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# SOME DOUBLE INTEGRAL RELATIONS INVOLVING THE $H$-FUNCTION OF SEVERAL COMPLEX VARIABLES

*By*

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The object of the present paper is first to give certain double integral relations involving the $H$-function of several complex variables due to H. M. Srivastava and R. Panda (See [4] and [5] ), and the nemploy these relations to evaluate certain double integrals involving the product of the (Fox's) $H$-function and the $H$-function of several complex variables of general arguments. Several possible applications provided by these results (and their various special cases) when viewed as two-dimensional integral transformations are also indicated briefly.

## INTRODUCTION AND THE MAIN RESULTS

For the $H$-function of several complex variables defined by Srivastava and Panda ([4]; see also [5], p 251) we derive the following double integral transformations :

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+x^v)^\sigma} \cos(2w \tan^{-1} y/x)$$

$$H^{m_1, n_1}_{p_1, q_1}\left[\frac{a_1 x^{2h}}{(x^2+y^2)^h} \middle| \begin{matrix} (\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1}) \end{matrix}\right]$$

$$.H_1 \begin{bmatrix} z_1\, x^{2\rho}\, (x^2+y^2)^{k_1-\rho} \\ z_2\, (x^2+y^2)^{k_2} \\ \vdots \\ z_r\, (x^2+y^2)^{k_r} \end{bmatrix} f(x^2+y^2)\, dx\, dy$$

$$= \frac{\pi}{2^{2\sigma+2}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s\, \phi(\eta_s)\, \alpha_1^{\eta_s}}{s!\, f_g'\, 2^{2h\eta_s}}$$

$$\int_0^\infty H^{0,\,0\,:\,(u',\,v'+1)\,;\,(u'',\,v'')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{A,\,C:\,[B'+1,\,D'+2]\,;\,[B'',\,D'']\,;\ldots;\,[B^{(r)},\,D^{(r)}]}$$

$$\left( \begin{matrix} [(a):\theta',\ldots,\theta^{(r)}]:[-2\sigma-2h\eta_s:2\rho]\,;\,[(b'):\phi']\,;\ldots;\,[(b^{(r)}):\phi^{(r)}]; \\ [(c):\Psi',\ldots,\Psi^{(r)}]:[(d'):\delta'];[-\sigma-h\eta_s\pm w:\rho];[(d''):\delta''];\ldots;(d^{(r)}):\delta^{(r)}]; \end{matrix} \right.$$

$$\left. z_1\, t^{-k_1}\, 4^{-\rho},\, z_2\, t^{k_2},\ldots,\, z_r\, t^{k_r} \right) f(t)\, dt. \tag{1.1}$$

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+y^2)^\sigma} \cos\,(2\, w\, \tan^{-1} y/x)$$

$$H^{m_1,\,n_1}_{p_1,\,q_1}\left[ \frac{\alpha_1\, x^{2h}}{(x^2+y^2)^h} \,\middle|\, \begin{matrix} (\alpha'_{p_1},\, e'_{p_1}) \\ (\beta'_{q_1},\, f'_{q_1}) \end{matrix} \right]$$

$$H_1 \begin{pmatrix} z_1\,(x^2+y^2)^{k_1-\rho}\, x^{-2\rho} \\ z_2\,(x^2+y^2)^{k_2} \\ \vdots \\ z_r\,(x^2+y^2)^{k_r} \end{pmatrix} f(x^2+y^2)\, dx\, dy =$$

$$= \frac{\pi}{2^{2\sigma+2}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s\, \phi(\eta_s)\, \alpha_1^{\eta_s}}{s!\, f'_g\, 2^{2h\eta_s}}$$

$$\int_0^\infty H^{0'\,0\,:\,(u'+1,\,v')\,;\,(u'',v'')\,;\,\ldots;\,(u^{(r)},\,v^{(r)})}_{A,\,C\,:\,[B'+2,\,D'+1]\,;\,[B'',\,D'']\,;\ldots;\,[B^{(r)},\,D^{(r)}]}$$

$$\begin{pmatrix} [(a):\theta',\dots,\theta^{(r)}] : [(b'):\phi'] : [1+\sigma+h\eta_s \pm w : \rho] ; \\ [(c):\Psi',\dots,\Psi^{(r)}] : [1+2\sigma+2h\eta_s : 2\rho] ; \end{pmatrix}$$

$$\left. \begin{matrix} [(b''):\phi''] ;\dots; [(b^{(r)}):\phi^{(r)}] ; \\ [(d'):\delta'] ;\dots; [(d^{(r)}) ; \delta^{(r)}] ; \end{matrix} \; z_1\, t^{k_1} 4^{\rho}, z_2\, t^{k_2}, \dots, z_r\, t^{k_r} \right) f(t)\, dt. \tag{1.2}$$

