\, y^{\rho_s}(1\ y^2)^{\sigma_s}$$

$$.\ \{(1-y^2)^{1/2}+\omega y\}^{\rho_s+2\sigma_s}\Big]\,dy$$

$$= exp\ \{\omega\rho\pi/2\}\sum_{k_1,\ldots,k_r=0}^{M_1k_1+\ldots+M_rk_r\leqslant n}(-n)_{M_1k_1+\ldots+M_rk_r}$$

$$.\ A(n;k_1,\ldots,k_r)\prod_{i=1}^{r}\left\{\frac{t_i^{k_i}\,exp\ \{\omega u_i k_i\pi/2\}}{k!}\right\}$$

$$\cdot\ H^{0\ \ ,N+2:\ m_1,n_1;\ldots;m_s,n_s}_{P+2,Q+1:\ p_1,q_1\ ;\ldots;p_s,\ p_s}\left[\begin{matrix} z_1\ exp\ \{\omega\ \rho_1\pi/2\} \\ \vdots \\ z_s\ exp\ \{\omega\ \rho_s\pi/2\}\end{matrix}\right|$$

$$(1-\rho-\sum_{i=1}^{r} u_i k_i;\rho_1,\ldots,\rho_s),\ (1-2\sigma-2\sum_{i=1}^{r} v_i k_i;\ 2\sigma_1,\ldots,2\sigma_s),$$

$$(b_j;\beta'_j,\ \ldots\ ,\beta^{(s)}_j)_{1,Q}\ ,\ (1-\rho-2\sigma-\sum_{i=1}^{r}(u_i+2v_i)k_i;\ \rho_1+2\sigma_1,\ldots,\rho_s+2\sigma_s),$$

$$\left.\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha^{(s)}_j)_{1,P} & :(c'_j,\gamma'_j)_{1,\ p_1} ;\ldots;(c^{(s)}_j,\gamma^{(s)}_j)_{1,\ p_s} \\ & :(d',\delta'_j)_{1,q_1}\ ;\ldots;(d^{(s)}_j,\delta^{(s)}_j)_{1,\ p_s}\end{matrix}\right] \quad \ldots (3.2)$$

The equation (3.2) is valid if

(i) $min\ \{Re(u_i),\ Re(v_i)\} \geqslant 0\ (i=1,\ldots,r);$
$min\ \{Re(\rho_j),\ Re(\sigma_j)\} \geqslant 0\ (j=1,\ldots,s);$

(ii) $Re(\rho)+\sum_{i=1}^{s}\rho_i\,\xi_i > 0$ and $Re(\sigma)+\sum_{i=1}^{s}\sigma_i\,\xi_i > 0$,

where $\xi_i = \min_{1\leqslant j\leqslant m_i}\left\{Re\ d^{(i)}_j/\delta^{(i)}_j\ )\right\}\ (i=1,\ldots,s).$

(iii) Conditions (i), (v) and (vi) stated with (2.1) are also satisfied.

III. If we take $M_i = 0\ (i = 2,\ldots r)$, $t_i = 1$, and replace $A(n;k_1,\ldots,k_r)$ by $A\ n;k)$ therein, we easily arrive at the following general integral:

$$\int_0^1\int_0^1 x^{\rho-1}\,y^{\rho-1}\,(1-x)^{d-1}\,(1-y^2)^{\sigma-1}\,\{(1-y^2)^{1/2}+\omega y\}^{\rho+2\sigma}$$

$$[ax+b(1\cdot x]^{-\rho-d}\ exp\ \{\ zax/\ [ax+b(1-x)]\ \}\ {}_2F_1\left[\alpha\ \beta;c;\frac{ax}{ax+b(1-x)}\right]$$

$$\cdot\ S^M_N\left[t\,y^u\,(1-y^2)^v\,\{(1-y^2)^{1/2}+\omega y\}^{u+2v}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\delta}\right]$$

$$\cdot\ H\Big[\ z_1\ y^{\rho_1}\,(1-y^2)^{\sigma_1}\left\{(1-y^2)^{1/2}+\omega y\right\}^{\rho_1+2\sigma_1}\left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_1},\ldots,$$

$$z_s y^{\rho_s} (1-y^2)^{\sigma_s} \{(1-y^2)^{1/2}+\omega y\}^{\rho_s+2\sigma_s} \left\{\frac{1-x}{ax+b(1-x)}\right\}^{\theta_s}\Bigg] dx\, dy$$

$$= \Gamma(c)\ exp\ \{-z+\omega(\pi/2)\rho\}\ a^{-c}\ b^{-d}$$

$$\cdot \sum_{p=0}^{\infty} \sum_{k=0}^{[n/m]} (-n)_{mk}\ A(n;k)\ \frac{t^k}{k!}\ b^{\delta k}\ exp\ \{uk\rho\ \pi/2\}\ \frac{z^p}{p!}$$

$$\cdot\ H^{0,\ N+4:m_1,n_1;\ldots;m_s,n_s}_{P+4,Q+3:p_1,q_1;\ldots;p_s,q_s}\left[\begin{matrix} z_1 b^{\theta_1}\ exp\ \{\omega\rho_1\pi/2\} \\ \vdots \\ z_s b^{\theta_s}\ exp\ \{\omega\rho_s\pi/2\}\end{matrix}\right|$$

$$(1-p-d-\delta k;\ \theta_1,\ldots,\theta_s),\ (1-p-c-d+\alpha+\beta-\delta k;\ \theta_1,\ldots,\theta_s),$$

$$(b_j;\beta'_j,\ldots,\beta^{(s)}_j)_{1,\theta},\ (1-c-d-p+\alpha-\delta k;\ \theta_1,\ldots,\theta_s),$$

$$(1-\rho-uk;\ \rho_1,\ldots,\rho_s),\ (1-2\sigma-2vk;\ 2\sigma_1,\ldots,2\sigma_s),$$
$$(1-c-d-p+\beta-\delta k;\ \theta_1,\ldots,\theta_s),$$

$$(a_j;\alpha'_j,\ldots,\alpha^{(s)}_j)_{1,P} \qquad :(c'_j,\gamma'_j)_{1,p_1};\ldots;$$
$$1-\rho-2\sigma-(u+2v)\,k;\rho_1+2\sigma_1,\ldots,\rho_s+2\sigma_s) \ :(d'_j,\delta'_j)_{1,q_1};\ldots;$$

$$\left.\begin{matrix}(c^{(s)}_j,\gamma^{(s)}_j)_{1,p_s} \\ (d^{(s)}_j,\delta^{(s)}_j)_{1,q_s}\end{matrix}\right] \qquad \ldots (3.3)$$

provided that the conditions easily obtainable from those mentioned with the main integral (2.1) are satisfied.

The integrals (3.1), (3.2) and (3.3) are also quite general in nature. By suitably specializing the arbitrary coefficients in the general class of polynomials and the parameters of the multivariable H-function, a number of (known and new) integrals can be evaluated.

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