Jñānabha, Vol. 9/10, 1980

(Dedicated to the memory of Professor Arthur Erdélyi)

FLUCTUATING FLOW OF A VISCO–ELASTIC FLUID PAST AN INFINITE PLANE, POROUS WALL WITH FLUCTUATING SUCTION IN SLIP FLOW REGIME

By

B. SIDDAPPA and G.S. PATEL

*Department of Mathematics, Karanatak University Post-Graduate, Centre, Gulbarga--***585105**, *India*

(Received : November 5, 1977; Revised : January 25, 1979)

ABSTRACT

This paper considers the Rivlin-Ericksen fluid flow past an infinite plane porous wall with fluctuating suction in slip flow conditions. It is observed that :

(i) The velocity profile is effected by the rarefaction parameter h_1 in the slip flow, but this is not the case in no slip flow.

(ii) If h_1 increases, the magnitude of the mean velocity increases for a fixed viscoelastic parameter K.

(iii) The main stream fluctuations cause the fluctuations in the skin friction.

(iv) As h_1 increases, the skin friction decreases for a fixed A.

(v) As A increases, the amplitude of the skin friction $|H|$ increases.

(vi) For large A a change in h_1 produces larger variations in $|H|$

(vii) For fixed h_1, as K increases, the magnitude of the mean velocity increases too.

1. Introduction

Lighthill [1] has studied the effects of fluctuations of the main stream velocity on the flow of an incompressible fluid past two dimensional bodies. Stuart [2] has obtained the exact solution of the Navier-Stokes equations for such an oscillatory flow over an infinite plane porous wall wiih constant suction. Siddappa [4] has extended Stuart's work for Rivlin-Ericksen visco-elastic fluid. Siddappa and Chetty [5] studied how Siddappa's results get modified when no slip boundary conditions are replaced by the velocity slip conditions. Here we extend this problem for the fluctuating suction.

2. Basic Equations

We consider the flow due to fluctuating main stream of Rivlin-Ericksen fluid flow past an infinite plane, porous wall with fluctuating suction at the surface.

Let $y=0$ be the wall. Let u and v be the velocity components along and normal to the wall.

The visco-elastic equations to the problem are

$$\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\alpha\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right) \tag{2.2}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y}+2\left(2\beta+\gamma\right)\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)=0 \tag{2.3}$$

where α is the kinematic viscosity, β is the kinematic visco-elasticity, γ is the kinematic cross-viscoelasticity.

3. Boundary Conditions

Th first order velocity slip condition is

(i) $$u=\left(\frac{2-f_1}{f_1}\right)L\left(\frac{\partial u}{\partial y}\right)=L_1\left(\frac{\partial u}{\partial y}\right) \text{ at } y=0 \tag{3.1}$$

(ii) $u=U(t)$

(iii) $\frac{\partial u}{\partial y}=0$

$$\left.\begin{array}{l} u=U(t) \\ \frac{\partial u}{\partial y}=0 \end{array}\right\} \text{ as } y\to\infty \tag{3.2}$$

which arises because of the symmetry about the axis of the pipe through the point at infinity.

Here f_1 is Maxwell's reflexion coefficient.

$L=\mu(\pi/2P\rho)^{\frac{1}{2}}$ is the mean free path, and is constant for an incompressible fluid, and $L_1=\left(\frac{2-f_1}{f_1}\right)L$

4. Mathematical Analysis

Considering (2.2) as $y\to\infty$, $u\to U(t)$ and the partial derivatives of u w.r.t. y tend to zero.

Therefore we have

$$\frac{dU}{dt}=-\frac{1}{\rho}\frac{dP}{dx}=-\frac{1}{\rho}\frac{dg}{dx} \tag{4.1}$$

and so by integrating we have

$$g\,(x,t)=-\rho x\frac{dU}{dt}+h\,(t) \tag{4.2}$$

where h (t) is at most a function of t.

Thus the pressure is given by

$$P(x,y,t)=\rho(2\beta+\gamma)\left(\frac{\partial u}{\partial y}\right)^2-\rho\, x\frac{dU}{dt}+h\,(t) \tag{4.3}$$

It is clear from (4.3) that the pressure of the fluid depends on y in addition to x and t, unlike in the classical viscous case. This is because of the viscoelastic nature of the fluid.