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+y^2)^\sigma} \cos\,(2w \tan^{-1} y/x)$$

$$H^{m_1, n_1}_{p_1, q_1} \left[ \frac{\alpha_1 x^{2h}}{(x^2+y^2)^h} \,\middle|\, \begin{matrix} (\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1}) \end{matrix} \right]$$

$$H_1 \begin{pmatrix} z_1\, x^{2\rho_1} (x^2+y^2)^{k_1-\rho_1} \\ \vdots \\ z_r\, x^{2\rho_r} (x^2+y^2)^{k_r-\rho_r} \end{pmatrix} f\,(x^2+y^2)\, dx\, dy$$

$$= \frac{\pi}{2^{2\sigma+2}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \frac{\phi\;(\eta_s)\, \alpha_1^{\eta_s}}{f'_g\; 2^{2h\eta_s}}$$

$$\int_0^\infty H^{0,\;1\; : (u', v') ;\dots; (u^{(r)}, v^{(r)})}_{A+1, C+2 : [B', D'] ;\dots; [B^{(r)}, D^{(r)}]}$$

$$\begin{pmatrix} [-2\sigma-2h\eta_s : 2 : 2\rho_1, \dots, 2\rho_r] ; [(a):\theta',\dots,\theta^{(r)}] : \\ [-\sigma \pm w - h\eta_s : \rho_1, \dots, \rho_r] ; [(c):\Psi',\dots,\Psi^{(r)}] : \end{pmatrix}$$

$$\left. \begin{matrix} [(b'):\phi'] ;\dots; [(b^{(r)}):\phi^{(r)}] , \\ [d'):\delta'] ;\dots; [(d^{(r)}):\delta^{(r)}] ; \end{matrix} \; z_1\, t^{k_1} 4^{-\rho_1}, \dots, z_r\, t^{k_r} 4^{-\rho_r} \right)$$

$$f\,(t)\, dt. \tag{1.3}$$

In (1.1) to (1.3), $w = 0, 1, 2,$ , $\sigma > 0$, $\alpha_1 > 0$, $h > 0$, $k_i > 0$, $i > 0$, $\forall i \varepsilon \{1,\dots,r\}$ ; and $f(z)$ is so chosen that the various integrals involved exist.

## 2. PROOF OF (1.1)

To establish (1.1), we start with the following integral formula :

$$\int_0^{\pi/2} \cos 2w\theta \, (\cos\theta)^{2\sigma} \, H^{m_1, n_1}_{p_1, q_1}\left[ \alpha_1 (\cos\theta)^{2h} \,\middle|\, \begin{matrix} (\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1}) \end{matrix} \right]$$

$$\cdot H_1 \begin{pmatrix} z_1 u_1^{k_1} (\cos)^{2\rho} \\ z_2 u_2^{k_2} \\ \vdots \\ z_r u_r^{k_r} \end{pmatrix} d\theta =$$

$$= \frac{\pi}{2^{2\sigma+1}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s \phi(\eta_s) \, \alpha_1^{\eta_s}}{s! \, f'_g \, 2^{2h\eta_s}}$$

$$\cdot H^{0,\,0\,:\,(u',\, v'+1)\,;\,(u'',\, v'')\,;\ldots;\,(u^{(r)},\, v^{(r)})}_{A,\,C\,:\,[B+1,\, D'+2]\,:\,[B'',\, D'']\,;\ldots;\,[B^{(r)},\, D^{(r)}]}$$

$$\left( \begin{matrix} [(a) : \theta', \ldots, \theta^{(r)}] : [-2\sigma - h\eta_s : 2\rho]; \\ [(c) : \Psi', \ldots, \Psi^{(r)}] : [(d') : \delta']; [-\sigma - h\eta_s \pm w : \rho]; \end{matrix} \right.$$

$$\left. \begin{matrix} [(b') : \phi'] ;\ldots; [(b^{(r)}), \phi^{(r)}] : \\ [(d''), \delta''] ;\ldots; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix} \; z_1 u_1^{k_1} 4^{-\rho}, z_2 u_2^{k_2}, \ldots, z_r u_r^{k_r} \right) \quad (2.1)$$

provided that $w = 0, 1, 2, \ldots$, $h > 0$, $\alpha_1 > 0$, $\sigma > 0$, $\rho > 0$, $k_i > 0$, $\forall\, i \in \{1, \ldots, r\}$ ;

