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THEORETICAL STUDY OF RELATIONSHIP BETWEEN DERMAL BLOOD FLOW AND SKIN SURFACE TEMPERATURE OF A HUMAN SUBJECT

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ABSTRACT

This paper deals with a direct mathematical relationship of skin surface temperature and blood mass flow rate in dermis and subdermal regions of a human body . The investigations are based on the mathematical formulation for heat migration in skin and subcutaneous tissues of a human body . This formulation incorporates the effects of blood mass flow rate, rate of metabolic heat generation and evaporation from the surface at a moderate atmospheric temperature . The variational finite element method has been employed to solve the problem dividing the entire region in four natural compartments, namely, stratum corneum, stratum germinativum, dermis and subcutaneous tissues . Then blood mass flow rate in dermis and subcutaneous part is varied independently . The effect of this variation on the skin surface temperature is studied for suitable numerical values of various physical and physiological quantities thus involved .

1. INTRODUCTION

The heat transfer process through human dermal regions is a complex phenomena due to various reasons . As the core of a human body remains at a fairly uniform temperature, the peripheral region undergoes frequent changes due to variations in environmental conditions. The important factors which account for these changes, are blood flow, metabolic heat generation, water or sweat evaporation from the outer skin surface etc. The region which is known as skin and sub-cutaneous tissues (SST), can broadly be divided into four almost parallel layers. The first two layers are stratum corneum and stratum germinativum and makes the epidermis. The other two layers are dermis and sub-cutaneous tissues. The biological parameters have different properties in these layers. The uppermost one, stratum corneum is made of dead cells and there is no blood flow and metabolic heat generation in this part. The second layer is also free from blood vessels but has some metabolic activity due to presence of some living cells near the lower interface. The next layer (dermis) contains various constituents such as blood capillaries, blood cells, lymphatics, nerves and glands. The subcutaneous layer is fully saturated with active cells and blood vessels. The changes taking place in these layers have their effect on the body surface temperature. In fact the body surface temperature is controlled by the internal as well as external parameters. Amongest the internal parameters, the most important one is blood flow. In this paper we investigate a direct relationship between blood mass flow rate in dermis and subcutaneous tissues, and the outer skin surface temperature.

Temperature distribution problems in SST have already been studied theoretically by Saxena ([2] and [3]), Saxena and Arya ([4] and [5]), and Saxena and Bindra [6]. In most of the cases they have used variational finite element method by dividing SST into its natural compartments and taking each one as an element having independent variation of parameters . We use same technique in this paper . Our problem deals with a one dimensional steady state case having heat loss due to radiation, convection and evaporation at the outer surface . Numerical results have been obtained only for a single case of atmospheric temperature and SST structure ;

2. MATHEMATICAL FORMULATION

We employ the following partial differential equation for temperature distribution in SST (see [3])

$$\rho c \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial \theta}{\partial x}\right) + MC_b (\theta_b - \theta) + S \tag{1}$$

along with the boundary condition

$$K \frac{\partial \theta}{\partial n} = h (\theta - \theta_a) + LE \tag{2}$$

where

θ = temperature at distance x from the outer skin surface,

ρ = density of the tissue,

c = specific heat of the tissue,

K = thermal conductivity of the tissue,

M = blood mass flow rate,

c_b = specific heat of blood,

θ_b = temperature of blood when it enters the tissue element

S = rate of metabolic heat generation,

t = time,

θ_a = atmospheric temperature,

h = coefficient of convection and radiation,

L = Latent heat of evaporation,

E = rate of evaporation

x = perpendicular distance from the outer skin surface .

n = normal to skin surface .

The terms on the right of equation (1) are respectively Fickian diffusion, perfusion due to blood flow, and the rate of metabolic heat generation . Here M and S may be functions of position and temperature . The equations (1) and (2) when written in one dimensional steady state case can be transformed into Euler-Lagrange's variational form and written as

$$I = \frac{1}{2} \int_0^d \left[K \left(\frac{\partial \theta}{\partial x} \right)^2 + M c_b \, (\theta_b - \theta)^2 - 2 S \theta \right] dx$$

$$+ \frac{h \, (\theta - \theta_a)^2}{2} + L E \theta \qquad (3)$$

where d is thickness of the entire region .