The equation of our interest is

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{dU}{dt}+\alpha\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial^2}{\partial y^2}\left\{\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right\} \tag{4.4}$$

Integrating (2.1) following Messiha [3] we have

$$V=-V_0\,(1+eAe^{i\omega t}) \tag{4.5}$$

negative sign indicates that the suction velocity is directed towards the plate. We look for the solution of the equation (4.4) following Lighthill [1] of the form

$$u=U_0[\phi_0(y)+\epsilon\phi_1(y)e^{i\omega t}] \tag{4.6}$$

The free stream velocity is

$$u=U(t)=U_0(1+\epsilon e^{i\omega t}) \text{ as } y\to\infty \tag{4.7}$$

substituting the expressions for u, U and v into (4.4)and equating the harmonic and the nonharmonic terms of the equation we get

$$-V_0\frac{d\phi_0}{dy}=\alpha\frac{d^2\phi_0}{dy^2}-\beta V_0\frac{d^3\phi_0}{dy^3} \tag{4.8}$$

and

$$i\omega\phi_1-V_0\frac{d\phi_1}{dy}=i\omega+\alpha\frac{d^2\phi_1}{d^2}+\beta i\omega\frac{d^2\phi_1}{dy^2}-\beta V_0\frac{d^3\phi_1}{dy^3}-\alpha A\frac{d^2\phi_0}{dy^2} \tag{4.9}$$

with the boundary conditions

$$\left.\begin{array}{l}\left.\begin{array}{l}\phi_0=L_1\dfrac{d\phi_0}{dy}\\[2ex] \phi_1=L_1\dfrac{d\phi_1}{dy}\end{array}\right\}\text{ at } y=0\\[4ex] \text{and}\\ \left.\begin{array}{l}\phi_0=\phi_1=1\\[1ex] \dfrac{d\phi_0}{dy}=\dfrac{d\phi_1}{dy}=0\end{array}\right]\text{ at } y\to\infty\end{array}\right\} \tag{4.10}$$

putting $\eta=\dfrac{V_0}{\alpha}y$. Equation (4.8), (4.9) and the boundary conditions (4.10) reduce to

$$K\frac{d^3\phi_0}{d\eta^3}+\frac{d^2\phi_0}{d\eta^2}+\frac{d\phi_0}{d\eta}=0 \tag{4.11}$$

$$K\frac{d^3\phi_1}{d\eta^3}+(1)-i\lambda k)\frac{d^2\phi_1}{d\eta^2}+\frac{d\phi_1}{d\eta}-i\lambda\phi_1=i\lambda+A\frac{d^2\phi_0}{d\eta^2} \tag{4.12}$$

and

$$\left.\begin{array}{l}\phi_0=h_1\dfrac{d\phi_0}{d\eta},\ \phi_1=h_1\dfrac{d\phi_1}{d\eta}\ \text{at}\ \eta=0\\[2ex]\phi_0=\phi_1=1,\ \dfrac{d\phi_0}{d\eta}=\dfrac{d\phi_1}{d\eta}=0\ \text{at}\ \eta\to\infty\end{array}\right\}\tag{4.13}$$

where $\lambda=\dfrac{\omega a}{V_0^2}$, $K=-\beta\dfrac{V_0^2}{a^2}$ and

$h_1=L_1\dfrac{V_0}{a}$ is the rarefaction parameter.

The solution of the equation (4.11) subject to the conditions (4.13) is

$$\phi_0=1-\frac{e^{-a_1\eta}}{1+a_1h_1}\tag{4.14}$$

where $a_1=\dfrac{1-\sqrt{1-4K}}{2K}$ for $K\leqslant 0$

and the other solution is

$$\phi_0=1-\frac{e^{-a_2\eta}}{1+a_2h_1}\tag{4.15}$$

where $a_2=\dfrac{1+\sqrt{1-4K}}{2K}$ for $0<K\leqslant\frac{1}{4}$.

The solution of (4.12) subject to the conditions (4.13) is

$$\phi_1=1-\frac{e^{-h\eta}}{1+hh_1}-\frac{Aa^2e^{-a\eta}}{(1+ah_1)\,f(-a)}\tag{4.16}$$

where $h=h_r+ih_1$ is given by the equation

$$f(h)=Kh^3+(1-i\lambda K)h^2+h-i\lambda=0\tag{4.17}$$

and $a=a_1$ for $K\leqslant 0$; $a=a_2$ for $0<K\leqslant\frac{1}{4}$.

The equation (4.17) is solved numerically [4].

When K=0, (4,14) and (4.15) become the asymptotic suction profile in the Newtonian fluid.

