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MHD HEAT AND MASS TRANSFER FLOW OVER A SEMI INFINITE INCLINED PLATE AT CONSTANT CONCENTRATION GRADIENT EMBEDDED IN A POROUS MEDIUM WITH HEAT SOURCE

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Abstract

We present analytical study of viscous incompressible fluid through porous medium with heat source and constant concentration gradient. The equations are solved analytically and results are obtained for velocity, temperature and concentration as well as skin friction coefficient and Nusselt number. The effect of angle of inclination, permeability parameter, magnetic field parameter, radiation parameter, heat source parameter, Schmidt number and Prandtl number are shown both in graphical and tabular form.
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1 Introduction

The porous media plays an important role in technology. The most important areas of technology that depended upon porous media are

(1) Hydrology, which is related to water movement in earth and sand structure such as dam, flow of well from water bearing formation, insulation of sea water in coastal areas, filter beds for purification of drinking water etc.
(2) Petroleum engineering which is mainly concerned with petroleum and natural gas production, exploration, well drilling and logging etc.
(3) In chemical engineering, filtering of gases and liquids and drying the bulk of goods are the important technologies based on flow through porous media.
(4) In medicine and biochemical engineering, biological membranes and flow of blood and other body fluids are few examples where the role of porous media is critical.

A detailed review of convective heat transfer in Darcy and non-Darcy porous medium is available in the books of Nield and Bejan [16] and Ingham and Pop [12]. Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux was studied by Acharya at al. [1]. Exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium was presented by Bhuvaneswari et al. [7]. AbdusSamad and Rahman [3] studied the effects of thermal radiation for unsteady MHD flow over a vertical plate immersed in porous medium.

Kandasamy and S.P.A Devi [15] considered suction and injection in laminar boundary-layer flow over a wedge with chemical reaction. H. Kumar ([13], [14]) considered the MHD heat and mass transfer flow over an isothermal inclined plate at constant concentration gradient and also studied the heat transfer over a stretching porous sheet in presence of heat source. Ahmed and Sarmah [6] analyzed the effect of magnetic field on a transient mixed convection flow through a porous medium bounded by a suddenly fixed infinite vertical plate, whereas Barik and Dash [8] investigated unsteady magnetohydrodynamic flow past an inclined porous heated plate.

The study of heat and mass transfer with chemical reaction is of considerable importance in food processing, Mechanical and Aerospace Engineering and in Bio engineering.Ganesan and Palani ([10], [11]) presented an analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux and also studied the natural convection effects on impulsively started inclined plate with heat and mass transfer. Seethamahalakshmi [19] investigated the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical moving plate embedded in porous medium. Shit and Haldar [17] presented the thermal radiation and Hall Effect on MHD heat and mass transfer flow over an inclined permeable stretching sheet. Anghel [2] discussed the combined heat and mass transfer over an inclined flat plate. Said et al. [18] and Tania and Samad [20] studied the natural convection flow between inclined isothermal plates with viscous dissipation and heat generation. Free convection heat and mass transfer flow with heat generation has been discussed by Alam et al. ([4], [5]). Chen [9] analyzed the heat and mass transfer flow with variable wall temperature and concentration.

In present study we analyzed the MHD heat and mass transfer flow of viscous electrically conducting fluid over an inclined porous plate embedded in a porous medium with heat source at a constant concentration gradient. Analytical method was used to obtain the solutions of ordinary differential equations governing the flow problem. It is found that fluid velocity and temperature increases with increasing in heat source parameter and heat transfer rate at the plate decreases in the presence of heat source parameter.

2 Mathematical Formulation

Consider a steady, two-dimensional laminar flow of an incompressible, viscous, electrically conducting fluid past a semi-infinite inclined porous plate embedded in a porous medium and subjected to a uniform magnetic field B_0 normal to the direction of flow. The physical configuration and coordinate system of problem is shown in Fig. 1. The x-axis is taken along the inclined plate and the y-axis normal to the plate.