$Re\,[\sigma + h\,\beta_j'/f_j' + d_j'/\delta_j' + \frac{1}{2}] > 0$, $|\arg \alpha_1| < \frac{1}{2}\pi T$,
$1 \leqq j \leqq m_1$, $1 \leqq j \leqq u'$

$$T = \sum_1^{n_1} (e_j') - \sum_{n_1+1}^{p_1} (e_j') + \sum_1^{m_1} (f_j') - \sum_{m_1+1}^{q_1} (f_j') > 0,$$

[ The result (2.1) can be easily established by using series representation for Fox's $H$-function [3] and appealing to the definition ( [4], p. 271, eq. (4.1) ) and a konwn result ( [1], p. 16, eq. (5) ) ].

Now, in (2.1) we replace $u_i \forall_i \in \{1, \ldots, r\}$ by $u^2$, multiply both the sides of the resulting equation by $uf(u^2)$, integrate it with respect to $u$ between the limits $0$ to $\infty$, interchange the order of integration and summations in the right hand side, to get

$$\int_0^\infty u\, f(u^2) \left\{ \int_0^{\pi/2} \cos(2w\theta) \cos^{2\sigma}\theta\, H_{p_1, q_1}^{m_1, n_1}\left[ \alpha_1 (\cos\theta)^{2h} \middle| \begin{matrix} (\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1}) \end{matrix} \right] \right.$$

$$\left. . \, H_1 \begin{pmatrix} z_1 \, u^{2k_1} (\cos\theta)^{2\rho} \\ z_2 \, u^{2k_2} \\ \vdots \\ z_r \, u^{2k_r} \end{pmatrix} d\theta \right\} du$$

$$= \frac{\pi}{2^{2\sigma+1}} \sum_{g=0}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s \phi(\eta_s)\, \alpha_1^{\eta_s}}{s!\, f'_g\, 2^{2h\eta_s}}$$

$$\int_0^\infty u\, H_{A, C : [B'+1, D'+2]; [B'', D'']; \ldots; [B^{(r)}, D^{(r)}]}^{0, 0 : (u', v'+1); (u'', v''); \ldots; (u^{(r)}, v^{(r)})}$$

$$\begin{pmatrix} [(a) : \theta, \ldots, \theta^{(r)}] : [-2\sigma - 2h\eta_s ; 2\rho] ; \\ [(c) : \Psi', \ldots, \Psi^{(r)}] : [(d') : \delta'] ; [-\sigma - h\eta_s \pm w : \rho] ; \end{pmatrix}$$

$$\begin{matrix} [(b') ; \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; \\ [(d'') : \delta''] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix} \quad z_1 u^{2k_1} 4^{-\rho},\ z_2 u^{2k_2}, \ldots,$$

$$z_r u^{2k_r} \Big) f(u^2)\, du\,, \tag{2.2}$$

where $f(u^2)$ is to be so chosen that the integrals involved in the above equation (2.2) exist.

On transforming the left-hand side of (2.2) into cartesian coordinates by the substitution $x = u \cos\theta$, $y = u \sin\theta$ and making a slight change of variables in the right hand side of (2.2), we easily arrive at (1.1).

The integral relations given by (1.2) and (1.3) can similarly be established.

### 3. Applications

If in the integral relationships (1.1) to (1 3), we set

$$f(t) = t^{\gamma-1} H_{p_2, q_2}^{m_2, 0}\left[\alpha_2 t^{\delta} \,\middle|\, \begin{matrix}(a''_{p_2}, e''_{p_2}) \\ (\beta''_{q_2}, f''_{q_2})\end{matrix}\right], \tag{3.1}$$

evaluate the $t$–integral by means of the known result [2, *p.* 122, *Eq.* (4.2)] rewritten in its equivalent form :

$$\int_0^{\infty} x^{\alpha-1} H_{p,q}^{m,0}\left[zx^{\delta} \,\middle|\, \begin{matrix}(A_p, \varepsilon_p) \\ (F_q, f_q)\end{matrix}\right] H_1\left[\begin{matrix} z_1 x^{h_1} \\ \vdots \\ z_r x^{h_r}\end{matrix}\right] dx$$

