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A NOTE ON SOME GENERATING FUNCTIONS

By

R. S. PAL

Reader in Department of Mathematics, Narain College,
Shikohabad, U. P.

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1. INTRODUCTION.

Recently, Dhawan [1] has proved the following three results :

(1.1) $$F_1\,[a, b, b; c; \{x-(x^2-1)^{1/2}\}t, \{x+(x^2-1)^{1/2}\}\,t\,] = \sum_{n=o}^{\infty} \frac{(a)_n\ C_n^{(b)}(x)}{(c)_n}\, t^n \quad |t|<1,$$

(1.2) $$\psi_2\,[1+a; 1+b, 1+a; \left(\frac{x-1}{2}\right)t, \left(\frac{x+1}{2}\right)t\,] = \sum_{n=o}^{\infty} \frac{P_n^{(a,b)}(x)}{(1+b)_n}\, t^n \quad |t|<1,$$

(1.3) $$\Xi_2\,[a,a;b;\{x-(x^2-1)^{1/2}\}t,\{x+(x^2-1)^{1/2}\}t] = \sum_{n=o}^{\infty} \frac{C_n^{(a)}(x)}{(b)_n}\, t^n \quad |t|<1,$$

and their particular cases have also been discussed by him.

Now in this paper, we shall establish some generating functions for Bessel polynomials [3] and Srivastava's polynomials [5] in terms of different hypergeometric functions of two variables.

2. GENERATING FUNCTIONS.

The confluent hypergeometric function ϕ_1 of two variables is defined as [2, p. 225]

$$\phi_1\,(a, b, c, x, y) = \sum_{m,n=o}^{\infty} \frac{(a)_{m+n}\,(b)_n\, x^m\, y^n}{(c)_{m+n}\, m\,!\, n\,!} \qquad |x|<1.$$

Hence we ave

$$\phi_1 (a+2, a+1, a+1, xt, -t) = \sum_{m,n=o}^{\infty} \frac{(a+2)_{m+n} (a+1)_n (xt)^m (-t)^n}{(a+1)_{m+n} \quad m! \quad n!}$$

$$= \sum_{n=o}^{\infty} \sum_{m=o}^{\infty} \frac{(a+2)_n (a+1)_n (-n)_m (-1)^{n-m} x^m t^n}{(a+1)_n \ (-a-n)_m \qquad m! \ n!}$$

$$= \sum_{n=o}^{\infty} \left(\frac{a+n+1}{a+1} \right) \left(1 - \frac{(a+n+1)}{n!} \right)^n$$

$${}_1F_1 \left[\begin{matrix} -n ; \\ 1-(a+n+1) ; \end{matrix} \quad -x \right] t^n$$

$$= \sum_{n=o}^{\infty} \left(\frac{a+n+1}{a+1} \right) L_n^{-(a+n+1)} (-x) \, t^n .$$

Now using the following relation due to Srivastava [5] :

$$L_n^{-(a+n+1)}(-x) = A_n^{(a)}(x) ,$$

we obtain

$$(2.1) \quad \phi_1 (a+2, a+1, a+1, xt, -t) = \sum_{n=o}^{\infty} \left(\frac{a+n+1}{a+1} \right) A_n^{(a)} (x) \quad t^n$$

$$= \sum_{n=o}^{\infty} A_n^{(a)} (x) \ t^n$$

$$+ \sum_{n=o}^{\infty} \left(\frac{n}{a+1} \right) A_n^{(a)} (x) \quad t^n \qquad |t| < 1,$$

which appears to be new.

Using the result due to Srivastava [5]

$$(1+t)^{-a-1}\, e^{xt} = \sum_{n=o}^{\infty} A_n^{(a)}(x)\; t^n ,$$

we derive from the above result

(2.2) $\left(a- [illegible] (a+2,\, a-1,\, a+1,\, xt, -t) - (1+t)^{-a-1}\, e^{xt}\right]$

$$= \sum_{n=o}^{\infty} n\, A_n^{(a)}(x)\; t^n \qquad |t| < 1.$$

Again the confluent hypergeometric function ϕ_3 of two variables [2, p. [illegible]] is defined as

$$\phi_3\,(b, c, x, y) = \sum_{m,n=o}^{\infty} \frac{(b)_m\, x^m\, y^n}{(c)_{m+n}\, m!\, n!} .$$

Hence we can write

$$\phi_3\left(a-1, c, \frac{xt}{b}, t\right) = \sum_{m,n=o}^{\infty} \frac{(a-1)_m \left(\frac{xt}{b}\right)^m t^n}{(c)_{m+n}\, m!\, n!}$$

$$= \sum_{n=o}^{\infty} \sum_{m=o}^{n} \frac{(a-1)_m\, (-n)_m \left(\frac{-x}{b}\right)^m t^n}{(c)_n\, m!\, n!}$$

$$= \sum_{n=o}^{\infty} \frac{1}{(c)_n\, n!}\; {}_2F_0\left[-n,\, a\text{-}1\text{-}n+n;\, -;\, -\frac{x}{b}\right] t^n .$$

Therefore

(2.3) $$\phi_3\left(a-1, c, \frac{xt}{b}, t\right) = \sum_{n=o}^{\infty} \frac{1}{(c)_n\, n!}\, y_n\,(x, a-n, b)\; t^n \qquad |t| < 1,$$

where y_n (x, a, b) are generalized Bessel polynomials [3].

Considering the following confluent hypergeometric function H_6 of two variables [2, p. 225]

$$H_6\,(a, c, x, y) = \sum_{m,n=o}^{\infty} \frac{(a)_{2m+n}\, x^m\, y^n}{(c)_{m+n}\, m!\, n!} \qquad |x| < \tfrac{1}{4},$$

and applying the above techniques, we also obtain

$$(2.4)\ H_6\ (a-1, c, \frac{xt}{b}, t) = \sum_{n=o}^{\infty} \frac{(a-1)_n}{(c)_n\ n!}\ y_n\ (x, a, b)\ t^n \qquad |t| < 1.$$

Particularly, for a=b=2, we have

$$(2.5)\qquad H_6\ (1, c, \frac{xt}{2}, t) = \sum_{n=o}^{\infty} \frac{1}{(c)_n}\ y_n\ (x)\ t^n \qquad |t| < 1,$$

where y_n (x) are simple Bessel polynomials.

We also obtain

$$(2.6)\ \psi_1\ (a,b',b',c, \frac{-xt}{b}, t) = \sum_{n=o}^{\infty} \frac{(a)_n}{(c)_n\, n!}\ y_n\ (x, 2-c-2n, b))\ t^n \qquad |t| < 1,$$

where

$$\psi_1\ (a, b, c, c', x, y) = \sum_{m,n=o}^{\infty} \frac{(a)_{m+n}\ (b)_m\ x^m\ y^n}{(c)_m\ (c')_n\ m!\ n!} \qquad |x| < 1.$$

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