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EFFECT OF SUSPENDED PARTICLES ON THE ONSET OF BÉNARD CONVECTION IN HYDROMAGNETICS THROUGH POROUS MEDIUM

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ABSTRACT

The effect of suspended particles on the onset of Bénard convection is studied in the presence of a uniform vertical magnetic field through porous medium. It is shown that the effects of suspended particles and medium permeability are destabilizing.

1. INTRODUCTION

A detailed account of the thermal instability of fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given in a treatise by Chandrasekhar [2]. Chandra [1] found that the instability depended on the depth of the layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed, if the layer depth is more than 10 mm. Chandra [1] observed that a convection which was different in character from that in deeper layers occurred at much lower gradients than predicted, if the layer depth was less than 7 mm and called this motion columnar instability. He added an aerosol to mark the flow pattern. Motivated by interest in fluid particle mixtures and columnar instability, Scanlon and Segel [5] considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh - number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles .

The medium has been considered to be nonporous in the abovs studies . Lapwood [3] studied the stability of convective flow in hydrodynamics in a porous medium using Rayleigh's procedure . Wooding [6] considered the Rayleigh instability of a thermal boundary layer in flow through a porous medium . When a fluid slowly percolates through the pores of the rock, the gross effcct is represented by Darcy's law which states that the usual viscous term in the equations of fluid motion is replaced by the resistence term $\left(\frac{\mu}{K_1,} \right) \vec{V}$, where μ is the viscosity of the fluid, k_1 the permeability of the medium and $\vec{V}$ is the velocity of the fluid .

The thermal instability of fluids through porous medium in the presence of suspended particles may find applications in various problems particularlarly in biomechanics, geophysics and chemical technology . It is therefore the motivation of this study to examine the effects of suspended particles on the onset of Bénard convection in hydromagnetics through porous medium .

2. FORMULATION OF THE PROBLEM

Consider an infinite horizontal gas-particle layer of thickness d bounded by the planes $z=0$ and $z=d$ and flowing through porous medium . This layer is heated from below such that a steady adverse temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained . The gas is assumed to be infinitely conducting . The system is acted on by a uniform vertical magnetic field $\vec{H}$ (0, 0, H) and a gravity force $\vec{g}$ (0, 0, -g) . The particles are assumed to be non-conducting.

The relevant equations and boundary conditions in their non-dimensional form governing the infinitesimal linear instability of the above physical system are given as follows :

$$(D^2-k^2-Bn) \left[L_1-L_2 (D^2-k^2) + \frac{L_2}{p} - L_2 Q D^2 \right] (D^2-k^2) W$$

$$= R (z_0 n + B) k^2 W , \qquad (1)$$

$$W = \frac{\partial^2 W}{\partial z^2} = \left(L_1 - L_2 \nabla^2 + \frac{L_2}{P} \right) \nabla^2 W - L_2 Q \nabla^2 \frac{\partial^2 W}{\partial z^2} = 0 ,$$

$$\text{at } z = 0 \text{ and } z = 1 \qquad (2)$$

where Q is the Chandrasekhar number, p is the permeability of the medium and various other symbols occuring in the above equations have the same meaning as in [5] .

3. DISPERSION RELATION AND DISCUSSION

The principle of the exchange of stabilities is valid for this problem as can be easily demonstrated by the usual Pellew and Southwell [4] technique . The marginal state is thus characterized by $n = 0$ and equation (1) and boundary conditions (2), then become

$$(D^2-k^2)^2 \left[D^2-k^2 + QD^2 - \frac{1}{P} \right] W = - k^2 R_C B W , \qquad (3)$$

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = 0 \text{ , at } z=0 \text{ and } z=1 \text{ .} \tag{4}$$

where R_C is the critical Rayleigh number . It can be shown that all the even derivatives of W vanish on the boundaries and characterzi_g the lowest mods is

$$W = A \; Sin \, \pi z \text{ ,} \tag{5}$$

where A is a constant .

Substituting the solution (5) in equation (3) , we obtrin

$$R_C = \frac{(\pi^2 + k^2)^2 \left(\pi^2 + k^2 + \frac{1}{P} + Q\pi^2 \right)}{k^2 (b+1)} \text{ .} \tag{6}$$

It is clear from equation (6) that the magnetic field has an inhibitng effect on the onset of Bénard convection . If R_C is critical Rayleigh number i_ the presence of suspended particles and $\overline{R_C}$ is the critical Rayleigh number in the absence of suspended particles, then

$$R_C = \frac{\overline{R_C}}{b+1} \text{ ,} \tag{7}$$

which shows that the effect of suspended particles is to destabilize the layer .

It follows from equation (6) that

$$\frac{dR_C}{db} = -\frac{(\pi^2 + k^2)^2 \left(\pi^2 + k^2 + \frac{1}{P} + Q\pi^2\right)}{k^2 (b + 1)^2}, \quad (8)$$

which is negative. This re-affirms the destabilising effect of suspended particles.

To find the role of permeability of the medium, we examine the nature of $\frac{dR_C}{dP}$.

It follows from equation (6) that

$$\frac{dR_C}{dP} = -\frac{(\pi^2 + k^2)^2}{k^2 (b+1) P^2}, \quad (9)$$

which is always negative. The effect of medium permeability is, therefore, to destabilize the layer.

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