We divide the whole region into four elements according to its structure . The epidermis is divided into two elements, The dermis is taken as third element and subdermal tissues as fourth . We assign values θ_r ($r = 0, 1, 2, 3, 4,$) to θ at interfaces at distances a_r ($r = 0, 1, 2, 3, 4$) . Thus θ_1, θ_2 and θ_3 are internal points on the temperature profile curve with θ_0 and θ_4 as end points . Here we assume θ_0 (skin surface temperature) to be unknown and $\theta_4 = \theta_b$ (blood temperature = body core temperature) Let $\theta^{(i)}$ denotes $\theta(x)$

for $a_{i-1} < x < a_i$ and I_1, I_2, I_3 and I_4 are the values of I in the four elements, then

$$I = \sum_{i=1}^{4} I_i \tag{4}$$

Let $K_i(x)$, $M_i(x)$, and $S_i(x)$ are the values of K, M and S in the i th element. Then taking linear values of $Q^{(i)}$ with respect to x in terms of nodal temperatures θ_r (r = 0, 1, 2, 3, 4) in (3) we get the following equations :

$$\left.\begin{aligned}
I_1 &= \frac{B_1^2}{2} K^{(1)} + \frac{h}{2} [\theta_0 - \theta_a]^2 + LE\theta_0 \\
I_2 &= \frac{B_2^2 K^{(2)}}{2} - A_2 S^{(2)} - B_2 N^{(2)} \\
I_3 &= \frac{B_3^2 K^{(3)}}{2} + \frac{C_b(\theta_b - A_3)^2 M^{(3)}}{2} \\
&\quad - B_3 C_b(\theta_b - A_3) P^{(3)} + \frac{C_b B_3^2 Q^{(3)}}{2} - A_3 S^{(3)} - B_3 N^{(3)} \\
I_4 &= \frac{B_4^2 K^{(4)}}{2} + \frac{C_b(\theta_b - A_4)^2 M^{(4)}}{2} \\
&\quad - B_4 C_b(\theta_b - A_4) P^{(4)} + \frac{C_b B_4^2 Q^{(4)}}{2} - A_4 S^{(4)} - B_4 N^{(4)}
\end{aligned}\right\} \tag{5}$$

where

$$B_i = \frac{\theta_i - \theta_{i-1}}{u_i}$$

$$A_i = \frac{a_i\theta_{i-1} - a_{i-1}\theta_i}{u_i} \qquad i = 1\ (1)\ 4$$

$$u_i = a_i - a_{i-1}$$

$$K^{(i)} = \int_{a_{i-1}}^{a_i} K_i\ (x)\ dx$$

$$S^{(i)} = \int_{a_{i-1}}^{a_i} S_i\ (x)\ dx$$

$$N^{(i)} = \int_{a_{i-1}}^{a_i} x\ S_i\ (x)\ dx \qquad i = 2\ (1)\ 4$$

$$M^{(i)} = \int_{a_{i-1}}^{a_i} M_i\ (x)\ dx$$

$$P^{(i)} = \int_{a_{j-1}}^{a_i} x\ M_i\ (x)\ dx \qquad i = 3,\ 4$$

$$Q^{(i)} = \int_{a_{i-1}}^{a_i} x^2\ M_i\ (x)\ dx$$

Now extremizing I with respect to θ_0, θ_1, θ_2 and θ_3 we get the following system of algebraic equations :

$$\left.\begin{aligned} &(R_1 + h)\ \theta_0 - R_1\theta_1 = D_1 \\ &- R_1\theta_0 + (R_1 + R_2)\ \theta_1 - R_2\ \theta_2 = D_2 \\ &- R_2\theta_1 + R_3\theta_2 + R_4\theta_3 = D_3 + F_1\theta_b \\ &R_4\theta_2 + R_5\theta_3 = D_4 + F_2\theta_b \end{aligned}\right| \qquad (6)$$

