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ON SOME FINITE INTEGRALS INVOLVING THE MULTIVARIABLE H - FUNCTION

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ABSTRACT

In this paper, we evaluate some finite integrals involving the products of Jacobi polynomials and the H-function of several complex variables .

1. INTRODUCTION

Srivastava and Panda [13] have defined the multivariable H-function as (see also [12] and [14])

$$H\begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} = H^{O,\varepsilon : (M', N') ; \dots ; (M^{(r)}, N^{(r)})}_{A, C : (B', D') ; \dots ; (B^{(r)}, D^{(r)})} \left(\begin{matrix} [(a) : \theta', \dots, \theta^{(r)}] : \\ [(c) : C', \dots, C^{(r)}] : \end{matrix} \right.$$

$$\left. \begin{matrix} [(b') : \phi'] ; \dots ; [(b^{(r)}) : \phi^{(r)}] ; \\ [(d') : \delta'] ; \dots ; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix} \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \right)$$

$$= \frac{1}{(2\pi i)^r} \int_{L_1} \dots \int_{L_r} V(s_1, \dots, s_r)\, U_1(s_1) \dots U_r(s_r)\, z_1^{s_1} \dots z_r^{s_r}$$

$$. \, ds_1 \dots ds_r \quad \dots (1.1)$$

where

$$V(s_1,\ldots,s_r)=\frac{\prod_{j=1}^{\varepsilon}\Gamma\left(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}s_i\right)}{\prod_{j=\varepsilon+1}^{A}\Gamma\left(a_j-\sum_{i=1}^{r}\theta_j^{(i)}s_i\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{i=1}^{r}C_j^{(i)}s_i\right)} \quad \ldots(1.2)$$

and

$$U_i(s_i)=\frac{\prod_{j=1}^{M^{(i)}}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\prod_{j=1}^{N^{(i)}}\Gamma\left(1-b_j^{(i)}+\phi_j^{(i)}s_i\right)}{\prod_{j=M^{(i)}+1}^{D^{(i)}}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\prod_{j=N^{(i)}+1}^{B^{(i)}}\Gamma\left(b_j^{(i)}-\phi_j^{(i)}s_i\right)}$$

$$i=1,\ldots,r \quad \ldots(1.3)$$

The multiple integral in (1.1) absolutely converges if

$$|\arg(z_i)|<\tfrac{1}{2}T_i\pi,\ i=1,\ldots,r \quad \ldots(1.4)$$

where

$$T_i=-\sum_{j=\varepsilon+1}^{A}\theta_j^{(i)}+\sum_{j=1}^{N^{(i)}}\phi_j^{(i)}-\sum_{j=N^{(i)}+1}^{B^{(i)}}\phi_j^{(i)}-\sum_{j=1}^{C}C_j^{(i)}$$

$$+\sum_{j=1}^{M^{(i)}}\delta_j^{(i)}-\sum_{j=M^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}>0,\ i=1,\ldots,r \quad \ldots(1.5)$$

The following notations and known results will be used throughout this paper :

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}=\begin{cases}1, & \text{if } n=0,\\ \alpha(\alpha+1)\ldots(\alpha+n-1), & \forall n\in\{1,2,3,\ldots\}.\end{cases}$$

$$\ldots(1.6)$$

Next for the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ [11, *p.* 254, Eq.(1)], we have

$$P_k^{(\alpha,\beta)}(t+\rho)\,P_k^{(\alpha,\beta)}(t-\rho) = \frac{(-1)^k (1+\alpha)_k (1+\beta)_k}{(k!)^2}$$

$$\cdot \sum_{r=0}^{k} \frac{(-k)_r (1+\alpha+\beta+k)_r}{(1+\alpha)_r (1+\beta)_r} P_r^{(\alpha,\beta)}(x)\, t^r \quad \ldots (1.7)$$

$$\rho^k P_k^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right) = \frac{(1+\alpha)_k}{k!} \sum_{r=0}^{k} \frac{(-k)_r}{(1+\alpha)_r} P_r^{(\alpha,\alpha)}(x)\, t^r \quad \ldots (1.8)$$

$$\frac{1}{\rho}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta} = 2^{-\alpha-\beta} \sum_{r=0}^{\infty} P_r^{(\alpha,\beta)}(x)\, t^r \quad \ldots (1.9)$$

