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SOME RESULTS ON THE H-FUNCTION OF SEVERAL COMPLEX VARIABLES INVOLVING THE GENERALIZED HYPERGEOMETRIC FUNCTION AND THE GENERALIZED PROLATE SPHEROIDAL WAVE FUNCTION

By

V. B. L. CHAURASIA and V. G. GUPTA

Department of Mathematics, University of Rajasthan,

Jaipur–302004,, India

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In this paper an integral involving generalized prolate spheroidal wave function, generalized hypergeometric function and the (Srivastava and Panda) H-function of several complex variables has been evaluated and an expansion formula for the product of the generalized hypergeometric function and the H-function of several complex variables has been established with the application of this integral. As the generalized hypergeometric function and the H-function of several complex variables are each of a very general nature, the results, on specializing the parameters, lead to many simpler results, some of which are known and others are believed to be new.

1. INTRODUCTION

The solution of the following differential equation

$$(1-x^2)\ \frac{d^2y}{dx^2} + [\ (\beta-\alpha) - (\alpha+\beta+2)\ x\ ]\ \frac{dy}{dx}$$

$$+ [\ \xi\ (s) - s^2\ x^2\ ]\ y = 0 \qquad \ldots(1)$$

is the generalized prolate spheroidal wave function (Gupta [5]) which has been denoted as

$$\phi_n^{\alpha,\beta}(s, x) = \sum_{p=0}^{\infty} R_{p,n}^{\alpha,\beta}(s)\, P_{p+n}^{(\alpha,\beta)}(x) \qquad \ldots(2)$$

where $s = 0$ and $\xi(0) = (n+p)(\alpha+\beta+n+p+1)$, $p \geqq 0$.

Recently, Srivastava and Panda ([11] and [12]) have defined the H– function of several complex variables by means of the multiple Mellin–Barnes integral (see also [10], p. 251, eqn. (C. 1) *et seq.*)

$$H_{A,\, C:\, (B',\, D');\ldots;(B^{(r)},\, D^{(r)})}^{0,\, \varepsilon:\, (u',\, v');\ldots;(u^{(r)},\, v^{(r)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\Psi',\ldots,\Psi^{(r)}]:\end{matrix}\right.$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\, z_1 \\ \vdots \\ [(d'):\delta'];\ldots;[(d^{(r)}),\delta^{(r)}];\, z_r\end{matrix}\right]$$

$$= (2\pi w)^{-r}\int_{L_1}\cdots\int_{L_r} U_1(s_1)\ldots U_r(s_r)\, V(s_1,\ldots,s_r)$$

$$\cdot\, z_1^{s_1}\ldots z_r^{s_r}\, ds_1\ldots ds_r,\ w = \sqrt{-1}, \qquad \ldots(3)$$

where

$$U_i(s_i) = \frac{\prod_{j=1}^{u^{(i)}}\Gamma[d_j^{(i)} - \delta_j^{(i)} s_i]\ \prod_{j=1}^{v^{(i)}}\Gamma[1-b_j^{(i)} + \phi_j^{(i)} s_i]}{\sum_{j=1+u^{(i)}}^{D^{(i)}}\Gamma[1-d_j^{(i)} + \delta_j^{(i)} s_i]\ \sum_{j=v^{(i)}+1}^{B^{(i)}}\Gamma[b_j^{(i)} - \phi_j^{(i)} s_i]}$$

$$i = 1, \ldots, r$$

$$V(s_1,\ldots,s_r) = \frac{\prod_{j=1}^{\varepsilon}\Gamma[1-a_j + \sum_{i=1}^{r}\theta_j^{(i)} s_i]}{\prod_{j=\varepsilon+1}^{A}\Gamma[a_j - \sum_{i=1}^{r}\theta_j^{(i)} s_i]\ \prod_{j=1}^{C}\Gamma[1-c_j + \sum_{i=1}^{r}\Psi_j^{(i)} s_i]}$$

an empty product is interpreted as 1, the coefficients $\theta_j^{(i)}$,

$j=1,\dots, A$, $\phi_j^{(i)}$, $j=1,\dots, B^{(i)}$, $\Psi_j^{(i)}$, $j=1,\dots, C$,

