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INVENTORY MODEL FOR SINGLE ITEM-MULTI SUPPLIER WITH SHORTAGES

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Abstract

This paper deals with the problem of developing an acquisition policy for solving the single-item multi-supplier problem with shortages and real world constraints. We evaluate the optimal quantity of item to be ordered to the different suppliers so as to optimize the total cost incurred. The optimal solution is obtained to indicate the best acquisition policy. In addition, the effects of shortages are also considered. By taking numerical example, the sensitivity analysis has been carried out to explore the effect of system parameters on optimal policy.

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1 Introduction

Traditionally there has always been some friction between organizations and their suppliers. This friction causes serious problems with loyalty and co-operation. Suppliers set rigid conditions, and as they have no guarantee of repeat business, they try to make as much profit from each sale as possible; organizations shop around to make sure they get best deal, and remind suppliers of the competition. The result is uncertainty about orders, constant changes in an organizations suppliers and customers, changing products, varying order sizes, different times between orders, uncertainty about repeat orders, changes in the costs, and so on.

Just-in-time production models have been developed in recent years in order to reduce the costs of diversified small-lot productions. Organizations using JIT rely on their suppliers being completely dependable and removing all uncertainty from supply. The only way of achieving this is through co-operation, recognizing that organizations and their suppliers both want a mutually beneficial trading arrangement. If they can agree conditions that satisfy both the organization and the supplier- with the feeling that they get best possible deal this is far better than working with unnecessary friction. The implication is that organizations should identify the best suppliers and always give order to that supplier. This reinforces the ideas behind strategic alliances and partnerships. With close co-operation, it can be difficult for suppliers to make the small, frequent delivery needed, and to co-ordinate their delivery with demands. Sometimes they respond by increasing their stock of finished goods to ensure the required pattern of delivery. The effect of this is simply to move stocks from an organizations raw material store to the suppliers finished goods store. This does not reduce overall costs and might even increase them. The aim of JIT is to eliminate stocks rather than move them to another point in the supply chain. And, again, the way to achieve this is through co-operation. Hax and Candea [10] indicated that unconstrained optimization of economic order quantity (EOQ) is often an unrealistic assumption. Burton [5] considered JIT and introduced repetitive sourcing strategy. The greatest flaw of the weights and criteria grading are open to subjectiveness. However, the clear advantage of these methods is that many important criteria that cannot be quantified are taken into consideration. For a summary of vendor selection criteria, we refer Weber et al. [19]. Benton [2] derived a heuristic procedure for treating PQD decisions under the conditions of multiple items, multiple suppliers and resource limitations.

Dickson [7] analyzed the supplier selection problem in supply chain management. Activity based costing approach was developed by Roodhooft and Konings [16]. They gave various aspects of vendor selection considering the total cost of ownership. Ghodsyapour and O'Brien [9] applied integrated analytical hierarchical process and linear programming to decision support system for supplier selection. Rosenblatt et al. [17] considered the problem of developing an acquisition policy. Specifically, given a set of potential suppliers, from whom should the firm buy the product, in what quantities, and how often? They provided an algorithm for multi-supplier single item system.

Yang and Wee [22] developed a production-inventory system model of a deteriorating item, taking into account the view of both the vendor and the multi-buyers. Minner [15] reviewed inventory models with multi supply options and discussed their contribution to supply chain management. He also discussed strategic aspects of supplier competition and the role of operational flexibility in global sourcing and outlined inventory models, which use several suppliers in order to avoid or reduce the effects of shortage situations. Li [11] discussed the multi-buyer joint replenishment problem (MJRP) of coordinating the replenishment of a group of items that are jointly ordered from a single supplier. Xu [20] considered a class of multi-period dynamic supply contracts in which a buyer orders a product from a supplier in each period and the supplier allows the buyer to cancel a portion of an outstanding order with penalty during a planning horizon. He used simulation to show how the supplier chooses cancellation costs that minimize his expected cost during the planning horizon.