In each of the formulas (1.7), (1.8), and (1.9), and throughout this paper, $\rho = (1-2xt+t^2)^{1/2}$. Formulas (1.7), (1.7) and (1.9) can be found, for example in [4, p. 945], [4, p. 946] and [6, p. 172], respectively .

$$\int_{-1}^{1} (1+x)^{\eta} (1-x)^{\alpha} P_n^{(\alpha,\beta)}(x) H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r} \end{pmatrix} dx$$

$$= \frac{{}^{\eta}2^{+\alpha+1}\,\Gamma(\alpha+n+1)}{n!} H^{0,\, \varepsilon+2 : (M', N') ; \ldots ; (M^{(r)}, N^{(r)})}_{A+2,\, C+2 : (B', D') ; \ldots ; (B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix} [(a) : \theta', \ldots, \theta^{(r)}],\ (-\eta : h_1, \ldots, h_r),\ (\beta-\eta : h_1, \ldots, h_r) : \\ [(c) : C', \ldots, C^{(r)}],\ (-\alpha-\eta-n-1 : h_1, \ldots, h_r),\ (\beta-\eta+n : h_1, \ldots, h_r) : \end{matrix} \right.$$

$$\left. \begin{matrix} [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; & 2^{h_1} z_1 \\ & \vdots \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; & 2^{h_r} z_r \end{matrix} \right) \quad \ldots (1.10)$$

Formula (1. 10) was given by Srivastava and Panda [13, p. 131, Eq. (2.2)] .

$$\int_{-1}^{1} (1-x)^{\eta} (1+x)^{\sigma} P_n^{(\alpha,\beta)}(x) H\begin{pmatrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{pmatrix} dx$$

$$= 2^{\eta+\sigma+1} \sum_{m=0}^{n} \frac{(-n)_m (\alpha+\beta+n+1)_m}{(\alpha+1)_m \, m!}$$

$$H^{0,\,\varepsilon+2 : (M', N') ; \ldots ; (M^{(r)}, N^{(r)})}_{A+2,\, C+1 : (B', D') ; \ldots ; (B^{(r)}, D^{(r)})}$$

$$\begin{pmatrix} [(a) : \theta', \ldots, \theta^{(r)}], (-\sigma : k_1, \ldots, k_r), (-m-\eta : h_1, \ldots, h_r) : \\ [(c) : C', \ldots, C^{(r)}], (-1-m-\eta-\sigma : h_1 + k_1, \ldots, h_r + k_r) : \end{pmatrix}$$

$$\left. \begin{matrix} [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; z_1\, 2^{h_1+k_1} \\ \vdots \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; z_r\, 2^{h_r+k_r} \end{matrix} \right) \quad \ldots (1.11)$$

The integral (1.11) can be evaluated by making use of a known result [7, p. 284, Fq. (2)] . In fact, (1.12) is an obvious special case of much more general results given by Srivastava and Singh [15, p. 172, Fq. (3.2)] .

2. MAIN INTEGRLS

The following integrals have been evaluated in this paper :

$$\int_{-1}^{1} (1+x)^{\eta} (1-x)^{\alpha} P_k^{(\alpha,\beta)}(t+\rho) P_k^{(\alpha,\beta)}(t-\rho) H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r} \end{pmatrix} dx$$

$$= 2^{\eta+\alpha+1} \frac{(-1)^k \Gamma(1+\alpha+k)(1+\beta)_k}{(k!)^2}$$

$$\sum_{R=0}^{k} \frac{(-k)_R\,(1+\alpha+\beta+k)_R}{(1+\beta)_R}\,\frac{t^R}{R!}$$

