# AN INTRODUCTION TO FUNDAMENTALS OF SOFT SET THEORY AND ITS HYBRIDS WITH APPLICATIONS

By
**[1]D. Singh, [2] I. A. Onyeozili, [3]J. N. Singh**
[1](Former Professor, Indian Institute of Technology, Bombay)
Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria.
E-mail:mathdss@yahoo.com
[2]Department of Mathematics, University of Abuja., Abuja, Nigeria.
E-mail:ijeozili@gmail.com
[3]Department of Mathematics and Computer Science,
Barry University,11300 NE Second Avenue
Miami Shores, FL 33161, USA
E-mail: singh_j_n@hotmail.com (corresponding author's e-mail)
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**Abstract**
This work is a comprehensive outline of fundamentals of soft set theory and some of its hybrids such as soft multiset, multi soft set, fuzzy soft set and intuitionistic fuzzy soft set. Most of the previous results and several new observations are given. Finally, a number of extant mathematical models constructed for various soft set theories to solve real-life problems are described and illustrated.

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## 1. Introduction

A number of real-life problems in areas such as engineering, medical sciences, social sciences, economics, management science, etc., involves imprecise and uncertain data which cannot be effectively analyzed by extant mathematical methods such as probability theory, fuzzy set theory, rough set theory, vague set theory, game theory, and interval mathematics, largely due to inadequacy of the parameterization tools associated with these theories (see [32] for details).

Molodtsov [32] in 1999 developed the concept of soft set theory as a new mathematical tool with adequate parameterization for dealing with problems involving uncertainties. Following Molodtsov [32 ], Maji *et al*. [27] were the first to elaborately describe various operations of soft sets and their basic properties. Pei and Miao [35], Ali *et al*. [2], Ge and Yang [19], Sezgin and Atagun [44], etc., modified and improved the findings of Maji *et al.* [27]. Ali *et al*. [2] also introduced new operations on soft sets along with a new notion of the complement of a soft set. Other contributions on the topic include [20, 38, 44, 45, 46, 47, 49, 50, 51]. In the sequel, various hybrid structures, formed by combining soft sets with multisets, fuzzy sets, intuitionistic fuzzy sets, etc., have been developed.
Alkhazaleh *et al*. [4] initiated the concept of soft multiset and discussed its basic operations. Pinaki Majumdar [37] and, Babitha and Sunil [7] redefined the notion of soft multiset and its operations using count function. Herawan *et al.* [21, 22, 23] introduced the concept of multi soft set, AND-and, OR-operations on multi soft sets which is applied for finding reducts and core of attributes in a multi-valued information system.

In recent years, a number of researchers have contributed toward fuzzification of soft set theory, leading to a more generalized concept. Maji *et al.* [24 ] in 2001 proposed the concept of fuzzy soft set and discussed some properties and results regarding fuzzy soft union, intersection, complement, etc., which were revised and improved by Ahmad and Kharal [1 ]. The notion of intuitionistic fuzzy soft set, a combination of soft sets and intuitionistic fuzzy sets was introduced and some basic operations were presented in [8, 25]. Other related works include [3, 14, 25, 28, 30, 36, 40, 54].

Various applications of the soft set theory were made by several authors. Maji *et al*. [26 ] gave a practical application of soft set reduction using the rough set theory of Pawlak [33]. Later, Cagman and Enginoglu [9] introduced new products on soft sets and applied them in decision-making problem. Cagman *et al.* [11] extended the work in [9] to fuzzy soft set theory and also applied it to solve decision-making problems. Other works on the application of soft set theory and related concepts include [7, 13, 16, 22, 29, 37, 40, 41, 49, 53,54].

In an attempt to broaden the application of soft set theory especially in decision-making, some researchers such as Cagman and Enginoglu [10,12 ], Manash *et al*. [29], Chetia and Das[15], Rajarajeswari and Dhanalakshima [39 ] and Herawan *et al*. [23] introduced the concept of soft matrix, fuzzy soft matrix, intuitionistic fuzzy soft matrix, multi soft matrix and illustrated their applications.

In this work, a comprehensive study of the fundamentals of soft set theory, some of its hybrids such as soft multiset, fuzzy soft set, etc., and their applications are presented. The paper is organized as follows: In section 2, basics of the soft set and some of its hybrids are reviewed. In section 3, various operations on soft sets and their properties are systematized. In Section 4, matrix representations of the soft set, soft multiset, fuzzy soft set and intuitionistic fuzzy soft set are presented. Finally, section 5 presents some applications of soft set theory to decision-making problems.

## 2. Soft set and its hybrids

This section reviews the notions of soft set, soft multiset, multi soft set, fuzzy soft set, and intuitionistic fuzzy soft set.