Bretthauer et al. [3] presented a model and solution methodology for production and inventory management problems that involve multiple resource constraints. Their model formulation was general, allowing organizations to handle a variety of multi-item decisions such as determining order quantities, production batch sizes, number of production runs, or cycle times. They presented efficient algorithms for solving both continuous and integer variable versions of the resource constrained production and inventory management model. Chan et al. [6] addressed the supplier-scheduling problem by considering the deliveries scheduling issue, once the optimal replenishment cycles are determined. They considered four large integer programming problems according to four different objectives in cost and resource minimization and solved them by converting them into network flow problems.

Lebacque et al. [12] suggested the methods which are aimed at maintaining the production rate of each type of part as smooth as possible and therefore holding small inventory and shortage costs. Burke et al. [4] analyzed single period, single product sourcing decisions under demand uncertainty. Their approach includes the product prices, supplier costs, supplier capacities, historical supplier reliabilities and firm specific inventory costs. Rong et al. [8] presented an EOQ model with two-warehouses for the perishable goods with fuzzy lead time and partially/ fully backlogged shortage. Again Madhavilata et al. [14] introduced two levels of storage for inventory of single item in their research work. Min et al. [13] developed inventory model in which items are deteriorating exponentially and shortages are allowed. Agrawal et al. [1] also considered an inventory system with two warehouses where demand rate is ramp type and deterioration rate is constant. Guchhait et al. [8] developed a model for inventory system with time dependent deteriorating items to determine the profit maximization. Xu.et al. [21] proposed an inventory system periodic review base stock with partial backlogging.

In this investigation we extend the model of Rosenblatt et al. [17]. In their model, shortages are not allowed, so we add the concept of shortages which are satisfied at the arrival epoch of the next replenishment. Since demand for an item may repeat either very frequently or after a long time, so the organization may face the problem of (i) how much to restock at each replenishment cycle, and (ii) how many intervals should a replenishment cycle be. To deal with this problem, this chapter presents an inventory model having real-world constraints with back order. The rest of the chapter is organized as follows. In the next section we describe our model. An optimal policy is given in section 3. Section 4 provides an acquisition policy from which we find that from which supplier we buy the product and how much quantity to buy. Section 5 is devoted to numerical illustrations. Finally section 6 presents conclusions and some recommendations for future research work.

2 The Model

A deterministic inventory model is developed for a single item, multi supplier, where each supplier has a limit on the average capacity of the items, which he can deliver per unit of time. Each of the suppliers has its own cost parameters and a finite long-run average capacity. The total cost is comprised of the sum of the total periodic purchasing, ordering, and inventory carrying, shortage and supplier management costs.

Ordering cost is a cost associated with ordering of raw material for production purposes. Advertisements, consumption of stationery and postage, telephone charges, telegrams, rent for space used by the purchasing department, travelling expenditures incurred, etc. constitute the ordering cost. The cost of purchasing total units of an item is known as total periodic purchasing cost. The carrying cost is associated with carrying (or holding) inventory. This cost generally includes the costs such as rent for space used for storage, interest on the money locked-up, insurance of stored equipment, production, taxes, depreciation of equipment and furniture used etc.. The penalty cost for running out of stock (i.e., when an item cannot be supplied on the customers demand) is known as shortage cost. This cost includes the loss of potential profit through sales of items and loss of goodwill, in terms of permanent loss of goodwill, in terms of permanent loss of customers and its associated lost profit in future sales.

The supplier management costs are fixed periodic costs incurred for every supplier who supplies a nonzero amount, independent of the supply volume and the number of orders. In general, any cost associated with maintaining a relationship with a supplier will constitute a part of supplier management costs.