where

$$R_1 = \frac{k^{(1)}}{a_1} \quad , \quad R_2 = \frac{k^{(2)}}{u_2}$$

$$R_3 = \frac{k^{(2)}}{u_2^2} + \frac{k^{(3)}}{u_3^2} + \frac{c_b\ M^{(3)}\ a_3^2}{u_3^2} - \frac{2c_bP^{(3)}a_3}{u_3^2} + \frac{c_bQ^{(3)}}{u_3^2}$$

$$R_4 = -\frac{k^{(3)}}{u_3^2} - \frac{c_bM^{(3)}a_2a_3}{u_3^2} + \frac{c_bP^{(3)}\ (a_3+a_2)}{u_3^2} - \frac{c_bQ^{(3)}}{u_3^2}$$

$$R_5 = \frac{k^{(3)}}{u_3^2} + \frac{c_bM^{(3)}a_2^2}{u_3^2} - \frac{2a_2c_bP^{(3)}}{u_3^2} + \frac{c_bQ^{(3)}}{u_3^2} + \frac{k^{(4)}}{u_4^2}$$

$$+ \frac{c_bM^{(4)}\ a_4^2}{u_4^2} - \frac{2c_bP^{(4)}a_4}{u_4^2} + \frac{c_bQ^{(4)}}{u_4^2}$$

$$D_1 = h\theta_a - LE \quad , \quad D_2 = \frac{S^{(2)}a_2}{u_2} - \frac{N^{(2)}}{u_2}$$

$$D_3 = \frac{N}{u_2} - \frac{S^{(2)}a_1}{u_2} + \frac{S^{(3)}a_3}{u_3} - \frac{N^{(3)}}{u_3}$$

$$D_4 = \frac{-S^{(3)}a_4}{u_3} + \frac{N^{(3)}}{u_3} + \frac{S^{(4)}a_4}{u_4}$$

$$F_1 = \frac{c_bM^{(3)}a^3}{u_3} - \frac{c_bP^{(3)}}{u_3}$$

$$F_2 = \frac{c_bP^{(3)}}{u^3} - \frac{c_bM^{(3)}a_2}{u_3} + \frac{K^{(4)}}{u_4^2} + \frac{c_bQ^{(4)}}{u_4^2} + \frac{c_bM^{(4)}a_4^2}{u_4^2} - \frac{2c_bP^{(4)}a_4}{u_4^2}$$

Solving the above system of algebraic equations (6) we have

$$\theta_0 = \frac{D_1G_1 + R_1R_2C_1 [R_5D_3 - R_4D_4 + V_1\theta_b] + R_1V_3V_4}{C_1G_1} \tag{7}$$

$$\theta_1 = \frac{R_2C_1 [R_5D_3 - R_4D_4 + V_1\theta_b] + V_3V_4}{G_1} \tag{8}$$

$$\theta_2 = \frac{V_2 [R_5D_3 - R_4D_4 + V_1\theta_b] + V_4R_2R_5}{G_1} \tag{9}$$

$$\theta_3 = \frac{(D_4 + F_2\theta_b) G_1 - R_4V_2 [R_5D_3 - R_4D_4 + V_1\theta_b] + R_4V_4R_2R_5}{G_1R_5} \tag{10}$$

where

$C_1 = R_1 + h,$ $\quad V_1 = R_5F_1 - R_4 F_2$

$V_2 = R_2C_1 + hR_1 ,$ $\quad V_3 = R_3R_5 - R_4^2$

$V_4 = D_2C_1 + D_1R_1 ,$ $\quad G_1 = V_2V_3 - R_2^2R_5C_1$

3. A SPECIAL CASE

Assuming that the variation of $K_i(x)$, $M_i(x)$ and $S_i(x)$ is insignificant with respect to x and taking

