ARTHUR ERDÉLYI

(1908—1977)

By

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The world of mathematics suffered an irreparable loss with the passing away of Professor Arthur Erdélyi, F. R. S., F. R. S. E., who died suddenly at his home in Edinburgh on Monday, December 12, 1977, at the age of 69, leaving behind a widow, Eva, and a stepson, David. He had apparently made a marvellous recovery after his surgery of 1973, though in private he often had to rest, and had been his usual self in the Department of Mathematics (University of Edinburgh) on the day of his death. He had his evening meal at home, and later was taken ill and died about 9 : 00 P. M.

Arthur Erdélyi was born on October 2, 1908 in Budapest, Hungary, the eldest of five children of Ignác and Frieda (*née* Roth) Diamant. After his father's death he was adopted by his mother's second husband, Paul Erdélyi. He received his primary and secondary education in Budapest from 1914 to 1926.

Upon completion of the school curriculum in Budapest, Erdélyi wanted to become a mathematician and was indeed offered a place in Budapest University to study mathematics. {This was not a small tribute to his ability in view of the quota on the admission of Jews imposed by the University under the *numerus clausus*.} Due to circumstances then obtaining in Hungary (regarding, for example, university appointment of Jews), however, he planned to become an engineer and then set up in private practice in Hungary. But the quota in engineering for Jews in Hungarian universities was nil. Consequently, after consulting L. Fejér at Budapest, Erdélyi enrolled at the Deutche Technische Hochschule in Brno, Czechoslovakia, to study electrical engineering. Two years leter, in 1928, he passed with distinction the first of the two "state examinations" (necessary in order

to obtain a degree at the Technische Hochschule; one in mathematics, physics, and related scientific subjects, and the other in professional subjects), but never completed work for the second. Such a situation was not at all uncommon in Central Europe in those days, especially because a university degree was not a prerequisite for a successful career then.

During his first year at Brno, Erdélyi was awarded both the first and second prizes in a mathematics competition organized by the Professor of Algebra and Geometry at the Technische Hochschule. At this stage in his life, Erdélyi realized that he could obtain employment as a mathematician if he stayed in Czechoslovakia and, following the advice of the Professor of Mathematical Analysis at the Technische Hochschule, he decided to devote himself to mathematics. He began active research in mathematics about 1930 and published his first paper in 1934.

In 1937 he matriculated at the German University of Prague, and in 1938 he submitted a collection of his published papers to the German University of Prague for the award of the degree of *Doctor rerum naturalium*. Under normal circumstances, a successful career as a professional mathematician in Czechoslovakia would have been assured.

Unfortunately, circumstances in Czechoslovakia were far from being normal and the German occupation of Czechoslovakia in 1938 put Erdéyi, as a Jew, in mortal danger. {As a matter of fact, the convocation ceremony of 1938 was the last *Promotion* which the German University of Prague held before it was taken over by the Nazis} Erdélyi was ordered to leave Czechoslovakia by the end of 1938 or else he was to risk internment in a concentration camp. Erdélyi was already in active correspondence with Professor Francesco G. Tricomi of the University of Torino (Italy) because of their related interests in research. Tricomi, however, discouraged Erdélyi's possible move to Italy because he felt that the surge of anti-Semitism in Italy militated against such a move. At the suggestion of Tricomi, Erdélyi then wrote to Professor E. T. (later Sir Edmund) Whittaker of the University of Edinburgh keeping in view the fact that he had by that time published over twenty papers, mostly devoted to the confluent hypergeometric function which Whittaker had discovered in 1904. Through the dedicated efforts of Whittaker and Professor S. Brodetsky of the University of Leeds, funds (sufficient to satisfy the British government regarding its policy of not issuing a visa to a refugee unless $£$ 400 per annum could be guaranteed for his/her support) were finally collected, and

Whittaker wrote about it to Erdélyi in December 1938. The offer came just in time, as evidenced by Erdélyi's letter of January 26, 1939 to Whittaker : *" . . . Necessity and danger compel me to trouble you once more . . . You know, perhaps, what it means today if a few is to be put on the German or Hungarian frontier. .."* In the last days of January 1939. Erdélyi was finally able to leave Czechoslovakia, and in February 1939 he duly appeared at the Mathe matical Institute of the University of Edinburgh, At Waverly railway station in Edinburgh, carrying a battered suitcase and very little money, Erdélyi was received by Barry Spain who had been deputed by Whittaker to give Erdélyi two pounds and arrange for his lodgings. At the same time Walter Ledermann was called in to act as an interpreter since Erdélyi spoke practically no English though his German was fluent. These welcoming signs marked the beginning of Erdélyi association with Edinburgh which was to flower into a major love of his life, sufficient to warrant his return from U. S. A. many years later at a considerable personal sacrifice.