The following notations are used for mathematical formulation of the inventory model:

M	: Total number of suppliers.
b_i	: Periodic capacity of item for i^{th} supplier.

d_i	:	Average periodic quantity ordered from supplier i.
C_i	:	The cost per item procured from i^{th} supplier.
D	:	The demand for item per period.
C_{mi}	:	The periodic supplier management cost incurred when using i^{th} supplier.
C_{hi}	:	The periodic holding cost associated with holding one unit from i^{th} supplier.
C_{si}	:	The fixed ordering cost associated with each order from i^{th} supplier.
Q_i	:	The order quantity of the item from i^{th} supplier.
q_i	:	Amount to be back-ordered.
y_i	:	An integer variable, set equal to 1 if supplier i is used and 0 otherwise.
$\frac{d_i}{Q_i}$	:	The number of orders from supplier i.
τ	:	Cycle time.
α_i	:	Number of orders placed per cycle by i^{th} supplier.
K	:	Maximum limit on inventory.

For all the above notations, $i = 1, 2, ..., M$. In our model, we allow the shortages, therefore the demand of the item is not fully met, it is greater than the ordered item. Hence,

$$\sum_{i=1}^{M} d_i \leq D. \tag{2.1}$$

Individual cost for the item from different suppliers is now evaluated before they are grouped together.

The total periodic purchasing cost from supplier $i = C_i d_i$.

The total ordering cost associated with supplier $i = C_{si} \times \frac{d_i}{Q_i}$.

The periodic holding cost for supplier i for the item $C_{hi}\left(\frac{Q_i - q_i}{2D}\right)^2 \times \frac{d_i}{Q_i}$.

The shortage cost for i^{th} supplier for the item $= S_i\left(\frac{q_i^2}{2D}\right) \times \frac{d_i}{Q_i}$.

The periodic supplier management cost associated with i^{th} supplier $= C_{mi} Y_i$.

The total average cost per period can be expressed as follows:

$TC =$ Purchasing cost + Ordering cost + Holding cost + Shortage cost +Management cost.

Therefore,

$$TC = \sum_{i=1}^{M} \left[C_i d_i + C_{si}\frac{d_i}{Q_i} + C_{hi}\left(\frac{(Q_i - q_i)^2}{2D}\right)\frac{d_i}{Q_i} + S_i\left(\frac{q_i^2}{2D}\right)\frac{d_i}{Q_i} + C_{mi}Y_i \right]. \tag{2.2}$$

3 Cost Optimization Problem

The purpose of this study is to minimize the total cost by simultaneously determining the values of Q_i and q_i optimally. According to equation (2.2), the total average cost function is a function of two variables Q_i and q_i. Our problem is to determine the values of Q_i and q_i which minimize the total cost. During the first part of the cycle all demand is met from the stock, so the amount sent to customers is $(Q_i - q_i)$. During the second part of the cycle all demand is back-ordered, so the shortage, q_i equals the unmet demand. After some manipulation the optimal order quantity (Q_i^*) and optimal amount to be back-ordered (q_i^*) is given by:

$$Q_i = \sqrt{\frac{2DC_{si}(S_i + C_{hi}}{C_{hi}S_i}} \tag{3.1}$$

and

$$q_i = \sqrt{\frac{2DC_{hi}DC_{si}}{(S_i + C_{hi})S_i}}. \tag{3.2}$$

4 The Acquisition Policy

The acquisition policy is maintained, which combines both the vendors selection and inventory management into the model. In this policy a firm can determine from which suppliers to buy, and what quantity to buy periodically.

