Jñānābha, Vol. 17, 1987

A RECURRENCE RELATION FOR THE *H*-FUNCTION OF SEVERAL COMPLEX VARIABLES

By

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(*Received* : *August 12, 1982* ; *Revised* [*Final*] : *May 22, 1987*)

ABSTRACT

The aim of the present note is to establish a recurrence relation for the *H*-function of several complex variables due to H. M. Srivastava and R. Panda (see [3] , [4] , and [5]) .

1. INTRODUCTION AND THE MAIN RESULTS

For the *H*-function of several complex variables defined by Srivastava and Panda ([4] and [5] ; see also [3], *p.* 251), we derive the recurrence relation :

$$(1)\quad H^{0,\epsilon:(u',v');\,..\,;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\,..\,;[B^{(r)},D^{(r)}]}\begin{pmatrix} z_1\\ \vdots\\ z_r\end{pmatrix}$$

$$= \frac{1}{2\pi i}\left\{ e^{i\pi a_{\epsilon+1}} H^{0,\epsilon+1:(u',v');\,..\,;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\,..\,;[B^{(r)},D^{(r)}]} \begin{pmatrix} z_1\, e^{-i\pi\,\theta'_{\epsilon+1}}\\ \vdots\\ z_r\, e^{-i\pi\,\theta^{(r)}_{\epsilon+1}}\end{pmatrix}\right.$$

$$\left. - e^{-i\pi a_{\epsilon+1}} H^{0,\epsilon+1:(u',v');\,..\,;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\,..\,;[B^{(r)},D^{(r)}]} \begin{pmatrix} z_1 e^{i\pi\theta'_{\epsilon+1}}\\ \vdots\\ z e^{i\pi\theta^{(r)}_{\epsilon+1}}\end{pmatrix}\right\},$$

where $i=\sqrt{-1}$, and the usual existence conditions for each H-function are assumed to be satisfied .

2. PROOF OF THE RESULT (1)

The recurrence relation (1) can be easily established by appealing to the definition ([4], *p*. 271, *Eq*. (4.1) ; [3], *p*. 251, *Eq*. (C.1) and the well-known relation (Rainville [2], *p*. 21, Theorem 8) :

$$(2)\quad \Gamma(z)\,\Gamma(1-z)=\frac{\pi}{\sin \pi z}=\frac{2\pi i}{e^{i\pi z}-e^{i\pi z}}$$

involving Γ-functions .

3. PARTICULAR CASES

For $r=2$, the result (1) reduces to a recurrence relation for the H-function of two variables obtained in [1] .

Several other interesting special cases of (1) (both known and unknown) can be obtained for simpler functions of one, two and more variables .

ACKNOWLEDGEMENTS

The authors are thankful to Professor H. M. Srivastava of the University of Victoria, Victoria, Canada, and also to Professor M. C. Gupta, for their kind help and suggestions in the preparation of this paper .

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