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SOME RESULTS ON FIXED POINTS FOR THREE MAPS

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In this note we present some results on fixed point theorem for three maps which generalize the results of Park and Rhoades [4] and Park [3] respectively. First we give the following definitions and notations.

Throughout the paper w stands for the set of all nonnegative integers and R_+ for the set of all non-negative real numbers.

A point $x_0 \varepsilon X$ is called regular for f, g and h, or, simply regular, if there exists a sequence $\{ x_n \} \subset X$ satisfying, $hx_{2n+1} = fx_{2n}$ and $hx_{2n+2} = gx_{2n+1}$ for each $n \varepsilon w$, and $\sup \{ d(hx_i, hx_j) \mid i, j \varepsilon w \} < \infty$. It should be noted that for $f(X) \cup g(X) \subseteq h(X)$, such a $\{x_n\}$ always exists.

Let $O(x_0) = \{ hx_n \mid n \varepsilon w \}$, and $\delta [O(x_0)]$ denote the diameter of $O(x_0)$.

Let $\delta(x,y) =$ diameter $\{ O(x) \cup O(u) \}$ where x and y are regular We have the following results :

Theorem 1. *Let* f, g, h *be self-maps of a metric space* (X,d) *such that* fh = hf, gh = hg, $\phi : R_+ \longrightarrow R_+$, ϕ *non-decreasing, continuous from the right, and satisfying* $\phi(t) < t$ *for each* t $\geqslant 0$. *Suppose there* *exists a regular point* $x_0 \in X$ *such that* $\{hx_n\}$ *has a cluster point* $a_0 \in X$, *which is regular.*

If

(1) $d(fx, gy) \leqslant \phi(\delta[O(x) \cup O(y)])$ *for each* $x, y \in \{x_n\} \cup \{a_n\} \cup ha_0$, *where* $\{a_n\}$ *is defined by* $ha_{2n+1} = fa_{2n}$ *and* $ha_{2n+2} = ga_{2n+1}$ *for* $n \in w$, *and*
(2) h *is continuous at* a_0,
then ha_0 *is a common fixed point for* f, g, h *and* $\{hx_n\}$ *converges to* a_0. *If* (1) *is satisfied for all regular points* $x, y \in X$, *then* ha_0 *is the unique common fixed point of* f, g and h.

The result can be easily proved by coupling the proof for two maps by Park and Rhoades [4] with the usual technique for the proof of fixed point for theree maps, given by Ganguly([1],[2]) and Singh[5].

Theorem 2. *Let* f, g, h *be self maps of a complete metric space* (X, d) *such that* $fh = hf$, $gh = hg$, h *is continuous, and* $\phi : R_+ \to R_+$, $\phi(t) < t$ *for each* $t > 0$. *If every point of* X *is a regular point and* (1) *is satisfied for all* $x, y \in X$, *then* f, g and h *have a unique common fixed point and* $\{hx_n\}$ *converges to the fixed point for each* $x \in X$.

The result follows from Theorem 1.

Remark 1. For $f = g$, we have Theorems 1 and 2 of Park and Rhoades [4].

Remark 2. From the logic of Park and Rhoades ([4], pp. 116–117) it follows that Theorem 1 of Canguly [1] is a special case of Theorems 2 and 1, respectively.

Now we state a different result which can also be easily proved.

Theorem 3. *Let* (X, d) *be a metric space,* f, g *and* h *be the same as above. Suppose there exists a regular point* $x_0 \in X$ *such that*

(1) $O(x_0$ *has a cluster point* $a_0 \in X$ *which is regular;*

(2) *for any* $\epsilon > 0$, *there exist* $\epsilon_0 < \epsilon$ *and* $\delta_0 > 0$ *such that for any* $x, y \in \{x_n\} \cup \{a_n\} \cup \{ha_0\}$,

$\epsilon \leqslant \delta(x, y) < \epsilon + \delta_0$ *implies* $d(fx, gy) \leqslant \epsilon_0$, *where* $\{a_n\}$ *is the same as in Theorem* 1, *and*

(3) *if* ha_0 *is continuous at* a_0,

then ha_c *is a common fixed point of* f, g *and* h, *and* $\{hx_n\}$ *converges to* a_0. *If* (2) *is satisfied for all regular points* $x, y \in X$, *then* ha_0 *is the unique common fixed point of* f, g *and* h.

Remark 3. For $f=g$, we have Theorem 2 (C δ') of Park [3].

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REFERENCES

[1] A. Ganguly, On an extension of the fixed point theorem of Kasahara, *Math. Sem.* Notes *Kôbe Univ.* **10** (1982), 307–310,

[2] A. Ganguly, On an extension of a fixed point theorem of Meir and Keeler for three mappings, Jñānābha **14** (1984), 157–161.

[3] S. Park, On general codtractive type conditions, *J Korean Math. Soc.* **17** (1980), 131–140.

[4] S. Park, and B. E. Rhoades, Extension of some fixed point theorems of Hegedus and Kasahara, *Math. Sem. Notes Kôbe Univ.* **9** (1981), 113–118.

[5] S L. Singh, On common fixed point theorems in *L*–Space, *Math. Sem. Notes Kôbe Univ.* **7** (1979), 91–97.