$\delta_j^{(i)}$, $j=1,\dots,D^{(i)}$, and $i=1,\dots, r$, are positive numbers, and,

$u^{(i)}$, $v^{(i)}$, A, $B^{(i)}$, C, $D^{(i)}$, are integers such that $0 \leqq \varepsilon \leqq A$, $0 \leqq u^{(i)} \leqq D^{(i)}$,

$C > 0$, and $0 \leqq v^{(i)} \leqq B^{(i)}$, $i=1,\dots, r$. The contour L_i in the complex plane s_i - plane is of the Mellin-Barnes type which runs from $-w\infty$ to $w\infty$ with indentations, if necessary, in snch a manner that all thc poles of

$\Gamma[d_j^{(i)} - \delta_j^{(i)} s_i]$, $j=1,\dots, u^{(i)}$, are to the right,

and those of $\Gamma[1 - b_j^{(i)} + \phi_j^{(i)} s_i]$, $j = 1,\dots, v^{(i)}$,

$$\Gamma[1 - a_j + \sum_{j=1}^{r} \theta_j^{(i)} s_i], \; j = 1,\dots, \varepsilon \text{, to the left,}$$

of L_i, the various parameters being so restricted that these poles are all simple and none of them coincide, and with the points $z_i = 0$, $i=1,\dots, r$, being tacitly excluded, the multiple integral in (3) converges absolutely if

$$|\arg(z_i)| < \tfrac{1}{2} T_i \pi, \quad i = 1,\dots, r,$$

where

$$T_i = -\sum_{j=\varepsilon+1}^{A} \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)}$$

$$- \sum_{j=1}^{C} \Psi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} > 0,$$

$$i = 1,\dots, r, \qquad \dots(4)$$

The following results will be required

$$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\sigma} P_n^{(\alpha,\beta)}(x)\; H_{A,\,C:\,(B',\,D');\,\ldots;\,(B^{(r)},\,D^{(r)})}^{0,\,\varepsilon:\,(u',\,v');\,\ldots;\,(u^{(r)},\,v^{(r)})} \begin{bmatrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{bmatrix} dx$$

$$= 2^{\rho+\sigma+1} \sum_{m=0}^{n} \frac{(-n)_m (\alpha+\beta+n+1)_m}{(\alpha+1)_m\, m!}$$

$$H_{A+2,\,C+1:\,(B',\,D');\,\ldots;\,(B^{(r)},\,D^{(r)})}^{0,\,\varepsilon+2:\,(u',\,v');\,\ldots;\,(u^{(r)},\,v^{(r)})} \left[\begin{matrix} [-\sigma: k_1, \ldots, k_r], \\ [(c): \Psi', \ldots, \Psi^{(r)}], \end{matrix}\right.$$

$$[-m-\rho: h_1, \ldots, h_r],\ [(a): \theta', \ldots, \theta^{(r)}]:$$

$$[-1-m-\rho-\sigma: h_1+k_1, \ldots, h_r+k_r]:$$

$$\left.\begin{matrix} [(b'): \phi'];\ldots;[(b^{(r)}): \phi^{(r)}]; & z_1\, 2^{h_1+k_1} \\ & \vdots \\ [(d'): \delta'];\ldots;[(d^{(r)}): \delta^{(r)}]; & z_r\, 2^{h_r+k_r} \end{matrix}\right] \qquad \ldots(5)$$

where $(\lambda)_m = \Gamma(\lambda+m)/\Gamma(\lambda)$, $\mathrm{Re}\,(\rho + \sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}) > -1$,

$Re\,(\sigma + \sum_{i=1}^{r} k_i\, d_j^{(i)}/\delta_j^{(i)}) > -1$, $h_i > 0$, $k_i > 0$, $T_i > 0$,

$|\arg(z_i)| < \frac{1}{2} T_i \pi$, $\alpha > -1$, $\beta > -1$.