Undoubtedly, the invitation to Edinburgh saved Erdélyi's life for the entire Jewish community in Brno was wiped out during the war. His two brothers and a sister died in concentration camps, but it is pleasing to record that his mother survived and lived in Israel for many years after the war and that Erdélyi was always a most devoteds on.

For the next few years, Erdélyi continued his work under a research grant from the University of Edinburgh and some financial support from the Society for the Protection of Science of Learing. His research publications continued unabated, and the University of Edinburgh recognized his widening international reputation by awarding him the degree of Doctor of Science in 1940. The following year Whittaker managed to persuade the University authorities to make Erdélyi an Assistant Lecturer in Mathematics. In 1942 Erdélyi was appointed Lecturer and, with his future reasonably secured, he married Eva Neuburg (daughter of Frederic and Helen Neuburg and second cousin of Max Perutz, F. R. S.) in Glasgow on November 4, 1942. The marriage brought him great joy and proved to be very happy and harmonious.

Erdélyi's association with Edinburgh was soon to be interrupted. On January 21, 1946, Professor Harry Bateman (1882—1946) of the California Institute of Technology died suddenly of coronary thrombosis, leaving behind him a mass of notes which he had intended to edit into a monumental work on special functions. Since Whittaker was at that time the

world's senior authority on special functions, and had known Bateman as a student at Trinity College, Cambridge, he was naturally asked by Caltech if he could recommend someone to supervise the editing and publication of the Bateman manuscripts. Without hesitation he recommended Erdélyi (who had just been elected a Fellow of the Royal Society of Edinburgh in 1945) as the most highly qualified expert for the proposed work. On July 1, 1947, Erdélyi took up a visiting professorship at Caltech, thus beginning his sixteen-year association with one of the outstanding centres of scientific learning in the world.

Erdélyi's initial appointment at Caltech was for one year, and he was assigned the task of evaluating the contents of the Bateman manuscripts and of determining the time needed to prepare them for publication. Upon completion his careful study, he reported that the job might take for him as many as fifteen years unless it could possibly be done by four highly trained mathematicians working over a period of four years. The alternative proposal was accepted by Caltech and followed up by an offer to Erdélyi of a premanent Full Professorship at Caltech, with directing the Bateman Manuscript Project as part of his duties. The University of Edinburgh responded by promoting him to Senior Lecturer, but the attraction of a Professorship at Caltech at double his Edinburgh salary proved to be irresistible, and after returing to Edinburgh for the academic session 1948–49, he resigned his position at Edinburgh and moved to U. S. A. He was joined at Caltech by W. Magnus from the University of Göttingen, Fritz Oberhettinger from the University of Mainz, and Francesco G. Tricomi from the University of Torino. Thus in 1949 the collaboration of the team of Erdélyi, Magnus, Oberhettinger and Tricomi on the Bateman Manuscript Project started, the famous team which produced the three volumes of *Higher Transcendental Functions* and the two volumes of *Tables of Integral Transforms*. These volumes were destined to be among the most widely cited mathematical works of all time and a basic source of reference for generations of mathematicians, physicists and engineers throughout the world.

The Bateman Manuscript Project marked a turning point in Erdélyi's development as a mathematician. Until this time most of his work was in the area of special functions, and although his results were often striking and elegant, his investigations were undertaken mainly for their own sake and not, in general, to illuminate other areas of mathematics. However, as the Bateman Manuscript Project neared completion around 1951, Erdélyi

became involved with investigations into various other areas, such as the analytical theory of singular partial differential equations (*cf.* [90] and [91]), diffraction theory ([104] and [109]), and the asymptotic expansions of solutions of certain classes of differential equations ([107] and [110]). The most important of these investigations was his work on asymptotics, and around 1950 Erdélyi and his co-workers began publishing a long series of papers on the asymptotic expansions of integrals and solutions to differential equations. Much of this work was summarized in a short paperback *Asymptotic Expansions* which Erdélyi published in 1956 and which soon became the standard work on the subject.