$$= (\delta z^{\alpha/\delta})^{-1} H_{A+q,\, C+p\, :\, [B',\, D'] ;\ldots; [B^{(r)},\, D^{(r)}]}^{0,\, m\, :\, (u',\, v') ;\ldots; (u^{(r)},\, v^{(r)})}$$

$$\left[\begin{matrix}[1-F_j-f_j\, a/\delta, f_j\, h_1/\delta, \ldots, f_j\, h_r/\delta]_{1,q} ; [(a) : \theta', \ldots, \theta^{(r)}] : \\ [1-A_j-\varepsilon_j\, \alpha/\delta, \varepsilon_j\, h_1/\delta, \ldots, \varepsilon_j\, h_r/\delta]_{1,p} ; [(c) : \Psi', \ldots, \Psi^{(r)}] ;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b') : \phi'] ;\ldots; [(b^{(r)}) ; \phi^{(r)}] ; z_1 z^{-h_1/\delta} \\ \vdots \\ [(d') : \delta'] ;\ldots; [(d^{(r)}) ; \delta^{(r)}] ; z_r z^{-h_r/\delta}\end{matrix}\right],$$

conditions of validity of (3.2) are given in [ 2, *p.*122 ] (3.2)

We obtain the following double integral relations :

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+y^2)^{\sigma-\gamma+1}} \cos(2w \tan^{-1} y/x)$$

$$.H_{p_1, q_1}^{m_1, n_1}\left[\frac{\alpha_1 x^{2h}}{(x^2+y^2)^h} \middle| \begin{matrix} (\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1}) \end{matrix}\right]$$

$$H_{p_2, q_2}^{m_2, 0}\left[\alpha_2 (x^2+y^2)^\delta \middle| \begin{matrix} (\alpha''_{p_2}, e''_{p_2}) \\ (\beta''_{q_2}, f''_{q_2}) \end{matrix}\right]$$

$$H_1\begin{pmatrix} z_1 x^{2\rho} (x^2+y^2)^{k_1-\rho} \\ z_2 (x^2+y^2)^{k_2} \\ \vdots \\ z_r (x^2+y^2)^{k_r} \end{pmatrix} dx\, dy$$

$$= \frac{\pi}{2^{2\sigma+2}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s \phi(\eta_s)\, \alpha_1^{\eta_s} (\delta\alpha_2^{\gamma/\delta})^{-1}}{s!\, f'_g\, 2^{2h\eta_s}}$$

$$H_{A+q_2,\, C+p_2:\, [B'+1,\, D'+2]\, :\, [B'',\, D'']\, ;\dots;\, [B^{(r)},\, D^{(r)}]}^{0,\, m_2\, :\, (u',\, v'+1);\, (u'',v'')\, ;\dots;\, (u^{(r)},\, v^{(r)})}$$

$$\left(\begin{matrix} [1-\beta_j''-f_j''\gamma/\delta : f_j'' k_1/\delta, \dots, f_j'' k_r/\delta]_{1,q_2} & ; [(a): \theta', \dots, \theta^{(r)}] : \\ [1-\alpha_j''-e_j''\gamma/\delta : e_j'' k_1/\delta, \dots, e_j'' k_r/\delta]_{1,p_2}; & (c): \Psi', \quad , \Psi^{(r)}] : \end{matrix}\right.$$

$$[-2\sigma-2h\eta_s : 2\rho]; \qquad [(b'):\phi'];\dots;[(b^{(r)}):\phi^{(r)}];$$

$$[(d'):\delta'];\ [-\sigma-h\eta_s \pm w:\rho];\ [(d''):\delta''];\dots;[(d^{(r)}):\delta^{(r)}];$$

$$\left. z_1 \alpha_2^{-k_1/\delta} 4^{-\rho},\ z_2 \alpha_2^{-k_2/\delta}, \dots,\ z_r \alpha_2^{-k_r/\delta} \right),$$

provided that $w=0, 1, 2, \dots, > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\sigma > 0$, $h > 0$,

$\rho > 0, \delta > 0; k_i > 0, \forall \; i \in \{1, \ldots, r\};$

(i) $\text{Re}\,[\gamma + \sum_{i=1}^{r} k_i \,(d_j^{(i)} \,/\, \delta_j^{(i)}) + \delta(\beta_j'' \,/\, f_j'')\,] >$

$\qquad 1 \leqq j \leqq u^{(i)} \qquad\qquad 1 \leqq j \leqq m_2$

$\text{Re}\,[\sigma + h(\beta_j' \,/\, f_j') + (d_j' \,/\, \delta_j') + \tfrac{1}{2}]\,;$