$$. \, H^{0,\,\varepsilon+2\,:\,(M',\,N');\,\ldots;\,(M^{(r)},\,N^{(r)})}_{A+2,\,C+2\,:\,(B',\,D');\,\ldots,\,(B^{(r)},\,D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],(-\eta:h_1,\ldots,h_r),(\beta-\eta:h_1,\ldots,h_r): \\ [(c):C',\ldots,C^{(r)}],(-\alpha-\eta-R-1:h_1,\ldots,h_r),(\beta-\eta+R:h_1,\ldots,h_r):\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\; z_1\,2^{h_1} \\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}],\; z_r\,2^{h_r}\end{matrix}\right) \qquad \ldots(2.1)$$

where

$\mathrm{Re}(\eta) > -1,\ \mathrm{Re}(\alpha) > -1,$

$h_i > 0,\ i = 1,\ldots,r$ and $|\arg(z_i)| < \frac{1}{2} T_i \pi,\ i = 1,\ldots,r,$

$T_i > 0$ and $\mathrm{Re}\left(\eta + \sum_{i=1}^{r} h_i\, d_j^{(i)} / \delta_j^{(i)}\right) > -1.$

$$\int_{-1}^{1} \rho^k (1-x)^\eta (1-x)^\alpha P_k^{(\alpha,\alpha)}\left(\frac{1-xt}{\rho}\right) H\begin{pmatrix} z_1\,(1+x)^{h_1} \\ \vdots \\ z_r\,(1+x)^{h_r}\end{pmatrix} dx$$

$$= \frac{2^{\eta+\alpha+1}\,\Gamma(1+\alpha+k)}{k!} \sum_{R=0}^{k} \frac{(-k)_R}{R!}\, t^R$$

$$H^{0,\,\varepsilon+2\,:\,(M',N');\,\ldots;\,(M^{(r)},\,N^{(r)})}_{A+2,\,C+2\,:\,(B',\,D');\,\ldots;\,(B^{(r)},\,D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],(-\eta:h_1,\ldots,h_r),(\alpha-\eta:h_1,\ldots,h_r): \\ [(c):C',\ldots,C^{(r)}],(-\alpha-\eta-R-1:h_1,\ldots,h_r),(\alpha-\eta+R:h_1,\ldots,h_r):\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];z_1\,2^{h_1}\\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];z_r\,2^{h_r}\end{matrix}\right) \qquad \ldots(2.2)$$

where,

$Re(\eta) > -1$, $Re(\alpha) > -1$, $h_i > 0, i = 1, \ldots, r,$

$|arg(z_i)| < \frac{1}{2} T_i \pi$, $T_i > 0$ and

$$Re\left(\eta + \sum_{i=1}^{r} h_i \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1, i = 1, \ldots, r$$

$$\int_{-1}^{1} \frac{1}{\rho} (1-t+\rho)^{-\alpha} (1+t+\rho)^{-\beta} (1+x)^{\eta} (1-x)^{\alpha} H\begin{pmatrix} z_1 (1+x)^{h_1} \\ \vdots \\ z_r (1+x)^{h_r}\end{pmatrix} dx$$

$$= 2^{\eta-\beta+1} \sum_{R=0}^{\infty} \frac{\Gamma(\alpha+R+1)}{R!} t^R H^{0,\, \varepsilon+2:\, (M', N');\, \ldots;\, (M^{(r)}, N^{(r)})}_{A+2,\, C+2;\, (B', D');\, \ldots;\, (B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],(-\eta:h_1,\ldots,h_r),(\beta-\eta;h_1,\ldots,h_r);\\ [(c):C',\ldots,C^{(r)}],(-\alpha-\eta-R-1;e_1,\ldots,h_r),(\beta-\eta+R:h_1,\ldots,h_r);\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];z_1\,2^{h_1}\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];z_r\,2^{h_r}\end{matrix}\right) \qquad \ldots(2.3)$$

where

$Re(\eta) > -1$, $Re(\alpha) > -1$, $h_i > 0,$

$|arg(z_i)| < \frac{1}{2} T_i\pi$, $T_i > 0$ and

$$Re\left(\eta + \sum_{i=1}^{r} h_i\, d_j^{(i)} / \delta_j^{(i)}\right) > -1, \quad i = 1, \ldots, r$$