The magnetic Reynolds number are assumed to be very small so that the induced magnetic field is considered to be negligible compared to the applied magnetic field. The governing equations of mass, momentum, energy and concentration for steady flow with Boussinesq's approximation are as follows

$$\frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + gB_T \cos\phi(T - T_\infty) + gB_C \cos\phi(c - c_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{k}u, \tag{2.2}$$

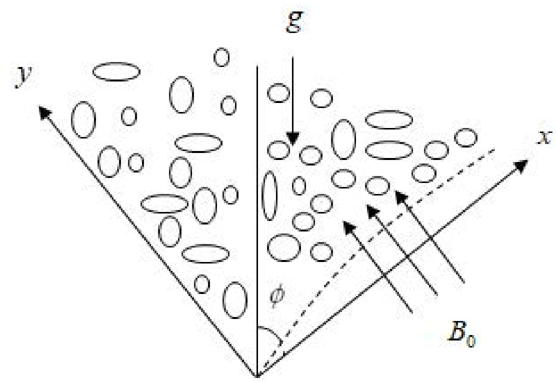

Figure 1: Physical configuration of the problem

$$v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_\infty), \tag{2.3}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}, \tag{2.4}$$

where u and v are corresponding velocity component along and perpendicular to the surface, ν is the kinematic viscosity, g is the acceleration due to gravity, B_T is the coefficient of volume expansion for the heat transfer and B_C is the volumetric coefficient of concentration expansion, k is the permeability of porous medium, ϕ is the angle of inclination, σ is the fluid electrical conductivity, $T$is the fluid temperature, T_∞ is the far field temperature , α is the thermal conductivity, Q heat source parameter, ρ is the density of the fluid, C_p is specific heat at constant pressure ,c is the species concentration,c_∞ is the far field concentration,D is the chemical molecular diffusivity, with boundary conditions

$$u = 0,\, v = v_w,\, T = T_w,\, -D\frac{\partial c}{\partial y} = m_w,\, at\, y = 0,$$

$$u \to 0,\, T \to T_\infty,\, c \to c_\infty,\, as\, y \to \infty. \tag{2.5}$$

The equation of continuity (1) gives

$$v = -v_w. \tag{2.6}$$

Introducing the following non-dimensional parameters in equations (2.2), (2.3) and (2.4)

$$Y = \frac{yv_w}{\nu}, \quad U = \frac{u}{u_w}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{c - c_\infty}{(m_w\nu/v_w D)}. \tag{2.7}$$

From the above dimensionless variables, we have

$$u = Uu_w, \quad T = \theta(T_w - T_\infty) + T_\infty$$

and

$$c = C(m_w\nu/v_w D) + c_\infty.$$

Using these relations, we further derive

$$\frac{\partial u}{\partial y} = u_w \frac{\partial U}{\partial Y}\frac{\partial Y}{\partial y} = u_w \frac{\partial U}{\partial Y}\frac{v_w}{\nu} = \frac{u_w v_w}{\nu}\frac{\partial U}{\partial Y},$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{v_w^2}{\nu}\frac{\partial U}{\partial Y}\right) = \frac{\partial}{\partial Y}\left(\frac{u_w v_w}{\nu}\frac{\partial U}{\partial Y}\right)\frac{\partial Y}{\partial y} = \frac{u_w v_w^2}{\nu^2}\frac{\partial^2 U}{\partial Y^2},$$

$$gB_T(T - T_\infty)\cos\alpha = gB_T(\theta(T_w - T_\infty) + T_\infty)\cos\alpha,$$

$$gB_C(c - c_\infty)\cos\alpha = gB_C C(m_w \nu / v_w D)\cos\alpha,$$

$$\frac{\partial T}{\partial y} = (T_w - T_\infty)\frac{v_w}{\nu}\frac{\partial \theta}{\partial Y},$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial Y}\left((T_w - T_\infty)\frac{v_w}{\nu}\frac{\partial \theta}{\partial Y}\right)\frac{\partial Y}{\partial y} = \frac{v_w^2}{\nu^2}(T_w - T_\infty)\frac{\partial^2 \theta}{\partial Y^2},$$