[$p_n^{(\alpha,\beta)}(x)$ is the well known Jacobi polynomial (Rainville [8], p. 254, eqn. (1)).].

The result (5) can be established by making use of a known result (Erdélyi *et al.* ([2], p. 284, eqn. (2)). The orthogonality property of the generalized prolate spheroidal wave function (Gupta [5], p. 107, eqn. (3. 1)).

$$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\beta} \phi_{n}^{\alpha,\beta}(s,x)\, \phi_{t}^{\alpha,\beta}(s,x)dx = N_{n,t}^{\alpha,\beta}\, \delta_{nt} \quad (6)$$

where

$$N_{n,t}^{\alpha,\beta} = \sum_{p=o}^{\infty} (R_{p,n}^{\alpha,\beta}(s))^2\, 2^{\alpha+\beta+1}$$

$$\frac{\Gamma(n+\alpha+1+p)\,\Gamma(n+p+\beta+1)}{(2n+2p+\alpha+\beta+1)\,\Gamma(n+p+1)\,\Gamma(n+p+\alpha+\beta+1)} \quad (7)$$

and δ_{nt} is the Kronecker delta.

THE MAIN INTEGRAL

$$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\sigma} \phi_{n}^{\alpha,\beta}(s,x)\; {}_MF_N\left[\begin{matrix} e_M \\ f_N \end{matrix};\; y(1-x)^{g}(1+x)^{w}\right]$$

$$\cdot\, H_{A,\,C\,:\,(B',\,D')\,;\ldots;\,(B^{(r)},\,D^{(r)})}^{0,\,\varepsilon\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}\left[\begin{matrix} z_1 (1-x)^{h_1}(1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r}(1+x)^{k_r}\end{matrix}\right]dx$$

$$= \sum_{p,\,q=0}^{\infty} 2^{\rho+\sigma+gq+wq+1}\, R_{p,n}^{\alpha,\beta}(s)\, \frac{(e_M)_q\, y^q}{(f_N)_q\, q\,!}$$

$$\sum_{m=0}^{n+p} \frac{(-n-p)_m\,(\alpha+\beta+n+p+1)_m}{(\alpha+1)_m\, m\,!}$$

$$H_{A+2,\,C+1\,:\,(B',\,C')\,;\ldots;\,(B^{(r)},\,D^{(r)})}^{0,\,\varepsilon+2\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}$$

$$\left[\begin{matrix}[-m-\rho-gq : h_1,\ldots,h_r],\ [-\sigma-wq : k_1,\ldots,k_r], \\ [(c) : \Psi',\ldots,\Psi^{(r)}],\ [-1-m-\rho-\sigma-gq-wq : h_1+k_1,\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}],[(b'):\phi'],\ldots,[(b^{(r)}):\phi^{(r)}]; \\ \ldots,h_r+k_r]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\;\begin{matrix} z_1\;2^{h_1+k_1} \\ \vdots \\ z_r\;2^{h_r+k_r}\end{matrix}\right],$$

...(8)

where $\text{Re}\left(\rho+\sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}\right) > -1$,

$\text{Re}\left(\sigma+\sum_{i=1}^{r} k_i\, d_j^{(i)}/\delta_j^{(i)}\right) > -1,\ h_i > 0,\ k_i > 0,\ T_i > 0,$

$|\arg z_i| < \frac{1}{2} T_i \pi,\ i = 1,\ldots,r,\ g > 0,\ w > 0,\ \alpha > -1,$

$\beta > -1,\ M \leqslant n\ (M = N+1$ and $|y| < 1)$.

PROOF

To establish (8), express the generalized prolate spheroidal function as given in (2) and the generalized hypergeometric function as infinite series (Rainville [8], p. 73, eqn. (2)), change the order of integration and summations (which is easily seen to be justified due to the absolute convergence of the integral and sums involved in the process) and then evaluate the resulting integral with the help of (5). We thus arrive at the right–hand side of (8).