The supplier can be selected on the basis of suppliers effective variable cost per unit item, which can be obtained as

$$f_i = C_i + \frac{2C_{hi} + S_i}{(C_{hi} + S_i)}\sqrt{\frac{C_{hi}S_iC_{si}}{2D\left(C_{hi} + S_i\right)}} - \frac{C_{hi}\left(C_{hi} + 2S_i\right)}{(C_{hi} + S_i)}\sqrt{\frac{C_{hi}C_{si}}{2DS_i\left(C_{hi} + S_i\right)}} + \sqrt{\frac{C_{hi}C_{si}\left(C_{hi} + S_i\right)}{2DS_i}} \tag{4.1}$$

A supplier (who is selected to supply units) will supply less than its capacity, therefore, any supplier who has a higher effective variable cost per unit item then this supplier will not be used. The objective function yields the following form:

$$\text{Min } TC = \sum_{i=1}^{M}(f_id_i + C_{mi}y_i). \tag{4.2}$$

In association to execute a cyclic schedule one has to determine the cycle length, τ. Now the number of orders placed with supplier i for item per cycle is determined as:

$$\alpha_i = \frac{d_i\tau}{Q_i^*}, \tag{4.3}$$

where Q_i^* is obtained from equation (3.1). But α_i might not be an integer, hence, we round off α_i to the respective geometrically close integer. Since the allocation from the various suppliers cannot change, we set the actual order size $Q_i' = Q_i^*\left(\alpha_i/\alpha_i'\right)$ so as to maintain $\alpha_i'Q_i' = d_i\tau_i$. Hence, $\alpha_i'Q_i'$ will be the actual amount of item supplied by i^{th} supplier during a cycle.

5 Numerical Illustrations

Table 5.1: Parameters and cost elements

Supplier	Cost per item (C_i)	Fixed ordering cost (C_{si})	Holding cost (C_{hi})	Periodic supplier management cost (C_{mi})	Shortage cost (S_i)	Periodic demand (d_i)
Supplier (1)	2.5	0.2	0.21	20	0.5	60
Supplier (2)	2.7	1.7	0.19	15	0.7	40

A single item multi supplier system with shortages and real world constraints is considered. The parameters and cost elements of suppliers are shown in Table 5.1. A commercial non-linear programming solver OPTIMIZATION TOOL in MATLAB 6.5 is used for solving this problem to obtain the optimal solution as $(Q_1, Q_2, q_1, q_2, TC) = (16.94, 47.70, 4.86, 10.18, 34409110)$.

From Table 5.2, we see that as the value of shortage cost (S_i) increases, the optimal order quantity (Q_i), optimal quantity to be back-ordered (q_i) and total cost (TC) decrease, which is quite obvious. Table 5.3 shows the effect of holding cost (C_{hi}) on optimal order size (Q_i), optimal amount to be back-ordered (q_i) and total cost (TC). It is clear from Table 5.3 that as the value of shortage cost (S_i) increases, the optimal order quantity (Q_i), optimal quantity to be back-ordered (q_i) and total cost (TC) decrease.

Table 5.4 displays the effect of ordering cost (C_{si}) on optimal order quantity (Q_i), optimal quantity to be back-ordered (q_i) and total cost (TC). An optimal order quantity, optimal quantity to be back-ordered and total cost increase as the value of ordering cost (C_{si}) increases. It is observed from Table 5.5 that as annual demand (D) increases, the optimal order quantity (Q_i), optimal quantity to be back-ordered (q_i) and total cost (TC) of the system increase.

Table 5.6 examines the effect of cost per item (C_i) on the total cost of the system. The total cost increases with the increase in the cost per item. From Table 5.7 we find that as periodic supplier management cost increases, the total cost of the system increases.

Table 5.2: Effect of shortage cost on optimal order size, optimal amount to be back-ordered and total cost

S_1	S_2	Q_1	q_1	Q_2	q_2	TC
0.50	0.70	16.45	4.86	47.70	10.18	34409110
0.70	0.90	15.74	3.63	46.55	8.11	33545150
0.90	1.10	15.33	2.90	45.81	6.75	32990410
1.10	1.30	15.06	2.41	45.29	5.77	32603550
1.30	1.50	14.87	2.07	44.90	5.05	32318200
1.50	1.70	14.74	1.81	44.60	4.48	32098980
1.70	1.90	14.63	1.61	44.37	4.03	31925250
1.90	2.10	14.54	1.45	44.17	3.67	31784180
2.10	2.30	14.47	1.32	44.01	3.36	31667330
2.30	2.50	14.42	1.21	43.88	3.10	31568960