$$\frac{\partial c}{\partial y} = \frac{m_w}{\nu}\frac{\partial C}{\partial Y},$$

$$\frac{\partial^2 c}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial c}{\partial y}\right) = \frac{\partial}{\partial Y}\left(\frac{m_w}{\nu}\frac{\partial C}{\partial Y}\right)\frac{\partial Y}{\partial y} = \frac{m_w v_w}{\nu D}\frac{\partial^2 C}{\partial Y^2}.$$

Now we substitute the values of above derivatives in equations (2.2), (2.3) and (2.4). By simplifying these equations, we obtain the following ordinary differential equations in terms of dimensionless variables

$$\frac{d^2 U}{dY^2} + \frac{dU}{dY} + Gr_1\theta + Gm_1 C - NU = 0, \tag{2.8}$$

$$\frac{d^2\theta}{dY^2} + \Pr\frac{d\theta}{dY} + \Pr S\theta = 0, \tag{2.9}$$

$$\frac{d^2 C}{dY^2} + Sc\frac{dC}{dY} = 0, \tag{2.10}$$

where $Gr_1 = Gr\cos\phi$, $Gm_1 = Gm\cos\phi$, $N = M + 1/K$.

Grashof number $Gr = \frac{gB_T(T_w - T_\infty)\nu}{u_w v_w^2}$,

Solutal Grashof number $Gm = \frac{gB_C m_w \nu^2}{u_w v_w^3 D}$,

Magnetic field number $M = \frac{B_0^2 \nu\sigma}{v_w^2 \rho}$,

Permeability parameter $K = \frac{\nu^2}{v_w^2}k$,

Prandtl number $\Pr = \frac{\rho\nu C_p}{k} = \frac{\nu}{\alpha}$,

Heat source parameter $S = \frac{Q\nu}{\rho C_p v_w^2}$,

Schmidt number $Sc = \frac{\nu}{D}$.

The boundary condition (2.5) turns into

$$U = 0,\ \theta = 1,\ \frac{\partial C}{\partial y} = -1,\ at\, Y = 0,$$

$$U \to 0,\, \theta \to 0,\, C \to 0,\, as\, Y \to \infty. \tag{2.11}$$

Solution of equations (2.8) to (2.10) with boundary condition (2.11) is as follows

$$U = m_5 e^{-m_2 Y} + m_3 e^{-m_1 Y} + m_4 e^{-ScY}, \tag{2.12}$$

$$\theta = e^{-m_1 Y}, \tag{2.13}$$

$$C = \frac{1}{Sc} e^{-ScY}, \tag{2.14}$$

where $m_1 = \frac{\Pr + \sqrt{\Pr^2 - 4\Pr S}}{2}$, $m_2 = \frac{1+\sqrt{1+4N}}{2}$, $m_3 = -\frac{Gr_1}{m_1^2+m_1-N}$, $m_4 = -\frac{Gm_1/Sc}{(Sc^2+Sc-N)}$, $m_5 = -(m_2 + m_3)$.

Skin friction coefficient is defined as

$$\tau = \frac{\tau_f}{\rho u_w v_w} = \left(\frac{dU}{dY}\right)_{Y=0} = -m_2 m_5 - m_3 m_1 - Scm_4, \tag{2.15}$$

where $\tau_f = \mu \left(\frac{du}{dy}\right)_{y=0}$, is wall shear stress.

Nusselt number is defined as

$$Nu = \frac{q_w \nu}{(T_w - T_\infty) k v_w} = -\left(\frac{d\theta}{dY}\right)_{Y=0} = -m_1, \tag{2.16}$$

where $q_w = -k\left(\frac{dT}{dy}\right)_{y=0}$, is local surface heat flux.

