Jñānābha, Vol. 13, 1983

A NOTE ON A SUBCLASS OF UNIVALENT FUNCTION

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(*Received: October 31, 1982; Revised : December 16, 1982*)

ABSTRACT

Let $S(\alpha, \beta)$ denote the class of functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic and univaleut in the unit disk $U = \{z : |z| < 1\}$ and satisfy the condition

$$\left| \frac{f'(z) - \beta}{1 - \beta} - \alpha \right| < \alpha \qquad (z \in U)$$

for $\alpha > \frac{1}{2}$ and $0 \leqslant \beta < 1$. It is the purpose of this paper to prove an argument theorem for the function $f(z)$ belonging to the class $S(\alpha, \beta)$

1. INTRODUCTION

Let S denote the class of functions

$$(1) \quad f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic and univalent in the unit disk $U = \{z : |z| < 1\}$.

Further, let $S(\alpha, \beta)$ denote the class of functions $f(z) \in S$ satisfying the condition

$$(2) \quad \left| \frac{f'(z) - \beta}{1 - \beta} - \alpha \right| < \alpha \qquad (z \in U)$$

for $\alpha > \frac{1}{2}$ and $0 \leqslant \beta < 1$.

This class $S(\alpha, \beta)$ was introduced by Goel and Sohi [3]. In particular, the class $S(\alpha, \beta)$ was studied by Goel ([1], [2]).

2. AN ARGUMENT THEOREM

Theorem. *Let a function $f(z)$ defined by (1) belong to the class $S(\alpha, \beta)$. Then*

$$(3) \quad |\arg\{f'(z)\}| \leqslant \sin^{-1}\left(\frac{(B-A)\,|z|}{1-AB\,|z|^2}\right)$$

for $z \in U$ where $A = (1-\alpha)/\alpha$ and $B = 1 - \beta + A\beta$.

PROOF. Let

$$(4) \quad g(z) = \frac{f'(z) - \beta}{\alpha(1-\beta)} - 1,$$

then $|g(z)| \leqslant 1$ for $z \in U$ and $g(0) = (1-\alpha)/\alpha$. Further let

$$(5) \quad h(z) = \frac{g(z) - g(0)}{1 - g(0)\,g(z)}$$

for $z \in U$. Then h(z) vanishes at the origin and $|h(z)| < 1$ for $z \in U$. Consequently, by Schwarz's lemma, we can write $h(z) = z\phi(z)$, where $\phi(z)$ is an analytic function in the unit disk U and satisfies $|\phi(z)| \leqslant 1$ for $z \in U$. Hence we get

$$(6) \quad f'(z) = \frac{1 + Bh(z)}{1 + Ah(z)}$$

$$= \frac{1 + Bz\phi(z)}{1 + Az\phi(z)},$$

where $(A = (1 - \alpha)/\alpha$ and $B = 1 - \beta + A\beta$. After a simple calculation, we obtain

$$(7) \qquad \left| f'(z) - \frac{1 - AB\,|z|^2}{1 - A^2\,|z|^2} \right| \leqslant \frac{(B - A)\,|z|}{1 - A^2\,|z|^2}$$

which implies that

$$(8) \qquad |\arg\{f'(z)\}| \leqslant \sin^{-1}\left(\frac{(B - A)\,|z|}{1 - AB\,|z|^2}\right)$$

for $z \in U$. This evidently completes the proof of the theorem.

ACKNOWLEDGEMENTS

I wish to express here my gratitude to Professor H. M. Srivastava for his direction over the obstacles of distance and time.

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