$$\int_{-1}^{1} (1-x)^{\eta} (1+x)^{\sigma} P_k^{(\alpha,\beta)} (t+\rho) P_k^{(\alpha,\beta)} (t-\rho) H\begin{pmatrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{pmatrix} dx$$

$$= 2^{\eta+\sigma+1} \frac{(-1)^k \Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}$$

$$\sum_{R=0}^{k} \sum_{m=0}^{R} \frac{(-k)_r (1+\alpha+\beta+k)_R}{\Gamma(1+\alpha+R)\Gamma(1+\beta+R)} t^R \frac{(-R)_m (\alpha+\beta+R+1)_m}{(\alpha+1)_m \, m!}$$

$$\cdot H^{0,\, \epsilon+2 : (M', N') ; \; . ; (M^{(r)}, N^{(r)})}_{A+2,\, C+1 : (B', D') ; \ldots ; (B^{(r)}, D^{(r)})}$$

$$\left(\begin{matrix} [(a) : \theta', \ldots \theta^{(r)}], (-\sigma : k_1, \ldots, k_r), (m-\eta : h_1, \ldots, h_r) : \\ [(c) : C', \ldots, C^{(r)}], (-1-m-\eta-\sigma : h_1+k_1, \ldots, h_r+k_r) : \end{matrix} \right.$$

$$\left. \begin{matrix} [(b) : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; z_1 2^{h_1+k_1} \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; z_r 2^{h_r+k_r} \end{matrix} \right) \quad \ldots (2.4)$$

where

$Re(\eta) < -1$, $Re(\sigma) < -1$, $k_i > 0$,

$h_i > 0$, $|arg(z_i)| < \frac{1}{2} T_i \pi$ and

$$Re\left(\eta + \sum_{i=1}^{r} h_i d_j^{(i)} / \delta_j^{(i)}\right) > -1, \quad Re\left(\sigma + \sum_{i=1}^{r} k_i d_j^{(i)} / \delta_j^{(i)}\right) > -1,$$

$T_i > 0$, $i = 1, \ldots, r$

$$\int_{-1}^{1} (1-x)^{\eta} (1+x^{\sigma} \rho^k P_k^{(\alpha,\alpha)} \left(\frac{1-xt}{\rho}\right) H\begin{pmatrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{pmatrix} dx$$

$$= \frac{2^{\eta+\alpha+1}\,\Gamma(1+\alpha+k)}{k!} \sum_{R=0}^{k} \sum_{m=0}^{r} \frac{(-k)_R}{\Gamma(1+\alpha+R)} t^R \frac{(-R)_m\,(2\alpha+R+1)_m}{(\alpha+1)_m\, m!}$$

$$.H^{0,\,\varepsilon+2\,:\,(M',N')\,;\,\ldots\,;\,(M^{(r)},\,N^{(r)})}_{A+2,\,C+1\,:\,(B',\,D')\,;\,\ldots\,;\,(B^{(r)},\,D^{(r)})}$$

$$\left(\begin{matrix} [(a):\theta',\ldots,\theta^{(r)}],\,(-\sigma:k_1,\ldots,k_r),\,(-m-\eta:h_1,\ldots,h_r): \\ [(c):C',\ldots,C^{(r)}],\,(-1-m-\eta-\sigma::h_1+k_1,\ldots,h_r+k_r): \end{matrix} \right.$$

$$\left. \begin{matrix} [(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\; z_1\, 2^{h_1+k_1} \\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}],\; z_r\, 2^{h_r+k_r} \end{matrix} \right) \qquad \ldots(2.5)$$

where

$\mathrm{Re}(\eta) > -1$, $\mathrm{Re}(\sigma) > -1$, $h_i > 0$, $k_i \geq 0$,

$|\arg(z_i)| < \frac{1}{2} T_i \pi$, and $\mathrm{Re}\left(\eta + \sum_{i=1}^{r} h_i\, d_j^{(i)} / \delta_j^{(i)}\right) > -1$.