As the years progressed and Erdélyi's international stature as a mathematician grew, he continued broadening and deepening his knowledge of mathematics, laying the foundation for the rest of his life's work. His last book *Operational Claulus and Generalized Functions* appeared in 1962, and he published his work on singular perturbation theory (*of.* [131], [134], [137], [143] and [144] and his fundamental papers on the asymptotic evaluation of certain classes of integrals ([130] and [149]) in the early 1960's.

During the years that Erdélyi spent at Caltech, Whittaker had retired and had been succeeded by Aitken. In 1963 the University of Edinburgh created a second Chair of Mathematics and, on account of Aitken's extremely poor health, the Department was in desperate need of a strong and effective leader. The position was advertised twice, but for various reasons, Erdélyi did not apply. Fortunately for the University of Edinburgh a catalyst, in the person of Professor Ian N. Sneddon of the University *of* Glasgow, was visiting Caltech while the second advertisement was running. In conversation with Sneddon, Erdélyi once remarked that he thought of the University of Edinburgh as his real *alma mater* because of how well he was reeived there as a refugee in 1939 and that, if the University invited him to occupy the chair, he would think about it very seriously. In spite of his failure to persuade Erdélyi to apply, Sneddon passed on the information by telephone to Edinburgh as soon as he returned to Glasgow three days after the conversation. Shortly thereafter Erdélyi was officially invited to occupy the chair, and after three months of correspondence, he finally accepted the invitation on October 1, 1963, the day before his fifty-fifth birthday. It was in July 1964 when, to the delight of his many friends in Edinburgh and elsewhere in Scotland, Erdélyi

returned to the University of Edinburgh where he eventually occupied (until his sudden death on December 12, 1977) the celebrated Chair of Mathematics held once by Mclaurin and, more recently, by Whittaker and Aitken.

Many academic honours were accorded to Arthur Erdélyi. In addition to those that have already been mentioned, he was elected a Foreign Member of the Academy of Sciences of Torino (Italy) in 1953 and a Fellow of the Royal Society in 1975. In 1977 he was awarded the Gunning Victoria Jubilee Prize of the Royal Society of Edinburgh, the only mathematician to receive this coveted prize in recent years other than Sir William Hodge. His service to the mathematical community included the Presidency of the Edinburgh Mathematical Society, Council Member of the American Mathematical Society, membership on various advisory bodies appointed by the National Academy of Sciences of U. S. A., and his active association in an editorial capacity with many journals such as :

> *Proceedings of the Edinburgh Mathematical Society,*
> *Journal of the Indian Mathematical Society,*
> *Mathematical Tables and Other Aids to Computation,*
> *Journal of Mathematics and Mechanics,*
> *Archive for Rational Mechanics and Analysis,*
> *Journal of the Society for Industrial and Applied*
> > *Mathematics,*
> *Journal of Mathematical Physics,*
> *Canadian Journal of Mathematics,*
> *Proceedings of the Royal Society of Edinburgh.*

He also found time to contribute innumerable reviews to *Mathematical Reviews*. His seventieth birhday was to have been celebrated by a special issue of *Applicable Analysis* and by the *Proceedings of the* 1978 *Dundee Conference on Differential Equations*. Both of these have now been dedicated to him as a lasting memorial to his work and the great influence which it continues to exert on current mathematical research.

The British citizenship (to which Erdélyi had become entitled by naturalization in (1947) did not diminish his interest in Jewish affairs. He

paid several visits to Israel and spent the academic year 1956-57 on sabbatical leave as Visiting Professor of Applied Mathematics at the Hebrew University, Jerusalem. He was a member of the *Friends of the Hebrew Univrsity* and, while in U. S. A., served on its Academic Council. For a period of time he also was Vice-President of the Edinburgh section of the British Zionist Association.