$\qquad 1 \leqq j \leqq m_1 \qquad\qquad 1 \leqq j \leqq u'$

(ii) $[\gamma + \sum_{i=1}^{r} k_i \,(b_j'^{(i)} - 1) \,/\, \phi_j^{(i)}] <$

$\qquad 1 \leqq j \leqq v^{(i)}$

$\text{Re}\,[\sigma + h\,(a_j' - 1) \,/\, e_j' + (b_j' - 1) \,/\, \phi_j' + \tfrac{1}{2}];$

$\qquad 1 \leqq j \leqq n_1 \qquad\qquad 1 \leqq j \leqq v'$

(iii) $|\arg a_1| < \tfrac{1}{2} T_1 \pi$, $|\arg a_2| < \tfrac{1}{2} T_2 \pi$, where

$$T_1 = \sum_{1}^{n_1} (e_j') - \sum_{n_1+1}^{p_1} (e_j') + \sum_{1}^{m_1} (f_j) - \sum_{m_1+1}^{q_1} (f_j') > 0,$$

$$T_2 = \sum_{1}^{p_2} (e_j'') - \sum_{1}^{m_2} (f_j'') + \sum_{m_2+1}^{q_2} (f_j'') > 0;$$

(iv) $m_1, n_1, p_1, q_1, m_2, p_2, q_2$ are integers such that

$0 \leqq n_1 \leqq p_1, 1 \leqq m_1 \leqq q_1, p_2 \geqq 0, 1 \leqq m_2 \leqslant q_2 : e_j' > 0\,;$

$j = 1, \ldots, p_1\,; f_j' > 0, j = 1, \ldots, q_1\,; e_j'' = 0, j = 1, \ldots, p_2,$

$f_j'' > 0, j = 1, \ldots, q_2.$

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+y^2)^{\sigma-\gamma+1}} \cos\,(2w \tan^{-1} y/x)$$

$$H\,{}^{m_1,\,n_1}_{p_1,\,q_1}\left[\frac{\alpha_1\, x^{2h}}{(x^2+y^2)^h}\,\middle|\,\begin{matrix}(\alpha'_{p_1}, e'_{p_1})\\(\beta'_{q_1}, f'_{q_1})\end{matrix}\right]$$

$$.\ H\,{}^{m_2,\,0}_{p_2,\,q_2}\left[\alpha_2(x^2+y^2)^\delta\,\middle|\,\begin{matrix}(\alpha''_{p_2}, e''_{p_2})\\(\beta''_{q_2}, f''_{q_2})\end{matrix}\right]$$

$$H_1\begin{pmatrix} z_1\, x^{-2\rho}\,(x^2+y^2)^{k_1+\rho} \\ z_2\,(x^2+y^2)^{k_2} \\ z_r\,(x^2+y^2)^{k_r}\end{pmatrix} dx\, dy$$

$$= \frac{\pi}{2^{2\sigma+1}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s\, \phi\,(\eta_s)\; a_1^{\eta_s}\,(\delta\, \alpha_2^{\gamma/\delta})^{-1}}{s!\, f'_g\; 2^{2h\eta_s}}$$

$$.\ H\,{}^{0,\, m_2\,:\,(u'+1,\, v')\,;\,(u'',\, v'')\,;\ldots;\,(u^{(r)},\, v^{(r)})}_{A+q_2\,C+p_2\,:\,[B+2,\, D'+1]\,:\,[B'',\, D'']\,;\ldots;\,[B^{(r)},\, D^{(r)}]}$$

$$\left(\begin{matrix}[1-\beta_j''-f_j''\,\gamma/\delta:\, f_j''\,k_1/\delta,\ldots,\, f_j''\,k_r/\delta]_{1,q_2} & :[\,(a)\,:\,\theta',\ldots,\,\theta^{(r)}]\,: \\ [1-\alpha''-e_j''\,\gamma/\delta:\, e''\,k_1/\delta\,\ldots,\, e_j''\,k_r/\delta]_{1\,p_2} & ;[(c)\,:\,\Psi',\ldots,\,\Psi^{(r)}]\,:\end{matrix}\right.$$

$$[\,-2\sigma-2h\eta_s\,:\,2\rho\,];\,[(b')\,:\,\phi'\,]\,;\ldots;\,[(b^{(r)})\,:\,\phi^{(r)}\,]\,,$$

$$[\,d')\,:\,\delta'];\,[-\sigma-h\eta_s \pm w\,:\,\rho];\,[(d'')\,:\,\delta'']\,;\ldots;\,[(d^{(r)})\,:\,\delta^{(r)}]\,;$$

$$\left. z_1\, 4^{-\rho}\, \alpha_2^{-k_1/\delta},\ z_2\, \alpha_2^{-k_2/\delta},\ldots,\, z_r\, \alpha_2^{-k_r/\delta} \right), \tag{3.4}$$