$K_i (x) = \mathbf{K}_i$ (constant) for $i = 1 (1) 4$

$M_i (x) = \mathbf{M}_i$ (constant) for $i = 1 (1) 4$

$S_i (x) = \mathbf{S}_i$ (constant) for $i = 1 (1) 4$

We have

$$\theta_0 = \frac{W_1m^2 + W_2m + b_{14}}{W_3m^2 + W_4m + b_{19}} \tag{11}$$

where

$$W_1 = \lambda^2 u_3^2 b_{10} + \lambda u_3 u_4 b_{11}$$

$$W_2 = \lambda u_3 b_{12} + u_4 b_{13}$$

$$W_3 = \lambda^2 u_3^2 b_{15} + \lambda u_3 u_4 b_{16}$$

$$W_4 = \lambda u_3 b_{17} + u_4 b_{18}$$

$$\mathbf{M} = \lambda \mathbf{M}_4 \,, \; M_3 c_b = \lambda \mathbf{M}_4 c_b = \lambda m$$

$$m = \mathbf{M} c_b$$

$$b_1 = \frac{(\mathbf{S}_3 u_3 - \mathbf{S}_4 u_4)}{12} G_2 \quad , \quad b_2 = \frac{\mathbf{S}_3 u_3 G_2}{6}$$

$$b_3 = \frac{\mathbf{S}_3 u_3 G_2 (2Y_3 + Y_4) + \mathbf{S}_4 u_4 G_2 Y_3}{2}$$

$$b_4 = R_1 \left(\frac{R_2}{3} + Y_3 + \frac{Y_4}{3} \right) V_4 \,, \quad b_5 = \left(\frac{R_2}{3} + \frac{Y_3}{3} \right) R_1 V_4$$

$$b_6 = R_1 R_2 (Y + Y_4) V_4 + R_1 y_3 Y_4 V_4$$

$$b_7 = \left\{ V_2 \left(\frac{R_2}{3} + \frac{Y_3}{3} + \frac{Y_4}{3} \right) - \frac{R_2^2 C_1}{3} \right\} D_1$$

$$b_8 = \left\{ V_2 \left(\frac{R_2}{3} + \frac{Y_3}{3} \right) - \frac{R_2^2 C_1}{3} \right\} D_1$$

$$b_9 = D_1 R_2 (Y_3 + Y_4)(V_2 - R_2 C_1) + D_1 V_2 Y_3 Y_4$$

$$b_{10} = \frac{R_1 V_4 + D_1 V_2 + G_2 \theta_b}{12} \,, \qquad b_{11} = \frac{R_1 V_4 + D_1 V_2 + G_2 \theta_b}{9}$$

$$b_{12} = b_1 + b_4 + b_7 + \left\{ \frac{D_2}{3} + \left(Y_3 + \frac{Y_4}{3} \right) \theta_b \right\} G_2$$

$$b_{13} = b_2 + b_5 + b_8 + G_2 \left(\frac{D_2}{3} + Y_3 \theta_b \right)$$

$$b_{14} = b_3 + b_6 + b_9 + G_2 \{ D_2 (Y_3 + Y_4) + \theta_b Y_3 Y_4 \}$$

$$b_{15} = \frac{C_1 V_2}{12} \quad , \quad b_{16} = \frac{C_1 V_2}{9}$$

$$b_{17} = C_1 \left\{ V_2 \left(\frac{R_2}{3} + Y_3 + \frac{Y_4}{3} \right) - \frac{R_2^2 C_1}{3} \right\}$$

$$b_{18} = C_1 \left\{ V_2 \left(\frac{R_2}{3} + \frac{Y_3}{3} \right) - \frac{R_2^2 C_1}{3} \right\}$$

$$b_{19} = C_1 R_2 (Y_3 + Y_4) (V_2 - R_2 C_1) + C_1 V_2 Y_3 Y_4$$

$$G_2 = R_1 R_2 C_1 \quad , \quad Y_3 = \frac{K_3}{u_3}$$

$$Y_4 = \frac{K_4}{u_4}$$

We use following values for numerical calculations :

$a_0 = 0.0$, $a_1 = 0.1$, $a_2 = 0.2$, $a_3 = 0.6$, $a_4 = 1.1$ cm.

$K_1 = 0.030$ cal / cm–min deg C

$K_2 = 0\ 030$ cal / cm–min deg C

$K_3 = 0.045$ cal / cm–min deg C

$K_4 = 0.060$ cal / cm–min deg C

$h = 0.009$ cal / cm^2–min deg C

$L = 579$ cal / gm

$\theta_a = 23^0C$

$E = 0.24, 0.48$ and 0.72×10^{-3} gm / cm^2–min

$\theta_b = 27^0C$

$s = 0.018$ cal / cm^3-min

$S_1 = 0,\ S_2 = 0.0012,\ S_3 = 0.0102$ and $S_4 = 0.018$ cal/cm^3–min.

