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# OBTAINING THE HORIZONTAL DEFLECTION OF A FALLING OBJECT WITHOUT THE KNOWLEDGE OF CORIOLIS FORCE

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## ABSTRACT

The horizontal deflection relative to the rotating earth is determined of an object dropped from rest from the top of a tower. A salient feature of the present quest is that the relevant equations of motion are formed without bringing in the concept of coriolis forcs .

## 1. INTRODUCTION

The well-known problem of horizontal or eastward deflection of a body dropped from rest from a certain height has been worked out in many text-books of classical dynamics ( see [2] and [4] ) introducing the idea of coriolis force that arises due to the interaction between the rotation of the Earth and the velocity of the body as observed relative to the Earth . J. M. Potgieter [1] while solving the problem in an alternativ method, first derived the equation of the path of the object with respect to an inertial frame (non-rotating) and then obtained the exact horizontal deflection by transforming the former equation to those with respect to a rotating frame relatively fixed to the earth . Stirling [3] derived the same result almost with the same technique as that of Potgieter [1] but only when the object is allowed to fall in the equatorial plane (zero latitude) .

Potigieter [1] tackled the problem in unnecessary details incorporating many redundant equations . However, the present design brings forth altogether a different method of attacking the same problem .

## 2. SOLUTION TO THE PROBLEM

Case 1. Let the particle fall from the top of a tower of height $h$ situated in the equatorial plane and $(x,y)$ be the instantaneous position of the particle with reference to a non-rotating frame XOY, O being the centre of the earth . If $g$ be the acceleration due to gravity, $\omega$ the angular velocity of the Earth about the polar axis and $a$ its radius then the equations of motion of the particle are given by

$$\frac{d^2x}{dt^2} = - g \cos \omega t \tag{1}$$

$$\frac{d^2y}{dt^2} = - g \sin \omega t \tag{2}$$

along with the initial conditions that at $t = 0$, $x = a + h$, *i. e.* $y=0$,

$$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = (a + h)\, \omega \text{ due to the earth's rotation} \quad .. (3)$$

The solutions of equations (1) and (2) with the help of (3) can be obtained as

$$x = a + h - g ( 1 - \cos \omega t ) / \omega^2 \tag{4}$$

$$y = ( a+h )\, \omega t - g ( - \sin \omega t ) / \omega \tag{5}$$

which, on expanding cos $(\omega t)$ and *sin* $(\omega t)$ in series aud neglecting the square and other higher terms of $\omega$, reduce to the form

$$x = a + h - \tfrac{1}{2} g t^2 \quad ... \tag{6}$$

$$y = (a+h)\,\omega t - \frac{g\omega t^3}{6} \quad \ldots \quad (7)$$

If $T$ be the time taken by particle to reach the surface of the earth then at $t=T$, $x=a\cos(wT)$ and $y=d$ (*say*) so that by the *use* of eqns (6) and (7) along with the foregoing approximation we get :

$$h=\tfrac{1}{2}\,gT^2 \quad (8)$$

$$d=(n+h)\;\omega T - \frac{g\omega T^3}{6} \quad (9)$$

Eliminating $T$ from (8), (9) and taking into consideration that the foot of the tower in the meantime has already moved eastward a distance equal to a $\sin(\omega T) \simeq a\omega T$, the eastward or horizontal dcf-lection, from the foot of the tower, of the object is given by

$$D=d-a\omega T = \frac{1}{3}\,g\omega T^3 = \frac{1}{3}\,2h\omega\left(\frac{2h}{g}\right)^{\frac{1}{2}} \quad \ldots \quad (10)$$