Suitable conditions of validity are easily obtainable from (1.2).

$$\int_0^\infty \int_0^\infty \frac{x^{2\sigma}}{(x^2+y^2)^{\sigma-\gamma+1}} \cos(2w \tan^{-1} y/x)$$

$$H_{p_1, q_1}^{m_1, n_1}\left[\frac{\alpha_1 x^{2h}}{(x^2+y^2)^h} \middle| \begin{matrix}(\alpha'_{p_1}, e'_{p_1}) \\ (\beta'_{q_1}, f'_{q_1})\end{matrix}\right]$$

$$. H_{p_2, q_2}^{m_2, 0}\left[\alpha_2 (x^2+y^2)^\delta \middle| \begin{matrix}(\alpha''_{p_2}, e''_{p_2}) \\ (\beta''_{q_2}, f''_{q_2})\end{matrix}\right]$$

$$. H_1\begin{pmatrix} z_1 \; x^{2\rho_1} \; (x^2+y^2)^{k_1-\rho_1} \\ \vdots \\ z_r \; x^{2\rho_r} \; (x^2+y^2)^{k_r-\rho_r}\end{pmatrix} dx\, dy$$

$$= \frac{\pi}{2^{2\sigma+2}} \sum_{g=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^s \phi(\eta_s)\, \alpha_1^{\eta_s} (\delta\, \alpha_2^{\gamma/\delta})^{-1}}{s!\, f'_g \, 2^{2h\eta_s}}$$

$$. H_{A+q_2+1,\, C+p_2+2 \,:\, [B', D'] ;\ldots; [B^{(r)}, D^{(r)}]}^{0,\, m_2+1 \,:\, (u', v') ;\ldots; (u^{(r)}, v^{(r)})}$$

$$\left(\begin{matrix}[1-B_j'' - f_j'' \gamma/\delta : f_j'' k_1/\delta, \ldots, f_j'' k_r/\delta]_{1,q_2} ; \\ [1-\alpha_j'' - e_j'' \gamma/\delta : e_j'' k_1/\delta, \ldots, e_j'' k_r/\delta]_{1,p_2} ;\end{matrix}\right.$$

$$[-2\sigma - 2h\eta_s : 2\rho_1, \ldots, 2\rho_r] :$$

$$[-\sigma - h\eta_s \pm w ; \rho_1, \ldots, \rho_r] ;$$

$$[(a) : \theta', \ldots, \theta^{(r)}] : [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ;$$

$$[(c) : \Psi', \ldots, \Psi^{(r)}] : [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ;$$

$$\left. z_1 4^{-\rho_1} \alpha_2^{-k_1/\delta}, \ldots, z_r 4^{-\rho_r} \alpha_2^{-k_r/\delta} \right) \qquad (3.5)$$

Suitable conditions of validity are easily obtainable from (1.3).

On specializing the parameters, Fox's H-function and the multivariable $H$-function may be transformed into $G$ functions, $E$-functions Lauricella's functions, Appell's functions, Kampé de Fériet's functions hypergeometric function, Legendre functions, Bessel functions and several other higher transcendental functions in one or more arguments Therefore, the double integral transformation for various other functions of one or more variables can be obtained as special cases of our results.

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**REFERENCES**

[1] Y.L. Luke, *The Special Functions and their Approximations*, Vol. I, Academic Press, New York and London, 1969.

[2] R. Panda, Integration of certain products associated with the $H$-function of several variables, *Comment. Math Univ. St. Paul* **26** (1977), 115-123.

[3] P. Skibiński, Some expansion theorem for the $H$-function, *Ann. Polon. Math.* **23** (1970), 125-138.

[4] H. M. Srivastava, and R. Panda, Some bilateral generating functions for a class of generalized hypergemetric polynomials *J. Reine Angew. Math.* **283/284** (1966) 265-274.

[5] H. M. Srivastava, K.C. Gupta and S. P. Goyal, *The H- Functions of One and Two Variables with Applications.South* Asian Publishers, New Delhi and Madras, 1982.