$m = M_4 c_b = 0.018$ cal / cm^3–min deg C .

4. CONCLUSION

The graphs have been plotted between outer skin surface temperature θ_0 and blood flow rate in dermis and subcutaneous tissues for different values of evaporation at the skin surface . The lower curve ($\lambda = 0$) gives an idea of variation in θ_0 when there is negligible blood flow in dermis due to construction of capillaries . The second curve ($\lambda = \frac{1}{2}$) indicates change in θ_0 when the blood flow rate is half of that in subcutaneous tissues . While the upper most curve ($\lambda = 1$) indicates variation in θ_0 when there is cent percent transfer of blood to this middle layer .

As shown in Fig. 1 all the curves are close to each other, but the gap between the curves [see Fig. 2,3] increases with the increase in rate of evaporation at the skin surface . In all the three figures, the curves diverge from a point at $m = 0$. This divergence seems to reverse for $m = 0.02$ in Fig. 1, for $m = 0.025$ in Fig. 2 and for $m = 0.0475$ in Fig. 3 (which is not visible in the figure) .

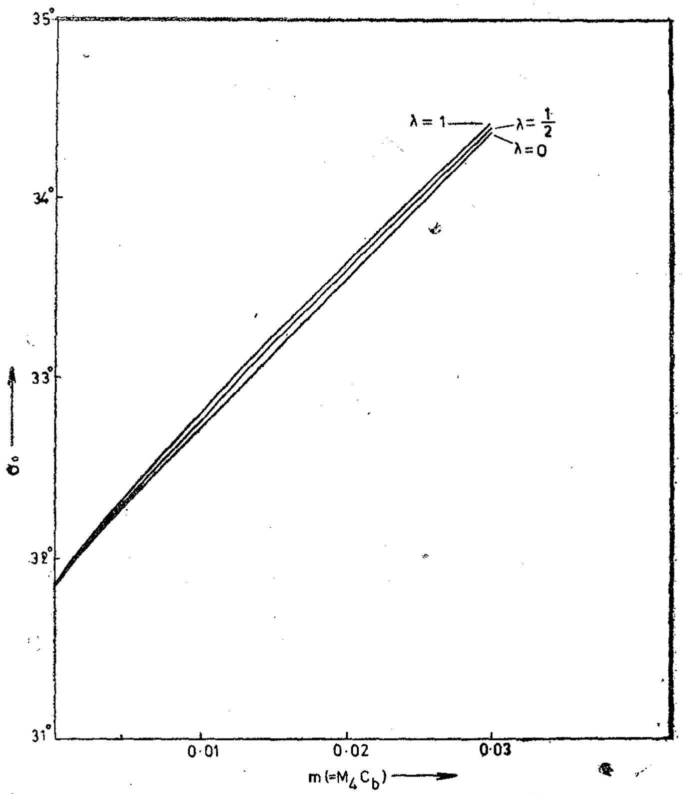

Fig.1:– Graph between θ_0 and m for $\lambda = 0, \frac{1}{2}$ and 1 ($\lambda = \frac{M_3}{M_4}$) and