**Case 2.** If the particle be dropped from the top of a tower situated in the northern hemisphere at the latitude $\lambda$ then because of the Earth's rotation about the polar axis, it initially acquires the velocity

$$\vec{V}_0 = \vec{\omega}\ x\,(a+h)\,\hat{r}$$

$$= \vec{k}\omega\ x\,(a+h)\,(\,i\cos\lambda + \mathrm{k}\sin\lambda\,)$$

$$= j\omega\,(\,a+h\,)\cos\lambda \quad \ldots \quad (11)$$

where the right-handed inertial frame $OXYZ$ consists of the $X$-axis whicn is the line of intersection of the equatorial plane with the plane passing through the polaraxis and the radius vector joining the centre

$O$ of the Eatth to the top of the tower, the Y-axis perpendicular to the former axis and the Z-axis coinncident with the polar axis, $(i.j.k)$ heing the unit vectors associated with the system of axes. Equation (11) reveals that the particle initially gains a velocity equal to $\omega(a+h) \cos \lambda$ and that too in tne eastward direction indicated by the $Y$-axis.

If $(x,y,z)$ be the position of the particle at any instnant of time $t$ with respect to this non-rotating frame fixed in space, the equations of motion can be written as

$$\frac{d^2x}{dt^2} = -\frac{qx}{r}, \frac{d^2y}{dt^2} = -\frac{qy}{r}, \frac{d^2z}{dt^2} = -\frac{qz}{r} \qquad \ldots (12)$$

$$( r^2 = x^2 + y^2 + z^2 )$$

where the initial conditions are

$$t=0,\ x=(a+h) \cos \lambda,\ y=0,\ z=(a+h) \sin \lambda,$$

$$\frac{dx}{dt} = \frac{dz}{dt} = 0 \text{ and } \frac{dy}{dt} = (a+h)\, \omega \cos \lambda \qquad \ldots (13)$$

It is a formadible task to obtain solutions to equations (12) subject to the initial conditions . However, approximate closed form solutions can be found out the equations, in the following way

If $w$ is considered to be so small as to be totally negligible, obviously the path of the particle is vertical, *i, e.,* one dimensional accounting for no deflection eastward or otherwise, which is tantamount to solving equations (12) wherein the parameters $x, y, z$ as a way of the first approximation are replaced by their initial values from (13). Because of this fiast approximation, equations (12) reduce to the form

$$\frac{d^2x}{dt^2} = -g \cos \lambda, \frac{d^2y}{dt^2} = 0, \frac{d^2z}{dt^2} = -g \sin\lambda \qquad \ldots (14)$$

which, as a consequence of (13) and neglecting $\omega$, yleld

$$x = (a + h)\cos\lambda - \tfrac{1}{2} g \cos\lambda\, t^2$$
$$y = 0$$
$$z = (a + h)\sin\lambda - \tfrac{1}{2} g \sin\lambda\, t^2 \qquad \cdots \quad (15)$$

The second approximate solutions to equations (14), which can be carried out by not neglecting $\omega$ in (13), become

$$y = (a + h)(\omega \cos\lambda)\, t \qquad (16)$$

where the expressions for $x$ and $z$ remain the same as in (16), In order to obtain the the third approximate solutions to the equations of set (12), we ban paproximate in its second equation $y$ and $r$ by $\omega\,(a+h)\cos\lambda\, t$ and $(a+h)$ respectively ; in view of the second approximation (16), initial conditions (13), and $\omega^2$ and other powers of $\omega$ being negligibly small, then, by virtue of (13). the integrations of (12) lead to

$$y = (a+h)\,\omega\,(\cos\lambda)\, t - (1/6)\, g\omega\,(\cos\lambda)\, t^3 \qquad \cdots \quad (17)$$
$$d = (a+h)\,\omega\,(\cos\lambda)\, T - (1/6)\, g\omega\,(\cos\lambda)\, T^3 \qquad \cdots \quad (18)$$
$$h = \tfrac{1}{2} g T^2 \qquad \cdots \quad (19)$$

whrre $T$ is the time taken by the particle to fall on the surface of the earth. Combiniug equations (18) and (19) on the lines of (8) and (9), the eastward deflection of the particle relative to the foot of the tower can be obtained as

$$D = d - aw\,(\cos\lambda)\, T$$
$$= (1/3)\, g\,(\omega \cos\lambda)\, T^2$$
$$= (1/3)\, 2h\,(\cos\lambda)\,(2h/g)^{1/2} \qquad \cdots \quad (20)$$