### 2.1 Soft Set

**Definition 2.1.1 [32 ]: Soft set**

Let $U$ be an initial universe set of objects and $E$ a set of parameters or attributes with respect to $U$. Let $P(U)$ denote the power set of $U$ and $A \subseteq E$. A pair ($F$, $A$) is called a soft set over $U$, where $F$ is mapping given by

$$F : A \to P(U).$$

In other words, a soft set over $U$ is a parameterized family of subsets of $U$. For $e \in A,\ F(e)$ may be considered as the set of e-elements or e-approximate elements of the soft set ($F$, $A$). Thus, ($F$, $A$), $A = \{e_1, e_2, e_3\} \subset E$ can be viewed as

$$(F, A) = \{F(e) \in P(U) : e \in E,\ F(e) = \varnothing \text{ if } e \notin A\}.$$

**Example 2.1[27]**

Assume that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universe set consisting of a set of six houses under consideration, $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters with respect to $U$, where each parameter $e_i, i = 1, 2, \ldots, 5$ stands for ‘expensive’, ‘beautiful’, ‘cheap’, ‘modern’, ‘wooden’, respectively, and $A = \{e_1, e_2, e_3\} \subset E$. Suppose a soft set ($F$, $A$) describes the attractiveness of the houses. Let $F(e_1) = \{h_2, h_4\}, F(e_2) = \{h_1, h_3, h_5\}$ and $F(e_3) = \{h_3, h_4, h_5\}$. Then the soft set (*F*,*A*) is a parameterized family $\{F(e_i): i=1,2,3\}$ of subsets of *U*, written as *(F ,A)* = *{e₁={h₂,h₄}, e₂={h₁,h₃,h₅}, e₃={h₃,h₄,h₅}}*. The soft set (*F*, *A*) can also be represented as a set of ordered pairs viz.,

$$(F,A)=\{(e_1,F(e_1)),(e_2,F(e_2)),(e_3,F(e_3))\}$$

$$\text{or,}(F,A)=\{(e_1,\{h_2,h_4\}),(e_2,\{h_1,h_3,h_5\}),(e_3,\{h_3,h_4,h_5\})\}.$$

Other notations for (*F*,*A*) are $F_A$ or $(F_A,E)$.

**Definition 2.1.2 [ 27 ]: Soft subset/soft equal**

Let (*F*,*A*) and (*G*,*B*) be two soft sets over a common universe *U*.

*(a) (F,A) is a soft subset of (G,B), denoted* $(F,A)\tilde{\subseteq}(G,B)$, *if*

*(i)* $A\subseteq B$*, and*

*(ii)* $\forall\ e\in A, F(e)$ and $G(e)$ *are identical approximations.*

*(b) (F,A) is soft equal to (G,B), denoted (F,A) = (G,B), if* $(F,A)\tilde{\subseteq}(G,B)$ *and* $(G,B)\tilde{\subseteq}(F,A)$.

Pei and Miao [35] pointed out that in, general, *F*(*e*) and *G*(*e*) may not be identical and modified the definition of the soft subset in the following way:

**Definition 2.1.3 [35 ]: Soft subset redefined**

For two soft sets (*F*, *A*) and (*G*, *B*) over a universe *U*, $(F,A)\tilde{\subseteq}(G,B)$ if

*(i)* $A\subseteq B$*, and*

*(ii)* $\forall\ e\in A,\ F(e)\subseteq G(e)$.

**Example 2.2**

Let $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}$ be a universe set and $E=\{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters. Let $A=\{e_1,e_3,e_5\}\subset E$ and $B=\{e_1,e_2,e_3,e_5\}\subset E$.

Suppose (*F*, *A*) and(*G*, *B*) are two soft sets over *U* where

$F(e_1)=\{u_2,u_4\}, F(e_2)=\{u_3,u_4,u_5\}, F(e_5)=\{u_1\}$ and

$G(e_1)=\{u_2,u_4\}, G(e_2)=\{u_1,u_3\}, G(e_3)=\{u_3,u_4,u_5\}, G(e_5)=\{u_1,u_4\}$. Then

$(F,A)\tilde{\subset}(G,B)$ since. But $(G,B)\tilde{\not\subset}(F,A)$. Hence $(F,A)\neq(G,B)$

**Definition 2.1.4 [27 ]: Not Set**

Let $E=\{e_1,e_2,e_3,\dots,e_n\}$ be a set of parameters. The "Not set of *E*', denoted $\neg E$, is defined by $\neg E=\{\neg e_1,\neg e_2,\neg e_3,\dots,\neg e_n\}$, where $\neg e_i$ means " not $e_i$", $\forall\ i=1,2,3,\dots,n$

**Definition 2.1.5 [27 ]: Null and Absolute Soft Sets**

Let (*F*, *A*) be a soft set over a universe $U$.