$\epsilon = 0\ 24 \times 10^{-3}$ gm/cm^2-min

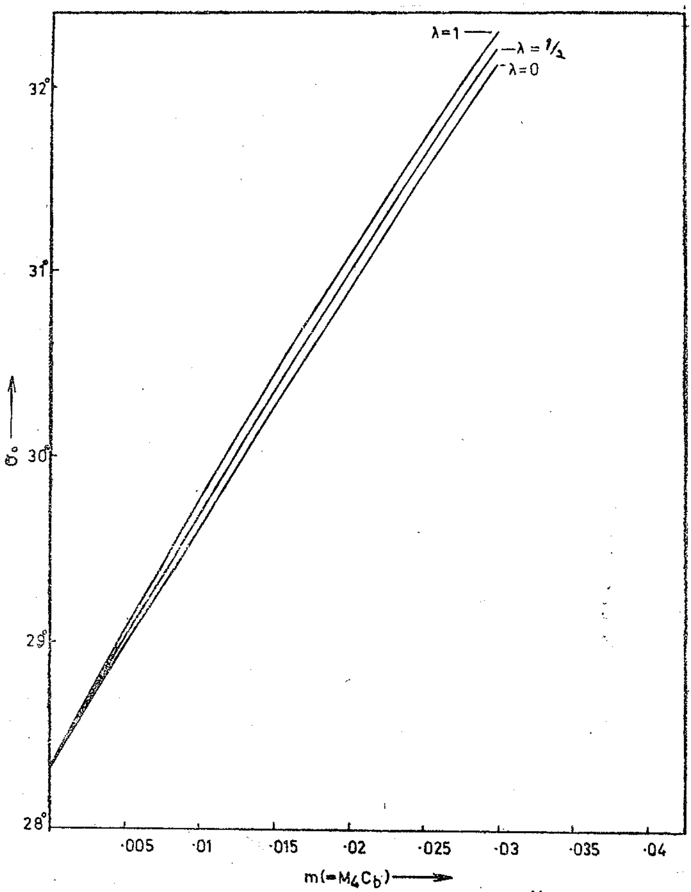

Fig. 2:—Graph between σ_0 and m for $\lambda = 0, \frac{1}{2}$ and 1 $(\lambda = \frac{M_3}{M_4})$ and

$E = 0 \cdot 48 \times 10^{-3}$ gm/cm^2-min

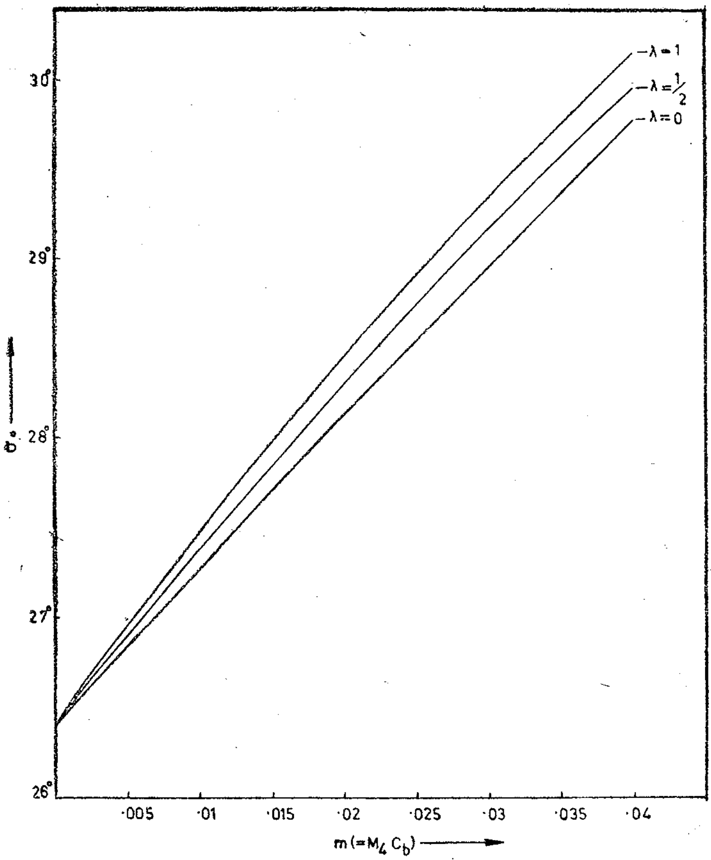

Fig 3 :— Graph between σ_0 and m for $\lambda = 0$, $\frac{1}{2}$ and 1 ($\lambda = \frac{M_3}{M_4}$) and $E = 0 \cdot 72 \times 10^{-3}$ gm/cm^2 min

From the above observations we make three conclusions :

1. For higher values of m change in blood flow doesnot have much effect on the skin surface temperature which is in conformity with the results derived by Saxena [3] .

2. At higher rates of evaporation, the contribution of heat transport due to blood flow is more . This is because of increase in temperature gradient due to increased cooling effect at the surface .

3. At zero blood flow rates the heat conduction plays an important role which is in conformity with the tneory used by Perl [1] .

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