To continue this approximation process, we need to replace in the second equation of (12) y by its expression from (17) and $r$ by its initial value as mentioned earlier, so as to obtain with the help of equations (13) and (19) the more accurate deflection .

$$D^1 = 1/3\; 2h\omega \left[ 1 - \frac{h}{20\,(a+h)} \right] (\cos\lambda) \left( \frac{2h}{g} \right)^{\frac{1}{2}} \qquad \cdots \quad (12)$$

The comparision of (20) with (21) reveals that since $h << a$, the former gives the most ideal result . However, from the theoretical point of view, further continuation of this process entails greater accuracy in determining the deflection of the particle ,

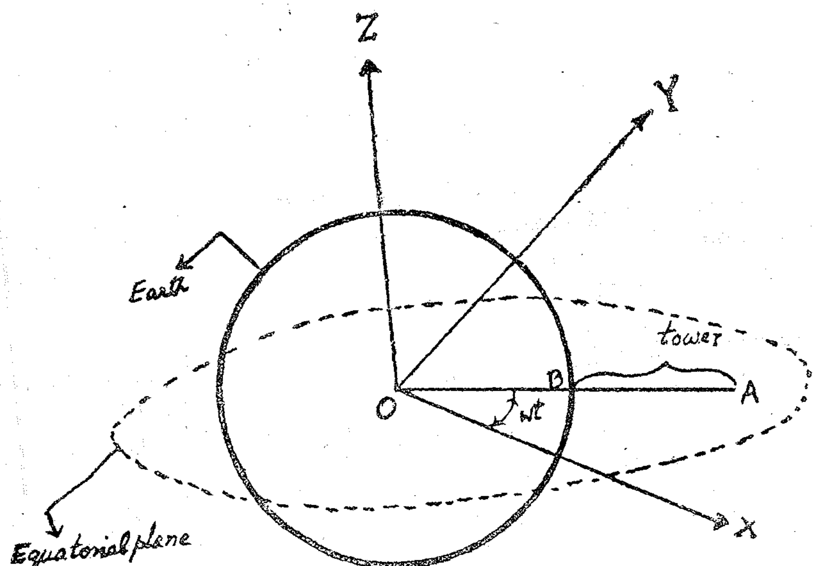


Fig 1. Horizontal deflection in case 1.

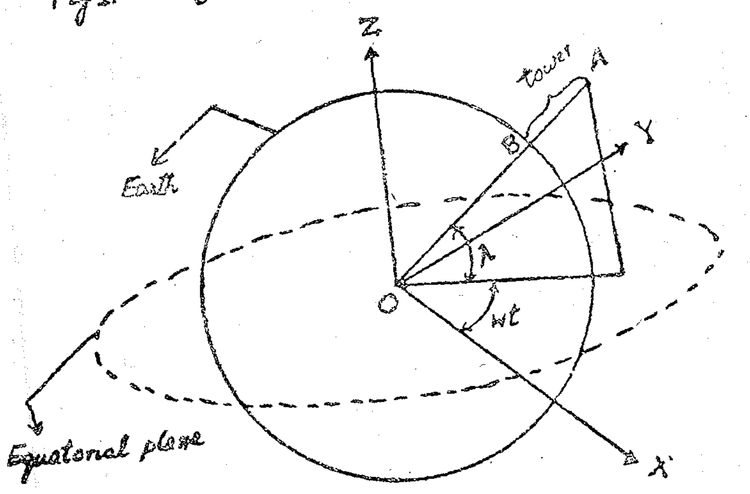


Fig 2. Horizontal deflection in case 2.

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