EXPANSION FORMULA

$$(1-x)^{\rho-\alpha}(1+x)^{\sigma-\beta}\,{}_MF_N\left[\begin{matrix}e_M\\ f_N\end{matrix};\,y\,(1-x)^g\,(1+x)^w\right]$$

$$\cdot\, H^{0,\,\varepsilon\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{A,\,C\,:\,(B',\,D')\,;\ldots;\,(B^{(r)},\,D^{(r)})}\left[\begin{matrix} z_1\,(1-x)^{h_1}\,(a+x)^{k_1}\\ \vdots\\ z_r\,(1-x)^{h_r}\,(1+x)^{k_r}\end{matrix}\right]$$

$$=\sum_{t,\,p,\,q=0}^{\infty}\frac{(e_M)_q\,y^q}{(f_N)_q\,q!}\,R^{\alpha,\,\beta}_{p,\,t}(s)\,2^{\rho+\sigma-\alpha-\beta+gq+wq}$$

$$\sum_{m=0}^{t+p} \frac{(-t-p)_m\,(\alpha+\beta+t+p+1)_m}{(\alpha+1)_m\, m\,!}$$

$$.\,H^{0,\,\varepsilon+2\,:\,(u',\,v')\,;\ldots;\,(u^{(r)},\,v^{(r)})}_{A+2,\,C+1\,:\,(B',\,D')\,;\ldots;\,(B^{(r)},\,D^{(r)})}$$

$$\left[\begin{matrix}[-\sigma - wq : k_1,\ldots,k_r],\\ [(c):\Psi',\ldots,\Psi^{(r)}],\end{matrix}\right.$$

$$[-m-\rho-gq : h_1,\ldots,h_r],\,[(a):\theta',\ldots,\theta^{(r)}]:$$

$$[-1-m-\rho-\sigma-wq-gq : h_1+k_1,\ldots,h_r+k_r]: \qquad (3)$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]\,;\,z_1\;2^{h_1+k_1}\\ \vdots \\ [(d):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\,;\,z_r\;2^{h_r+k_r}\end{matrix}\right]$$

$$\left\{ \phi_t^{\alpha,\,\beta}\,(s,x) \Big/ (R_{p,\,t}^{\alpha,\,\beta}(s))^2 \right.$$

$$\left.\frac{\Gamma(t+\alpha+1+p)\,\Gamma(t+\beta+1+p)}{(2t+2p+\alpha+\beta+1)\,\Gamma(t+p+1)\,\Gamma(t+p+\alpha+\beta+1)}\right\}, \qquad \ldots(9)$$

where $\mathrm{Re}\,(\rho+\sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}) > -1$,

$\mathrm{Re}\;(\sigma+\sum_{i=1}^{r} k_i\, d_j^{(i)}/\delta_j^{(i)}) > -1,\ h_i > 0,\ k_i > 0,$

$g > 0,\ w > 0,\ T_i > 0,\ |\arg z_i| < \frac{1}{2}\, T_i\,\pi,\ \alpha > -1,$

$\beta > -1$ and $M \leqslant N$ ($M = N+1$ and $|y| < 1$),

$i = 1,\ldots, r\,;\ j = 1,\ldots, u^{(i)}.$

PROOF

Let

$$f(x) = (1-x)^{\rho-\alpha} (1+x)^{\sigma-\beta} {}_MF_N \left[\begin{matrix} e_M \\ f_N \end{matrix} ; y (1+x)^w (1-x)^g \right]$$

$$\cdot H \begin{matrix} 0, \varepsilon : (u', v') ;\ldots; (u^{(r)}, v^{(r)}) \\ A, C : (B', D') ;\ldots; (B^{(r)}, D^{(r)}) \end{matrix} \left[\begin{matrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{matrix} \right]$$

$$= \sum_{t=0}^{\infty} E_t \, \phi_t^{\alpha, \beta} (s, x), \ (-1 < x < 1) \qquad \cdots(10)$$

Equation (10) is valid since $f(x)$ is continuous and of bounded variation in the open interval $(-1, 1)$.