The unperturbed thickness δ is given by

$$\delta = \int_{y=0}^{\infty} \left(1-\frac{u}{U}\right) dy, \text{ where U is the value of u at } y=\infty \cdot$$

$$\text{Thus } \delta = \frac{a}{V_0} \int_{\eta=0}^{\infty} \left[1 - \left\{ 1 - \frac{e^{-a\eta}}{1+ah_1} \right\} \right] d\eta,$$

whence

$$\delta = \frac{a}{V_0(1+ah_1)a} = \sqrt{\left(\frac{\lambda a}{wa^2(1+a\,h_1)^2} \right)}$$

As h_1 increases, thickness decreases for $K \leqslant 0$, and also for $0 < K \leqslant \frac{1}{4}$; and when $h_1=0$, it reduces to the result of Siddappa [4].

The total velocity component parallel to the wall is given by

$$u=U_0\left[1-\frac{e^{-a\eta}}{1+ah_1} + \varepsilon\left(1 - \frac{e^{-h\eta}}{1+hh_1} - \frac{Aa^2\;e^{-a\eta}}{(1+ah_1)f(-a)} \right) e^{iwt} \right] \quad (1.18)$$

Thus the velocity profile is effected by the rarefaction parameter h in the slip flow. This is not valid in the case of no slip flow since $h_1=0$.

If $h_1 \to \infty$, the velocity (4·18) become $u=U_0\ [1+\varepsilon e^{i\omega t}]$ which is the main stream velocity. This does not happen in the case of no slip flow.

In slip flow, because of the boundary conditions, the shear stress at the wall is proportionol to the slip velocity at the wall and given by

$$\tau_\omega = \rho\alpha\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho\alpha}{L_1}(u)_{y=0}$$

$$= \rho V_0 U_0 \left[1 + \frac{a}{1+ah_1} + \varepsilon\left(\frac{h}{1+hh_1} - \frac{Aa^2}{h_1(1+ah_1)f(-\alpha)} \right) e^{iwt} \right.$$

$$= \rho V_0 U_0 \left[\frac{a}{1+ah_1} + a\,\varepsilon\,|\,H\,|\,e^{i(\omega t+\theta)} \right] \quad (4\;19)$$

The non-dimensional skin friction is given by

where $\tau_0 = \dfrac{a}{1+ah_1}$

$$\text{mean friction} \frac{\tau}{V_0 U_0 \rho} = \tau_0 + \epsilon \mid H \mid e^{i(wt+\theta)} \tag{4.20}$$

$$\mid H \mid = \sqrt{H_r^2 + H_i^2} \tag{4.21}$$

$$\theta = \tan^{-1} (H_i/H_r) \tag{4.22}$$

$$H_r = \frac{h_r + h_i \left(h_r^2 + h_i^2 \right)}{\left(1+h_1 h_r \right)^2 + h_1^2 h_r^2} + \frac{Aa^2(1+ah_1)}{h_1^2 \left(1+ah_1 \right) + a^2\lambda^2}$$

$$H_i = \frac{h_r}{\left(1+h_1 h_r \right)^2 + h_1^2 h_i^2} - \frac{Aa^3\lambda}{h_1^2 \left(1+ah_1 \right) + a^2\lambda^2}.$$

DISCUSSION

It is observed from (4.20) that the mean friction τ_0 decreases with the increasing values of slip parameter h_1. We observe from equation (4.19) that the main stream fluctuations cause the fluctuations in the skin friction.

For a fixed h_1 it is observed that $\mid H \mid$ increase with increase in the suction parameter. Also as $h_1 \to \infty$, $\mid H \mid$ vanishes asymptotically, for a fixed A, $\mid H \mid$ decreases for the increase in h_1. It is observed that:

(i) As K increases for a fixed h_1 velocity increases.

(ii) For fixed K as h_1 increases velocity increases.

REFERENCES

[1] M. J. Lighthill, The response of laminar skin friction and heat transfer to fluctuations in the stream velocity, Proc. Roy, Soc. London Ser. A **224** (1954), 1–23.

[2] J. T. Stuart, A solution of the Navier–Stokes and energy equations illustrating the reponse of skin friction and temperature of an infinite plate thermometer to fluctuations in the stream velocity, Proc. Roy. Soc. London Ser. A **231** (1955), 116–130.

[3] S.A.S. Messiha, Laminar boundary layer in oscillatory flow along an infinite flat with variable suction, Proc. Cambridge Philos. Soc. **26** (1966), 329–337.

(4) B. Siddappa, Fluctuating flow of a visco-elastic fluid past an infinite plane, porous wall with constant suction, Jñānabha Sect. A, 3 (1973), 35-43.

[5] B. Siddappa and S. S. Chetty, Fluctuating flow of a visco-elastic fluid past an infinite plane, porous wall wilh constant suction in slip flow regime, Jñānabha Sect. A **5** (1975), 9–15.