3 Results and discussion

In order to get the effects of permeability parameter, magnetic field parameter, Schmidt number, Prandtl number, heat source parameter and angle of inclination on velocity temperature and concentration we plot the Fig.1-Fig.8. For numerical calculation, here we consider $\Pr = 0.71$, $Sc = 0.60$, $S = 0.1$, $K = 1$, $M = 1, Gr = 1$ $Gm = 0.5$ and $\phi = \pi/6$.

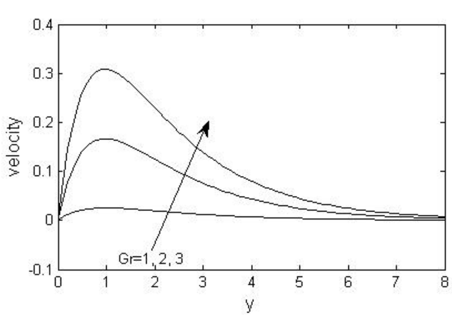

Figure 2: Velocity profile for different values of Gr

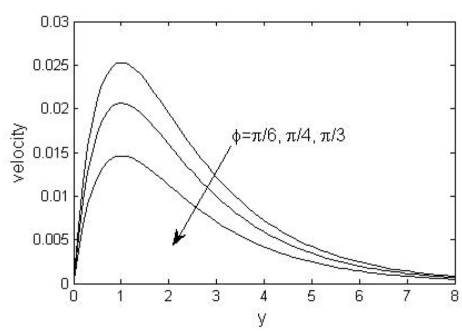

Figure 3: Velocity profile for different values of ϕ

Fig.2 presents the velocity profile for various values of thermal Grashof number (Gr). The Grashof number (Gr) approximates the ratio of buoyancy to viscous force acting on fluid. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. The influence of angle of inclination on velocity profile is displayed in Fig.3.

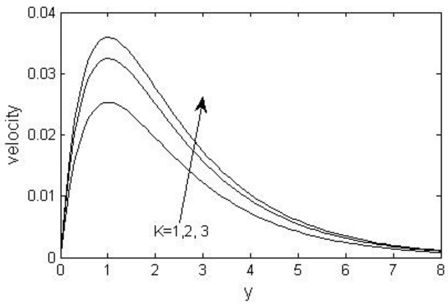

Figure 4: Velocity profile for different values of K

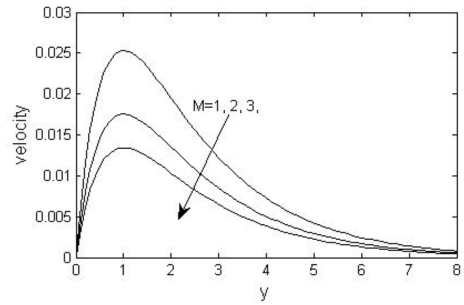

Figure 5: Velocity profile for different values of M

The velocity decreases as angle of inclination (ϕ) increases this is due to the reduction in thermal buoyancy force.

Fig.4 illustrates the variation of velocity profile for various values of the permeability parameter (K). The velocity increases with an increase in permeability parameter K. The effect of magnetic field parameter (M) on velocity distribution is shown in Fig.5. As expected the velocity decrease with an increase in magnetic field parameter$M$, due to Lorentz force, which opposes the flow.