Now multiply both sides of (10) by

$$(1-x)^\alpha (1+x)^\beta \phi_n^{\alpha, \beta} (s, x)$$

and integrate with respect to x from -1 to 1. Change the order of integration and summation (which is permissible) on the right,

$$\int_{-1}^{1} (1-x)^\rho (1+x)^\sigma {}_MF_N \left[\begin{matrix} e_M \\ f_N \end{matrix} ; y (1-x)^g (1+x)^w \right] \phi_n^{\alpha, \beta} (s,x)$$

$$. H \begin{matrix} 0, \varepsilon : (u', v') ;\ldots; (u^{(r)}, v^{(r)}) \\ A, C : (B', D) ;\ldots; B^{(r)}, D^{(r)} \end{matrix} \left[\begin{matrix} z_1 (1-x)^{h_1} (1+x)^{k_1} \\ \vdots \\ z_r (1-x)^{h_r} (1+x)^{k_r} \end{matrix} \right] dx$$

$$= \sum_{t=0}^{\infty} E_t \int_{-1}^{1} (1-x)^\alpha (1+x)^\beta \phi_n^{\alpha, \beta} (s, x) \, \phi_t^{\alpha, \beta} (s, x) \, dx \qquad \ldots(11)$$

Using the orthogonality property for the generalized prolate spheroidal wave functions (6) on the right-hand side and the result (8) on the left-hand side of (11), we obtain

$$E_l = \sum_{p,q=0}^{\infty} \frac{(e_M)_q\, y^q}{(f_N)_q\, q\,!}\; R_{p,\,l}^{\alpha,\,\beta}(s)\; 2^{\rho+\sigma+gq+wq+1}$$

$$\sum_{m=0}^{l+p} \frac{(-l-p)_m\,(\alpha+\beta+l+p+1)_m}{(\alpha+1)_m\; m\,!}$$

$$.\,H^{\,0,\;\varepsilon+2\;:\;(u',\,v')\;;\ldots;\;(u^{(r)},\,v^{(r)})}_{\,A+2,\;C+1\;:\;(B',\,D')\;;\;\;;\;(B^{(r)},\,D^{(r)})}\left[\begin{matrix}[-\sigma-wq:\,k_1,\;\;,k_r\;,\\ [(c)\;:\;\Psi',\ldots,\Psi^{(r)}],\end{matrix}\right.$$

$$[-m-\rho-gq\;:\;h_1\,,\ldots,\,h_r],\;[(a)\;:\;\theta'\,,\;\;,\,\theta^{(r)}]\;;$$

$$[-1-m-\rho-\sigma-gq-wq\;:\;h_1+k_1\,,\;\;,\;h_r+k_r]\;;$$

$$\left.\begin{matrix}[(b'):\,\phi']\;,\ldots,\;[(b^{(r)})\;:\;\phi^{(r)}]\;;\;z_1\;\;2^{h_1+k_1}\\ \vdots \\ [(d'):\,\delta']\;;\;\;;\;[(d^{(r)})\;:\;\delta^{(r)}];\;z_r\;\;2^{h_r+k_r}\end{matrix}\right]$$

$$.\left\{\phi_l^{\alpha,\,\beta}(s,\,x)/N_{n,\,l}^{\alpha,\,\beta}\;\;\delta_{n,\,l}\right\} \qquad \ldots(12)$$

With the help of (11) and (12), in view of (7), the expansion formula (9) is obtained.

SPECIAL CASES

(i) Letting $y=0$ in (8), we get

$$\int_{-1}^{1} (1-x)^{\rho}\,(1+x)^{\sigma}\,\phi_n^{\alpha,\,\beta}(s,\,x)$$

$$.\,H^{\,0,\;\varepsilon:\;(u',\,v')\;;\ldots;\;(u^{(r)},\,v^{(r)})}_{\,A,\;C\;:\;(B',\,D')\;;\ldots;\;(B^{(r)},\,D^{(r)})}\left[\begin{matrix}z_1\,(1-x)^{h_1}\quad(1+x)^{k_1}\\ \vdots\\ z_r\,(1-x)^{h_r}\quad(1-x)^{k_r}\end{matrix}\right]dx$$