$\mathrm{Re}\left(\sigma + \sum_{i=1}^{r} k_i\, d_j^{(i)}\right) > 1$, $i = 1, \ldots, r$

$$\int_{-1}^{1} (1-x)^{\eta}\,(1+x)^{\sigma}\,\frac{1}{\rho}\,(1-t+\rho)^{-\alpha}\,(1+t+\rho)^{-\beta}\, H\left(\begin{matrix} z_1\,(1-x)^{h_1}\,(1+x)^{k_1} \\ z_r\,(1-x)^{h_r}\,(1+x)^{k_r} \end{matrix} \right) dx$$

$$= 2^{-\alpha-\beta+\eta+\sigma+1} \sum_{R=0}^{\infty} \sum_{m=0}^{R} t^R \frac{(-R)_m\,(\alpha+\beta+R+1)_m}{(\alpha+1)_m\, m!}$$

$$.\,H^{0,\,\varepsilon+2\,:\,(M',\,N'')\,;\,\ldots\,;\,(M^{(r)},\,N^{(r)})}_{A+2,\,C+1\,:\,(B',\,D')\,;\,\ldots\,,\,(B^{(r)},\,D^{(r)})}$$

$$\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],(-\sigma:k_1,\ldots,k_r),(-m-\eta:h_1,\ldots,h_r):\\ [(c):C',\ldots,C^{(r)}],(-1-m-\eta-\sigma:h_1+k_1,\ldots,h_r+k_r):\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\ z_1\,2^{h_1+k_1}\\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\ z_r\,2^{h_r+k_r}\end{matrix}\right) \quad \ldots (2.6)$$

where

$Re\,(\eta) > -1$, $Re\,(\sigma) > -1$, $h_i > 0$, $k_i > 0$,

$|\arg(z_i)| < \frac{1}{2} T_i \pi, > 0$ and

$$Re\,(\eta + \sum_{i=1}^{r} h_i\, d_j^{(i)} / \delta_j^{(i)}) > -1,$$

$$Re\,(\sigma + \sum_{i=1}^{r} k_i\, d_j^{(i)} / \delta_j^{(i)}) > -1,\ i = ,\ldots, r$$

Proof : To prove (2.1), multiply both the sides of (1.7) by

$(1+x)^\eta (1-x)^\alpha\ H\begin{pmatrix} z_1\ (1+x)^{h_1} \\ \vdots \\ z_r\ (1+x)^{h_r}\end{pmatrix}$ and integrating both sides with respect to x between the limits -1 to 1, we get

$$\int_{-1}^{1} (1+x)^\eta (1-x)^\alpha P_k^{(\alpha,\beta)}(t+\rho) P_k^{(\alpha,\beta)}(t-\rho)\, H\begin{pmatrix} z_1\ (1+x)^{h_1} \\ \vdots \\ z_r\ (1+x)^{h_r}\end{pmatrix} dx$$

$$= \int_{-1}^{1} \frac{(-1)^k (1+\alpha)_k\ (1+\beta)_k}{(k\,!)^2} \sum_{R=0}^{k} \frac{(-k)_R}{(1+\alpha)_R} \frac{(1+\alpha+\beta+k)_R}{(1+\beta)_R}$$

$$.\ P_k^{(\alpha,\beta)}(x)\ t^R (1+x)^\eta (1-x)^\alpha\ H\begin{pmatrix} z_1\ (1+x)^{h_1} \\ \vdots \\ z_r\ (1+x)^{h_r}\end{pmatrix} dx \quad \ldots (2.7)$$

Now, on interchanging the order of integration and summation in the right-hand sidə of (2.7), which is permissible as the series involved is finite and integrals involved are absolutely couvergent and on evaluating the x-integral thus obtained with the help of (1.10), we get the desired result under the conditions mentioned therein .

The proofs of the formulas (2.2) to (2,6) can be peveloped by proceeding on similar lines with the help of the result (1.8) and (1.10), (1.9). and (1.10), (1.9) and (1.11), (1.8) and (1.11), (1.9) and (1.11), respectively.

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