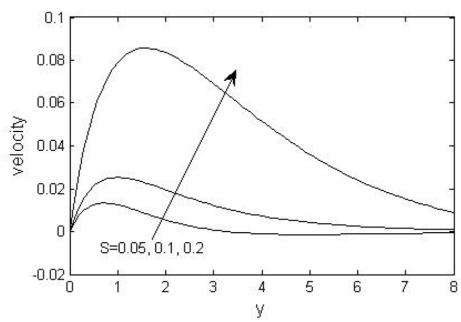

Figure 6: Velocity profile for different values of S

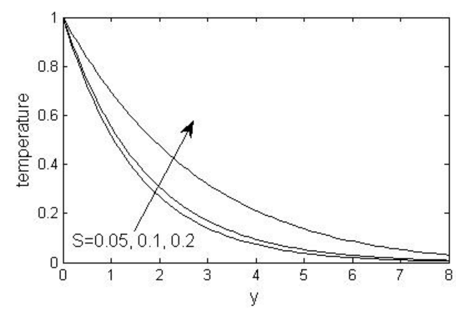

Figure 7: Temperature profile for different values of S

Fig.6 and Fig.7 depict the velocity and temperature profile for different values of the heat source parameter (S). It is observed that an increase in the heat source parameter S results in an increase in velocity and temperature profile.

Fig.8 and Fig. 9 display the effect of Prandtl number (Pr) and Schmidt number (Sc) on temperature and concentration profile. It is observed that temperature and concentration experience reduction for increasing value of Prandtl number Pr and Schmidt number Sc. Prandtl number Pr and Schmidt number Sc signifies the viscosity with thermal diffusivity. This implies that heavier species with low diffusion causes significant fall in temperature and concentration distribution. The numerical results for skin friction (τ) and Nusselt number are entered in Table 1 and Table 2. The effects of various parameters on skin friction (τ) at the surface are shown in Table 1. The skin friction (τ) decreases as Pr, M and K increase but increases with an increase in Sc and ϕ. From Table 2 it can be seen that Nusselt number (Nu) at the surface increases due to Pr and decreases as S increases.

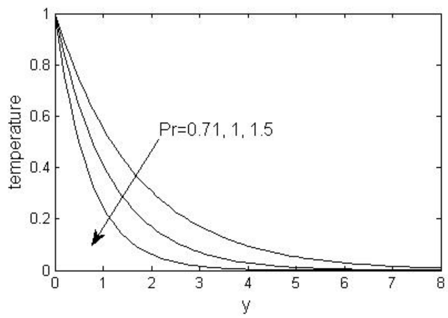

Figure 8: Temperature profile for different values of Pr

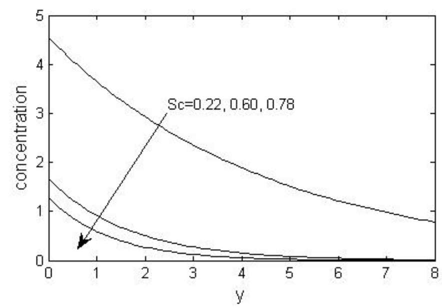

Figure 9: Concentration profile for different values of Sc

Table 1: Skin fiction coefficient for different values of Pr, Sc,M,ϕ and K.

Pr	Sc	M	ϕ	K	τ
1	0.22	3	$\pi/6$	1	-1.2681
5	0.22	3	$\pi/6$	1	-2.0206
1	0.6	3	$\pi/6$	1	-0.0169
1	0.6	1	$\pi/6$	1	-0.0223
1	0.6	3	$\pi/4$	1	-0.0138
1	0.6	3	$\pi/6$	2	-0.0183

Table 2: Nusselt number for different values of Pr and S

Pr	S	N_u
1	0.1	0.8873
5	0.1	4.8979
1	0.2	0.7236

4 Conclusion

- The velocity profile increases with the increase of thermal Grashof number (Gr) and heat source parameter (S) as well as permeability parameter (K) of the medium and decreases with the increase of magnetic field parameter (M) and inclination angle (ϕ).
- Skin friction increases with an increase in Schmidt number (Sc) and inclination angle (ϕ) and it shows the reverse effect in case of Prandtl number (Pr), magnetic field parameter (M) and permeability parameter (K).
- The temperature of the fluid increases with the heat source parameter (S) and decreases with the increase in Prandtl number (Pr).
- The rate of heat transfer i.e. Nusselt number (Nu) increases with an increase of Prandtl number (Pr) and decreases with an increase of heat source parameter (S).

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