Erdélyi attracted many invitations to visit mathematical centres elsewhere. He travelled widely and lectured frequently in Europe, North America and Australia. These include his Visiting Professorship at the University of Melbourne in 1970, and the invited lectures at the 1954 International Congress of Mathematicians in Amsterdam, the 1961 Summer Research Institute of the Canadian Mathematical Congress in Montreal, and the 1974 International Conference on Fractional Calculus and Its Applications in New Haven. It was at the New Haven conference in June 1974 when I first met Erdélyi personally. I am fortunate in having been in correspondence with him for over a decade prior to our first meeting and having remained in contact with him afterwards. During the academic year 1975–76, while on sabbatical leave at the University of Glasgow, I had many opportunities of seeing Arthur Erdélyi during my frequent visits to Edinburgh and elsewhere in Scotland, especially at the meetings of the Edinburgh Mathematical Society. During the last few weeks in his life, Erdléyi and I were involved, as external referees, in the evaluation af a certain case for promotion to Full Professor at a Canadian univrsity.

One cannot leave the subject of Erdélyi's contributions without mentioning that the work, which, he initiated and encouraged, has been carried on by his Ph. D. students, R.H. Owens (1952), P G. Rooney (1952), C. A. Swanson (1957), J. Rice (1959), T. Beohme (1960), D. W. Willett (1963), J. W. Macki (1964), D. L Colton (1967), J. Wimp (1968), J. Searl (1969), and A. McBride (1971). On the other hand, one just cannot overemphasize the importance of Erdélyi's classic paper of 1956 ([120]) in which he used operators of fractional integration. He continued using there operators in all his future work on singular equations of the GASPE type. In 1958 Erdélyi and Copson [123] made use of fractional integration operators and the Mellin transform to study a singular hyperbolic equation with intersecting singular lines, and in 1965 Erdélyi ([154] and [155]) returned to his study of fractional integration and the generalized axially symmetric potential equation. His last paper on this subject appeared in

1970 ([163]) in which he applied fractional integration operators to study the Euler-Poisson-Darboux equation.

Although Erdélyi used fractional integration in various papers on special functions published in 1939 and 1940 ([50], [57] and [58], his first major contribution in this area appeared in [59] and [60], partly in collaboration with H. Kober. In these papers Erdélyi and Kober introduced and studied certain *homogeneous* modifications of the Riemann-Liouville and Weyl fractional integrals and discussed their connections with the Hankel transform. Their results involving these generalized fractional integration operators, usually called Erdélyi-Kober operators, lay dormant for over twenty years, until in 1961 Erdélyi and Ian Sneddon came together as lecturers at the Summer Research Institute of the Canadian Mathematical Congress in Montreal. While delivering a series of lectures on mixed boundary value problems, Sneddon remarked that a unified treatment of the dual integral equations that arose in such problems depended on the development of certain relationships between Hankel transforms and fractional integration operators. This, of course, was precisely the topic discussed by Erdélyi and Kober in [59], and hence the chance remark by Sneddon resulted in a joint paper [139] in which Erdélyi and Sneddon solved, in a systematic and unified way, a rather general class of dual integral equations which occur frequently in a number of branches of mathematical physics. In his later years, Erdélyi (influenced by Zemanian's book *Generalized Integral Transformations*) began to extend fractional calculus to generalized functions (*cf.* [166], [167] and [170]). This work on fractional integrals of generalized functions was subsequently extended further to include the Stieltjes transform (*cf.* [175]), and during his final days Erdélyi was actively developing and refining his investigations in this area.

Arthur Erdélyi leaves behind him the memory of a mathematician with the astonishing range of interests, one who made no compromise with the highest standards of his profession, one who served his discipline with devotion and distinction, one who made significant and lasting contributions to his fields of expertise, and one who inspired many of his colleagues and students to carry out independent researches for themselves. He will indeed be remembered for his great intellectual gifts and research contributions, for his courtesy and kindliness and, above all, for his intensely human personality with a keen sense of humour and a great love of art, of music, of the countryside and of children.

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The photograph of Arthur Erdélyi was taken in 1957 in Jerusalem when he was spending his sabbatical year 1956-57 at the Hebrew University there; it has been reproduced with the technical assistance by Mr. Elisabeth Grambart of Media and Technical Services, University of Victoria.

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