Table 5.3: Effect of holding cost on optimal order size, optimal amount to be back-ordered and total cost

C_{h1}	C_{h2}	Q_1	q_1	Q_2	q_2	TC
0.21	0.19	16.45	4.86	47.70	10.18	34409110
0.31	0.29	14.46	5.53	40.72	11.93	29424930
0.41	0.39	13.33	6.00	36.84	13.18	26653510
0.51	0.49	12.59	6.36	34.35	14.14	24865460
0.61	0.59	12.07	6.63	32.59	14.90	23608060
0.71	0.69	11.68	6.85	31.28	15.53	22672180
0.81	0.79	11.37	7.03	30.27	16.05	21946850
0.91	0.89	11.13	7.19	29.46	16.49	21367340
1.01	0.99	10.94	7.32	28.79	16.87	20893220
1.11	1.09	10.77	7.43	28.24	17.20	20497860

Table 5.4: Effect of ordering cost on optimal order size, optimal amount to be back-ordered and total cost

C_{s1}	C_{s2}	Q_1	q_1	Q_2	q_2	TC
0.20	1.70	16.45	4.86	47.70	10.18	34409110
0.30	1.80	20.14	5.96	49.08	10.48	38964810
0.40	1.90	23.26	6.88	50.43	10.77	43906640
0.50	2.00	26.00	7.69	51.74	11.04	49190800
0.60	2.10	28.49	8.43	53.01	11.32	54787200
0.70	2.20	30.77	9.10	54.26	11.58	60673360
0.80	2.30	32.89	9.73	55.48	11.84	66831580
0.90	2.40	34.89	10.32	56.67	12.10	73247370
1.00	2.50	36.77	10.88	57.84	12.35	79908590
1.10	2.60	38.57	11.41	58.99	12.59	86804810

Table 5.5: Effect of annual demand on optimal order size, optimal amount to be back-ordered and total cost

D	Q_1	q_1	Q_2	q_2	TC
100.00	16.45	4.86	47.70	10.18	34409110
110.00	17.25	5.10	50.03	10.68	43667100
120.00	18.02	5.33	52.25	11.15	54278160
130.00	18.75	5.55	54.39	11.61	66302510
140.00	19.46	5.76	56.44	12.05	79797880
150.00	20.14	5.96	58.42	12.47	94819830
160.00	20.80	6.15	60.33	12.88	111422000
170.00	21.44	6.34	62.19	13.28	129656100
180.00	22.06	6.53	63.99	13.66	149572500
190.00	22.67	6.71	65.75	14.04	171220000

Table 5.6: Effect of cost per item on total cost

C_1	C_2	TC
2.50	2.70	34409110
3.00	3.20	34409160
3.50	3.70	34409210
4.00	4.20	34409260
4.50	4.70	34409310
5.00	5.20	34409360
5.50	5.70	34409410
6.00	6.20	34409460
6.50	6.70	34409510
7.00	7.20	34409560

Table 5.7: Effect of periodic supplier management cost on total cost

C_{m1}	C_{m2}	TC
20	15	34409110
25	20	34409120
30	25	34409130
35	30	34409140
40	35	34409150
45	40	34409160
50	45	34409170
55	50	34409180
60	55	34409190
65	60	34409200

Conclusions

A single item multi supplier EOQ model with real world constraints has been investigated. We have dealt with an inventory problem wherein the shortage occurs. Our study will be helpful to determine an acquisition policy to determine from which supplier to buy, and what quantity to buy from each supplier periodically. The model formulated with shortages under more realistic assumptions provides insight for many big firms and malls.

For future research on this problem, it would be of interest to add the effect of more realistic demand rate in the model. The incorporation of some more realistic factors, such as quantity discounts, inflation etc., can further enrich our inventory model

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