$$= \sum_{p=0}^{\infty} R_{p,\,n}^{\alpha,\,\beta}(s)\;2^{\rho+\sigma+1}\;\sum_{m=0}^{n+p}\frac{(-n-p)_m\,(\alpha+\beta+n+p+1)_m}{(\alpha+1)_m\;m\,!}$$

$$\cdot H \begin{matrix} 0,\ \varepsilon+2 : (u', v') ;\ldots; (u^{(r)}, v^{(r)}) \\ A+2,\ C+1 : (B', D') ;\ldots; (B^{(r)}, D^{(r)}) \end{matrix} \left\{ \begin{matrix} [-\sigma : k_1 ,\ldots, k_r], \\ [(c) : \Psi' ,\ldots, \Psi^{(r)}], \end{matrix} \right.$$

$$\begin{matrix} [-m-\rho : h_1 ,\ldots, h_r], [(a) : \theta' ,\ldots, \theta^{(r)}] : \\ [-1-m-\rho-\sigma : h_1 + k_1 ,\ldots, h_r + k_r] : \end{matrix}$$

$$\left. \begin{matrix} [(b) : \phi] ;\ldots; [(b^{(r)}) : \phi^{(r)}] ; \\ [(d') : \delta'] ;\ldots; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix} \begin{matrix} z_1 \; 2^{h_1 + k_1} \\ \vdots \\ z_r \; 2^{h_r + k_r} \end{matrix} \right\} \qquad \ldots(13)$$

where $Re\ (\rho + \sum_{i=1}^{r} h_i\, d_j^{(i)}/\delta_j^{(i)}) > -1,$

$$Re\ (\sigma + \sum_{i=1}^{r} k_i\, d_i^{(i)}/\delta_j^{(i)}) > -1,\ h_i > 0,\ k_i > 0,$$

and of course, $T_i > 0$, $|\arg z_i| < \frac{1}{2} T_i \pi$, $\alpha > -1$,

$\beta > -1$, $i = 1 ,\ldots, r$; $j = 1 ,\ldots, u^{(i)}$.

(ii) Setting $s=0$, $x=1-2\xi$ and using Saalschütz's theorem (Slater [9], p. 49), in (13) we get a result recently given by Srivastava and Panda ([12], p. 131, eqn. (2. 2)).

(iii) For $r=2$, we get from (8) the corresponding integral involving the H-function of two variables.

(iv) Taking $y=0$ and $\theta' = \theta'' = \Psi' = \Psi'' = \phi' = \phi'' = \delta' = \delta'' = 1$ in case (iii) above, we find a known result obtained by Mishra ([6], p. 158, eqn. 5. 6. 2).

(v) Setting $A = 0 = C = \varepsilon$, $v'' = B'' = d'' = 0$, $u'' = D'' = \delta'' = 1$, $z_2 \to 0$ and applying a known transformation formula Chaurasia ([1], p. 18, eqn. (1. 5. 4.)), the integral in case (iii) can be reduced to an integral involving Fox's H-function,

(vi) Letting $y=0$ in case (v) we arrive at another result given by Mishra ([6], p. 157, eqn. (5. 6. 1)).

(vii) Taking $r=2$ in (9), we easily get the corresponding expansion formula involving the H-function of two variables (Mittal and Gupta [7]).

(viii) Putting $y=0$ and $\theta'=\theta''=\Psi'=\Psi''=\phi'=\phi''=\delta'=\delta''=1$ in case (vii) and using a known formula (Rainville [8], p. 24, eqn. (2)), we obtain a result recorded by Mishra ([6], p. 166, eqn. (5. 7. 5)).

(ix) Setting $A=0=C=\varepsilon$, $v''=B''=d''=0$, $u''=D''=\delta''=1$, $z_2 \to 0$ and using a transform formula (Chaurasia [1], p. 18, eqn. (1. 5. 4)) in case (vii), we get an expansion formula involving Fox's H-function.

(x) When $y=0$, the result in case (ix) reduces to a known result (Mishra [6], p. 165, eqn. (5. 7. 4)).

Several other interesting special cases of our results can be deduced.

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