*(a) (F, A) is called a null soft set, denoted* $\tilde{\emptyset}$*, if* $\forall\ e\in A,\ F(e)=\emptyset$ *(null set).*

*This definition is confusing since the parameter set for* $\tilde{\emptyset}$ *is not clear. Accordingly, Ali et. al.[2] provided the following definition:*

*(F, A) is called a relative null softest (with respect to the parameter set A), denoted* $\tilde{\emptyset}_A$*, if* $\forall\ a\in A,\ F(a)=\emptyset$.

*(b) (F, A) is called an absolute soft set, denoted* $\tilde{A}$*, if* $\forall\ e\in A,\ F(e)=U$.

*In line with the modification provided above, Ali et. al.[2] redefined the whole softest as follows:*

*(F, A) is called a relative whole soft set, denoted* $U_A$, *if* $\forall\ \alpha \in A, F(\alpha) = U$ .

*In fact, this is equivalent to the one given in [27] since* $\tilde{A}$ *is with respect to the parameter set A (see [19] for details).*

**Definition 2.1.6 [27]: Complement Soft Set**

The complement of a soft set (*F*, *A*), denoted $(F, A)^C$, is defined as, $(F, A)^C = \left(F^C, \neg A\right)$ where $F^C : \neg A \to P(U)$ is a mapping given by $F^C(\alpha) = U - F(\neg\alpha)\ \forall \alpha \in \neg A$.

Later, Ali et al. [2] introduced a new notion of complement called "relative complement", which is defined in the next definition.

**Definition 2.1.7 [2]: Relative Complement**

The relative complement of a soft set (*F*, *A*) denoted by $(F, A)^r$ is defined by $(F, A)^r = \left(F^r, A\right)$ where $F : A \to P(U)$ is a mapping given by $F^r(\alpha) = U - F(\alpha),\ \forall\ \alpha \in A$

.

It is to be noted that a number of definitions provided in [27] were required to be modified since several related results were found counter-intuitive (see [2, 9, 11, 19, 27]).

## 2.2 Soft Multiset

The notion of soft multiset ( soft *m*set for short) was first introduced by Alkhazaleh *et al*. [4 ] and defined in the following way.

**Definition 2.2.1 [4 ] : Soft *m*set**

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \varnothing$ and let $\left\{E_{U_i} : i \in I\right\}$ be a collection of sets of parameters.

Let $U = \prod_{i \in I} P\left(U_i\right)$ where $P\left(U_i\right)$ is the power set of $U_i, E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$ .

A pair (*F*, *A*) is called a soft multiset over *U* where *F* is a mapping given by $F : A \to U$ .

**Example 2.4 [4 ]**

Suppose that $U_1, U_2$ and $U_3$ are three universes. Let a soft *m*set (*F*, *A*) describe the *attractiveness* of 'houses', 'cars' and 'hotels' which Mr. *X* says is considering for *accommodation* purchase, *transportation* purchase, and *venue* purchase to hold a wedding celebration, respectively.

Let $U_1 = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, $U_2 = \{c_1, c_2, c_3, c_4, c_5\}$ and $U_3 = \{v_1, v_2, v_3, v_4\}$ .

Let $E_{U_i} = \left\{E_{U_1}, E_{U_2}, E_{U_3}\right\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \left\{e_{U_1,1} = \text{expensive},\ e_{U_1,2} = \text{cheap},\ e_{U_1,3} = \text{beautiful},\right.$$

$$\left. e_{U_1,4} = \text{wooden},\ e_{U_1,5} = \text{in green surroundings}\right\},$$

$$E_{U_2} = \{ e_{U_2},1 = \text{expensive},\ e_{U_2},2 = \text{cheap},\ e_{U_2},3 = \text{model2000 and above},$$

$$e_{U_2},4 = \text{black},\ e_{U_2},5 = \text{Made in Japan},\ e_{U_2},6 = \text{ Made in Malaysia} \}$$

and

$$E_{U_3} = \{ e_{U_3},1 = \text{expensive},\ e_{U_3},2 = \text{cheap},\ e_{U_3},3 = \text{majestic},$$

$$e_{U_3},4 = \text{in Kuala Lumpue},\ e_{U_3},5 = \text{in Kajang} \}.$$

Let $U = \prod_{i=1}^{3} P(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$ such that

$$A = \{ a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),$$

$$a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),\ a_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),$$

$$a_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),\ a_6 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}),$$

$$a_7 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}),\ a_6 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2}) \}.$$

