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EXPANSION FORMULAS FOR MEIJER'S G—FUNCTIONS OF TWO VARIABLES IN SERIES OF CIRCULAR FUNCTIONS

By

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In this paper the author has obtained three expansion formulae for generalized Meijer's G—functions of two variables in series of circular functions of two variables in series of circular functions using known Fourier series. Further these results have been employed to deduce some finite integrals and a recurrence relation involving generalized Meijer's G—functions in two variables.

1. INTRODUCTION

Generalized Meijer's G—function of two variables

Recently, Agarwal [(2), p. 537] has defined an extension of Meijer's G—function in two variables by means of a double Mellin-Barnes contour integral in the form

$$(1.1)\quad G\begin{bmatrix} x \\ y \end{bmatrix} \equiv G^{p,\,q,\,s,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{matrix} (a) \\ (c)\ ;\ (e) \\ (b) \\ (d)\ ;\ (f) \end{matrix} \\ y & \end{array}\right] =$$

$$= \frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} \phi\,[\xi+\eta]\;\psi\,[\xi,\eta]\; x^{\xi}\, y^{\eta}\, d\xi\, d\eta$$

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where (a) denotes the sequence of A parameters $a_1, a_2, \ldots\ldots, a_A$, i. e. there are A of the a parameters, B of the b parameters and so on,

$$\phi[\xi+\eta] = \frac{\prod_{j=1}^{p} \Gamma[1-a_j+\xi+\eta]}{\prod_{j=p+1}^{A} \Gamma[a_j-\xi-\eta] \prod_{j=1}^{B} \Gamma[b_j+\xi+\eta]},$$

$$\psi[\xi,\eta] = \frac{\prod_{j=1}^{q} \Gamma[c_j+\xi] \prod_{j=1}^{r} \Gamma[d_j-\xi] \prod_{j=1}^{s} \Gamma[e_j+\eta] \prod_{j=1}^{t} \Gamma[f_j-\eta]}{\prod_{j=q+1}^{C} \Gamma[1-c_j-\xi] \prod_{j=r+1}^{D} \Gamma[1-d_j+\xi] \prod_{j=s+1}^{E} \Gamma[1-e_j-\eta] \prod_{j=t+1}^{F} \Gamma[1-f_j+\eta]},$$

$o \leqslant p \leqslant A$, $o \leqslant q \leqslant C$, $o \leqslant r \leqslant D$, $o \leqslant s \leqslant E$, $o \leqslant t \leqslant F$.

The Meijer's G-function in two arguments that we discuss here is a slight var... o. he one defined by Agarwal [(2)]. We introduce the generalized Meijer's G-function of two variables as

$$(1.2) \quad G\begin{bmatrix} x \\ y \end{bmatrix} \equiv G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\;B,\,[D,\,F]}\left[\begin{matrix} x \\ y \end{matrix}\middle|\begin{matrix} (a) \\ (c)\;;\;(e) \\ (b) \\ (d)\;;\;(f) \end{matrix}\right] =$$

$$= \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \phi[\xi+\eta]\, \psi[\xi,\eta] \times^{\xi} y^{\eta}\, d\xi\, d\eta$$

where

$$\phi[\xi+\eta] = \frac{\prod_{j=1}^{p} \Gamma[1-a_j+\xi+\eta] \prod_{j=1}^{l} \Gamma[b_j-\xi-\eta]}{\prod_{j=p+1}^{A} \Gamma[a_j-\xi-\eta] \prod_{j=l+1}^{B} \Gamma[1-b_j+\xi+\eta]},$$

and $\psi[\xi,\eta]$ remains the same as given in (1.1)a nd $o \leqslant p \leqslant A$, $o \leqslant l \leqslant B$, $o \leqslant q \leqslant C$, $o \leqslant r \leqslant D$, $o \leqslant s \leqslant E$, $o \leqslant t \leqslant F$, the sequences of parameters $a_1, a_2, \ldots\ldots, a_p$; $b_1, b_2, \ldots\ldots, b_l$; $c_1, c_2, \ldots\ldots, c_q$; $d_1, d_2, \ldots\ldots, d_r$; $e_1, e_2, \ldots e_s$, and $f_1, f_2, \ldots\ldots, f_t$ are such that none of the poles of the integrand coincide, and the paths of integration are indented, if necessary, in such a manner that all the poles of $\Gamma[d_j-\xi]$, $j = 1, 2, \ldots\ldots, r$, $\Gamma[f_k-\eta]$, $k = 1, 2, \ldots\ldots, t$ and $\Gamma[b_j-\xi-\eta]$, $j = 1, 2, \ldots\ldots, l$, lie to the right and those of $\Gamma[c_j+\xi]$, $j = 1, 2, \ldots\ldots, q$, $\Gamma[e_k+\eta]$, $k = 1, 2, \ldots\ldots, s$ and $\Gamma[1-a_j+\xi+\eta]$, $j = 1, 2, \ldots\ldots, p$, lie to the left of the imaginary axis.

