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CERTAIN RESULTS INVOLVING A CLASS OF GENERALIZED LAURICELLA FUNCTIONS

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1. INTRODUCTION

Certain addition and multiplication theorems of Lauricella functions have been discussed by H. Exton [1]. Our aim, in the present note, is to establish some results concerning the generalized Lauricella functions by an application of a fractional differential operator .

2. Notations and Definitions

We consider the following special class of the generalized Lauricella functions due to Srivastava and Daoust ([2, p. 454] ; see also [1, p. 107]) :

$$F^{1:2;\ldots;2}_{0:2;\ldots;2}\left(\begin{matrix} a : b_1, \lambda_1 ; \ldots ; b_n, \lambda_n ; \\ - : c_1, \mu_1 ; \ldots ; c_n, \mu_n ; \end{matrix} \; x_1 , \ldots , x_n\right)$$

$$= \sum_{m_1,\ldots,m_n=0}^{\infty} \frac{(a)_{m_1+\cdots+m_n} (b_1)_{m_1} (\lambda_1)_{m_1} \cdots (b_n)_{m_n} (\lambda_n)_{m_n} x_1^{m_1} \cdots x_n^{m_n}}{(c_1)_{m_1} (\mu_1)_{m_1} \cdots (c_n)_{m_n} (\mu_n)_{m_n} (1)_{m_1} \cdots (1)_{m_n}}$$

Also, the fractional differential operator D_x^{δ} is defined by (see, e. g., [3, p, 289])

$$= \sum_{p_1, \ldots, p_n = 0}^{\infty} \frac{(a)_{p_1+\ldots+p_n} (b_1)_{p_1} \ldots (b_n)_{p_n} y_1^{p_1} \ldots y_n^{p_n}}{(c_1)_{p_1} \ldots (c_n)_{p_n} (1)_{p_1} \ldots (1)_{p_n}}$$

$$. F_A^{(n)} (a+ p_1+ \ldots+p_n, b_1+p_1, \ldots, b_n+p_n ; c_1+p_1, \ldots, c_n + p_n; x_1, \ldots, x_n).$$

Multiplying both sides of (4.1) by $x_1^{\lambda_1-1} \ldots x_n^{\lambda_n-1} y_1^{\mu_1-1} \ldots y_n^{\mu_n-1}$ and applying the operators $D_{x_1}^{\delta_1} \ldots D_{x_n}^{\delta_n} D_{y_1}^{\nu_1} \ldots D_{y_n}^{\nu_n}$ on both sides, we get the result (3.1) after some simplification.

Similarly, results involving Lauricella functions $F_B^{(n)}$, $F_C^{(n)}$, $F_D^{(n)}$ may easily be extended.

Proof of (3.2): The multiplication theorem for the Lauricella function $F_A^{(n)}$, given by *H.* Exton [1], is

(4.2) $F_A^{(n)} (a, b_1, \ldots, b_n ; c_1, \ldots, c_n ; x_1 y_1, \ldots, x_n y_n) =$

$$= \sum_{p_1, \ldots, p_n = 0}^{\infty} \frac{(a)_{p_1+\ldots+p_n} (b_1)_{p_1} \ldots (b_n)_{p_n} x_1^{p_1} (y_1-1)^{p_1} \ldots x_n^{p_n} (y_n-1)^{p_n}}{(c_1)_{p_1} \ldots (c_n)_{p_n} (1)_{p_1} \ldots (1)_{p_n}}$$

$$, F_A^{(n)} (a+p_1+\ldots+p_n, b_1+p_1, \ldots, b_n + p_n; c_1+p_1, \ldots c_n +p_n; x_1, \ldots, x_n)$$

Multiplying both sides of (4.2) by $x_1^{\lambda_1-1} y_1^{\mu_1-1} \ldots x_n^{\lambda_n-1} y_n^{\mu_n-1}$ and applying the operators $D_{x_1}^{\delta_1} D_{y_1}^{\nu_1} \ldots D_{x_n}^{\delta_n} D_{y_n}^{\nu_n}$ on both sides, we get, after some simplification, the result (3.2).

Similarly, the known results for Lauricella functions $F_B^{(n)}$, $F_C^{(n)}$ and $F_D^{(n)}$ may be extended .

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