The integral (1.2) converges if

$$2(p+l+q+r) > [A+B+C+D]$$

$$2(p+l+s+t) > [A+B+E+F]$$

$$|\arg(x)| < [p+l+q+r-\tfrac{1}{2}(A+B+C+D)]\pi.$$

$$|\arg(y)| < [p+l+s+t-\tfrac{1}{2}(A+B+E+F)]\pi.$$

The following results are required in the present investigation.

Fourier series by Parashar [(11), p. 1084, (2.2)] and MacRobert [(9), p. 79; (10), p. 143]:

$$(1.3)\quad \sqrt{\pi}\,\frac{\Gamma(s+1)}{\Gamma(s+\frac{1}{2})}\,(\cos\tfrac{1}{2}\theta)^{2s} = 1 + 2\sum_{r=1}^{\infty}\frac{(-1)^r(-s)_r}{(s+1)_r}\cos r\theta,$$

$$\left\{0 \leqslant \theta \leqslant \pi \quad, \operatorname{Re}(s) > -\tfrac{1}{2}\right\},$$

$$(1.4)\quad \frac{\sqrt{\pi}}{2}\,\frac{\Gamma(2-s)}{\Gamma(3/2-s)}\,(\sin\theta)^{1-2s} = \sum_{r=0}^{\infty}\frac{(s)_r}{(2-s)_r}\sin(2r+1)\theta,$$

$$\left\{0 \leqslant \theta \leqslant \pi,\ \operatorname{Re}(s) \leqslant -\tfrac{1}{2}\right\},$$

$$(1.5)\quad \sqrt{\pi}\,\frac{\Gamma(1-s)}{\Gamma(\frac{1}{2}-s)}\,(\sin\tfrac{1}{2}\theta)^{-2s} = 1 + 2\sum_{r=1}^{\infty}\frac{(s)_r}{(1-s)_r}\cos r\theta,$$

$$\left\{0 \leqslant \theta \leqslant \pi,\ \operatorname{Re}(s) \leqslant \tfrac{1}{2}\right\},$$

Relations [(4), p.3 (3) and p.5, (14)]:

$$(1.6)\quad \frac{\Gamma(-z+n)}{\Gamma(-z)} = (-1)^n\,\frac{\Gamma(z+1)}{\Gamma(z-n+1)}, \text{ and}$$

$$(1.) \quad \sqrt{\pi}\ \Gamma(2z) = 2^{2z-1}\ \Gamma(z)\ \Gamma(z+\tfrac{1}{2}).$$

Some researchers have made an attempt to unify and to extend developments of some special functions into Fourier series. Carlson and Greiman [(6) have given a cosine series for Gegenbauer's function. MacRobert [(9) and (10)] has given a cosine and a sine series for the E-functions. Roop Narain [(12)] and Jain R. N. [(7)] have obtained Fourier series for Meijer's G-functions. Parashar [(11)] and Anandani P. [(3)] have given Fourier series for H-functions. Recently, Shah Manilal [(13), (14) and (15)] has established :—(i) Some Fourier series for Generalized Hypergeometric Polynomials which include Fourier series for polynomials of Bedient, Hermite, Laguerre and generalized Sister Celine [(16)], (ii) Some results on Fourier series for H-functions and (iii) Fourier series for generalized Meijer functions.

The object of this paper is to establish three expansion formulas for generalized Meijer's G-functions of two variables in series of sine and cosine functions. Some finite integrals and a recurrence relation for Meijer's G-functions of two variables have been derived with the help of these formulae. Certain known and interesting results are also obtained on specializing the parameters, as particular cases of these formulae.

2. Expansion Formulae for Generalized Meijer's G-functions :

We establish the following three results :

$$(2.1) \quad \sqrt{\pi}\ G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{array}{c|c} x\cos^2\tfrac{1}{2}\theta & \begin{array}{c}(a)\\ (c)\ ;\ (e)\\ (b)\\ (d)\ ;\ (f)\end{array}\\ y\cos^2\tfrac{1}{2}\theta & \end{array}\right] =$$

$$= G^{p+1,\,q,\,s,\,l,\,r,\,t}_{A+1,\,[C,\,E],\,B+1,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}\tfrac{1}{2}\ ,\ (a)\\ (c)\ ;\ (e)\end{array}\\ y & \begin{array}{c}(b)\ ,\ o\\ (d)\ ;\ (f)\end{array}\end{array}\right] +$$

$$+\ 2\sum_{\mu=1}^{\infty}\cos\mu\,\theta\ G^{p+2,\,q,\,s,\,l,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}\tfrac{1}{2},\ o,\ (a)\\ (c)\ ;\ (e)\end{array}\\ y & \begin{array}{c}(b),\mu\ ,\ -\mu\\ (d)\ ;\ (f)\end{array}\end{array}\right]$$

where $o \leqslant \theta \leqslant \pi$ and valid under the conditions stated in (1.2).

$$(2.2)\quad \frac{\sqrt{\pi}}{2}\,\sin\theta\; G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{array}{c|c} \frac{x}{\sin^2\theta} & \begin{array}{c}(a)\\ (c)\;;\;(e)\end{array} \\ \frac{y}{\sin^2\theta} & \begin{array}{c}(b)\\ (d)\;;\;(f)\end{array}\end{array}\right]$$

$$= \sum_{\mu=o}^{\infty} G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\mu+1,\,(a),\,\mu+2\\ (c)\;;\;(e)\end{array} \\ y & \begin{array}{c}3/2,\,(b),\,1\\ (d)\;;\;(f)\end{array}\end{array}\right]\sin(2\mu+1)\,\theta$$

valid for $o \leqslant \theta \leqslant \pi$ and conditions referred to (1.2).

$$(2.3)\quad \sqrt{\pi}\; G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{array}{c|c} \frac{x}{\sin^2\frac{1}{2}\theta} & \begin{array}{c}(a)\\ (c)\;;\;(e)\end{array} \\ \frac{y}{\sin^2\frac{1}{2}\theta} & \begin{array}{c}(b)\\ (d)\;;\;(f)\end{array}\end{array}\right]$$

$$= G^{p,\,q,\,s,\,l+1,\,r,\,t}_{A+1,\,[C,\,E],\,B+1,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}(a)\;,\;1\\ (c)\;;\;(e)\end{array} \\ y & \begin{array}{c}\frac{1}{2}\;,\;(b)\\ (d)\;;\;(f)\end{array}\end{array}\right] +$$

$$+\;2\sum_{\mu=1}^{\infty} G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\mu+1,\,(a),\,\mu+1\\ (c)\;;\;(e)\end{array} \\ y & \begin{array}{c}\frac{1}{2},\,(b)\;,\;1\\ (d)\;;\;(f)\end{array}\end{array}\right]\cos\mu\,\theta$$

where $o \leqslant \theta \leqslant \pi$ and valid under the given conditions (1.2).

Proof :

To establish (2.1), expressing the Meijer's G-function on the left as Mellin-Barnes type of integral (1.2), we obtain

$$\sqrt{\pi}\;\frac{1}{(2\,\pi\, i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} \phi\,[\xi+\eta]\;\psi\,[\xi,\,\eta]\times {}^{\xi}\;y^{\eta}\,(\cos\tfrac{1}{2}\,\theta)^{2\xi+2\eta}\,d\,\xi\,d\,\eta\,.$$

Now using (1.3) and (1.6), the expression reduces to

$$\frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\phi[\xi+\eta]\,\psi\,[\xi,\eta]\; x^{\xi}\,y^{\eta}\left\{\frac{\Gamma(\frac{1}{2}+\xi+\eta)}{\Gamma(1+\xi+\eta)}\right.$$

$$\left.+\;2\sum_{\mu=1}^{\infty}\frac{\Gamma(\frac{1}{2}+\xi+\eta)\,\Gamma(1+\xi+\eta)}{\Gamma(1-\mu+\xi+\eta)\,\Gamma(1+\mu+\xi+\eta)}\cos\mu\theta\right\}d\xi\,d\eta\,.$$

On chainging the order of integration and summation, the above expression takes the form

$$\frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\phi[\xi+\eta]\,\psi\,[\xi,\eta]\,\frac{\Gamma(\frac{1}{2}+\xi+\eta)}{\Gamma(1+\xi+\eta)}\; x^{\xi}\,y^{\eta}\,d\xi\,d\eta$$

$$+\;2\sum_{\mu=1}^{\infty}\cos\mu\theta\,\frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\phi[\xi+\eta]\,\psi\,[\xi,\eta]$$

$$\frac{\Gamma(\frac{1}{2}+\xi+\eta)\,\Gamma(1+\xi+\eta)}{\Gamma(1-\mu+\xi+\eta)\,\Gamma(1+\mu+\xi+\eta)}\; x^{\xi}\,y^{\eta}\,d\xi\,d\eta$$

which yields the expres sion on the right of (2.1) in view of (1.2).

Regarding the interchange of the order of integration and summation, it is observed that :—(i) the double contour integral converges under the conditions given in (1.2), (ii) the series

$$\sum_{\mu=1}^{\infty}\frac{\Gamma(\frac{1}{2}+\xi+\eta)\,\Gamma(1+\xi+\eta)}{\Gamma(1-\mu+\xi+\eta)\,\Gamma(1+\mu+\xi+\eta)}\cos\mu\theta \qquad \text{is}$$

uniformly convergent when $o \leqslant \theta \leqslant \pi$ and (iii) Meijer's G-function in two variables is a continuous function of x and y for all values of $x \geq x_0 \geq 0$ and $y \geq y_0 \geq 0$. Hence the order of integration and summation is easily justified due to Bromwich [(5), p. 500].

Expansion formulae (2.2) and (2.3) are proved in an analogous manner by using (1.4) and (1.5).

3. Corollaries :

From (2.1), we have

$$(3.1)\quad \int_0^{\pi} G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{matrix} x\cos^2\frac{1}{2}\theta \\ y\cos^2\frac{1}{2}\theta \end{matrix}\left|\begin{matrix} (a) \\ (c)\ ;\ (e) \\ (b) \\ (d)\ ;\ (f) \end{matrix}\right.\right]\cos\gamma\theta\, d\theta$$

$$= \sqrt{\pi}\, G^{p+2,\,q,\,s,\,l,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{matrix} x \\ y \end{matrix}\left|\begin{matrix} \frac{1}{2},\ o,\ (a) \\ (c)\ ;\ (e) \\ (b),\ \gamma, -\gamma \\ (d)\ ;\ (f) \end{matrix}\right.\right].$$

Similarly (2.2) yields

$$(3.2)\quad \int_0^{\pi} G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{matrix} \dfrac{x}{\sin^2\theta} \\ \dfrac{y}{\sin^2\theta} \end{matrix}\left|\begin{matrix} (a) \\ (c)\ ;\ (e) \\ (b) \\ (d)\ ;\ (f) \end{matrix}\right.\right]\sin(2\gamma+1)\theta\sin\theta\, d\theta$$

$$= \sqrt{\pi}\, G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,F]}\left[\begin{matrix} x \\ y \end{matrix}\left|\begin{matrix} -\gamma+1,\ (a),\ \gamma+2 \\ (c)\ ;\ (e) \\ 3/2,\ (b)\ ,\ 1 \\ (d)\ ;\ (f) \end{matrix}\right.\right].$$

Also (2.3) leads to

$$(3.3)\quad \int_0^{\pi} G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{matrix} \dfrac{x}{\sin^2\frac{1}{2}\theta} \\ \dfrac{y}{\sin^2\frac{1}{2}\theta} \end{matrix}\left|\begin{matrix} (a) \\ (c)\ ;\ (e) \\ (b) \\ (d)\ ;\ (f) \end{matrix}\right.\right]\cos\gamma\theta\, d\theta$$

$$= \sqrt{\pi}\, G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{matrix} x \\ y \end{matrix}\left|\begin{matrix} -\gamma+1,\ (a),\ \gamma+1 \\ (c)\ ;\ (e) \\ \frac{1}{2},\ (b)\ ,\ 1 \\ (d)\ ;\ (f) \end{matrix}\right.\right].$$

Now the integral (3.3) can be written

$$2\int_{o}^{\pi/2} \cos(2\gamma\varphi)\; G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,F]}\left[\begin{array}{c|c} \frac{x}{\sin^2\varphi} & \begin{array}{c}(a)\\(c)\;;\;(e)\end{array} \\ \frac{y}{\sin^2\varphi} & \begin{array}{c}(b)\\(d)\;;\;(f)\end{array}\end{array}\right] d\varphi$$

and, since this is equal to the same integral from o to π, (3.3) can be written

$$(3.4)\quad \int_{o}^{\pi} \cos(2\gamma\varphi)\; G^{p,\,q,\,s,\,l,\,r,\,t}_{A,\,[C,\,E],\,B,\,[D,\,F]}\left[\begin{array}{c|c} \frac{x}{\sin^2\varphi} & \begin{array}{c}(a)\\(c)\;;\;(e)\end{array} \\ \frac{y}{\sin^2\varphi} & \begin{array}{c}(b)\\(d)\;;\;(f)\end{array}\end{array}\right] d\varphi$$

$$= \sqrt{\pi}\; G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\gamma+1,\;(a),\;\gamma+1\\(c)\;;\;(e)\end{array} \\ y & \begin{array}{c}\tfrac{1}{2};\;(b)\;,\;1\\(d)\;;\;(f)\end{array}\end{array}\right]$$

Hence, on writing $\sin(2\gamma+1)\,\theta\,\sin\theta$ in (3.2) in the form

$$\tfrac{1}{2}\,\{\cos 2\gamma\theta - \cos(2\gamma+2)\,\theta\}$$

and applying (3.4), we obtain the recurrence relation

$$(3.5)\quad G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\gamma+1,\;(a),\;\gamma+1\\(c)\;;\;(e)\end{array} \\ y & \begin{array}{c}\tfrac{1}{2},\;(b)\;,\;1\\(d)\;;\;(f)\end{array}\end{array}\right]$$

$$-\; G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\gamma,\;(a),\;\gamma+2\\(c)\;;\;(e)\end{array} \\ y & \begin{array}{c}\tfrac{1}{2},\;(b)\;,\;1\\(d)\;;\;(f)\end{array}\end{array}\right] =$$

$$= 2\; G^{p+1,\,q,\,s,\,l+1,\,r,\,t}_{A+2,\,[C,\,E],\,B+2,\,[D,\,F]}\left[\begin{array}{c|c} x & \begin{array}{c}-\gamma+1,\;(a),\;\gamma+2\\(c)\;;\;(e)\end{array} \\ y & \begin{array}{c}3/2,\;(b)\;,\;1\\(d)\;;\;(f)\end{array}\end{array}\right]$$

which can be easily varified by comparing coefficients in case $A=B=C=D=E=F=o$ and using (1.7).

4. Particular Cases :

We know that the function $G\begin{bmatrix} x \\ y \end{bmatrix}$ is in a more generalized from which not only yields the Meijer's G-function or the product of two G-functions, as its specialized cases, but it also includes most of the commonly used functions in two argument e. g. Kampé de Fériet's [(8)] double hypergeometric functions which in turn, lead to the Appell functions [(1)] F_1, F_2, F_3 and F_4 the Whittaker function of two variables.

In (2.2), (2.3), (3.2), (3.3) and (3.5), setting $A=p$, $l=B$, $E=s$, $t=l$, $f_1=o$, and replacing $A+C$ by A, $B+D$ by B, $A+q$ by K together with the appropriate changes in the parameters etc., and then making $y\to o$, we can obtain the well-known results on Fourier series and integrals for Meijer's G-functions due to Roop Narain [(12), p. 149, (1.1), (1.2) and p. 151, sec. 3] and a recurrence relation [(11), p. 1085, (2.10)] :

$$(4.1) \qquad 2\,G^{m+1,\,n+1}_{p+2,\,q+2}\left(x \,\middle|\, \begin{matrix} 1-r,\ a_p,\ 2+r \\ 3/2,\ b_q,\ 1 \end{matrix} \right)$$

$$= G^{m+1,\,n+1}_{p+2,\,q+2}\left(x \,\middle|\, \begin{matrix} 1-r,\ a_p,\ 1+r \\ 1/2,\ b_q,\ 1 \end{matrix} \right) - G^{m+1,\,n+1}_{p+2,\,q+2}\left(x \,\middle|\, \begin{matrix} -r,\ a_p,\ 2+r \\ \frac{1}{2},\ b_q,\ 1 \end{matrix} \right).$$

If we make use of the relation [(4), p. 215] :

$$G^{q,\,1}_{p+1,\,q}\left(x \,\middle|\, \begin{matrix} 1,\ a_1,\ldots\ldots,a_p \\ b_1,\ldots\ldots\ldots,b_q \end{matrix} \right) = E\left(\begin{matrix} b_1,\ldots\ldots,\ b_q \\ a_1,\ldots\ldots,\ a_p \end{matrix} ;x \right)$$

where E (.) denotes Mac Robert's E-function [(4), p. 203], the formulae can be reduced to the Fourier series for E-functions [(9), p. 79, (1) and (2)] and a recurrence relation associated with E-functions [(11), p. 1085, (2.11)].

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