Suppose that

$$F(a_1) = (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\}),$$

$$F(a_2) = (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\}),$$

$$F(a_3) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \phi),$$

$$F(a_4) = (\{h_1, h_4, h_6\}, \phi, \{v_1, v_4\}),$$

$$F(a_5) = (\{h_1, h_4\}, \{c_1, c_2\}, \{v_1\}).$$

$$F(a_6) = (\{h_1, h_4, h_5\}, \{c_1, c_2\}, \{v_1, v_2, v_2, v_4\}),$$

$$F(a_7) = (\{h_1, h_4\}, \varnothing, \{v_2\}), \text{and}$$

$$F(a_8) = (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}).$$

Then we can view the soft set $(F, A)$ as consisting of the following collection of approximations:

$$(F, A) = \{ (a_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})), (a_2, (\{h_2, h_3, h_6\}, \{c_1, c_2, c_4, c_5\}, \{v_2\})),$$

$$(a_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \phi)), (a_4 (\{h_1, h_4, h_6, \phi\}, \{v_1, v_4\})),$$

$$(a_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})),\ (a_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \{v_1, v_2, v_3, v_4\})),$$

$$(a_7, (\{h_1, h_4\}, \phi, \{v_3\})),\ (a_8, (\{h_2, h_3, h_6\}, \{c_1, c_2\}, \{v_1, v_4\})) \}.$$

Pinaki Majumdar [37 ] redefined the notion of soft *m*set using count function as follows:

**Definition 2.2.2 [37 ] :Soft *m*set**
Let *U* be a universe set, *E* a set of parameters and *J* the set of all non-negative integers. Let $\tilde{P}(U)$ be the collection of all multisets defined on *U* and $A \subseteq E$.
A triple $\langle F, A, C_F \rangle$ is a soft multiset characterized by its soft count function $C_F : A \to J^U$ which is defined as $C_F(e) = C_F^e \in J^U$, $C_F^e : U \to J$ is the parameterized count function and $F : A \to \tilde{P}(U)$ is defined such that corresponding to each e ϵ E, every element u∈$U$ occurs exactly $C_F^e(U)$ times in *F*(*e*).

**Example 2.5[37]**
Let the universe set *U* and the parameter set *E* be given as follows:

$$U = \{u_1, u_2, u_3, u_4\}$$

and

$$E = \{e_1, e_2, e_3\}.$$

Define $F : E \to \tilde{P}(U)$ as follows:

$$F(e_1) = \left\{\frac{2}{u_1}, \frac{3}{u_2}, \frac{1}{u_3}, \frac{4}{u_4}\right\},$$

$$F(e_2) = \left\{\frac{4}{u_1}, \frac{4}{u_2}, \frac{5}{u_4}\right\}, \text{ and}$$

$$F(e_3) = \left\{\frac{2}{u_1}, \frac{1}{u_2}, \frac{1}{u_3}\right\}.$$

Then $\langle F, E, C_F \rangle$ is a soft multiset, characterized by the soft count function $C_F$ given by the parameterized count functions $C_F^{e_1}, C_F^{e_2}, C_F^{e_3} : U \to J$ which are defined as follows:

$C_F^{e_1}(u_1) = 2,\ C_F^{e_1} = (u_2) = 3,\ C_F^{e_1}(u_3) = 1, C_F^{e_1}(u_4) = 1,$

$C_F^{e_2}(u_1) = 4,\ C_F^{e_2} = (u_2) = 4,\ C_F^{e_2}(u_3) = 0, C_F^{e_2}(u_4) = 5,$ and

$C_F^{e_3}(u_1) = 2,\ C_F^{e_3} = (u_2) = 1,\ C_F^{e_3}(u_3) = 1, C_F^{e_3}(u_4) = 0.$

Babitha and Sunil [6] gave an alternative definition of soft *m*set in the following manner:

**Definition 2.2.3 [6 ]: Soft *m*set**
Let *U* be a universe *m*set, *E* a set of parameters and $A \subseteq E$. Then an ordered pair (*F*, *A*) is called a soft *m*set where *F* is a mapping given by $F : A \to PW(U)$ (The set of all whole submsets of *U*).

**Example 2.6**
Let $U = \left\{\frac{k_1}{u_1}, \frac{k_2}{u_2}, \frac{k_3}{u_3}, \frac{k_4}{u_4}, \frac{k_5}{u_5}, \frac{k_6}{u_6}\right\}$ be the universe *m*set and $E = \{e_1, e_2, e_3, e_4\}$ the set of parameters with respect *U*.

Let $A=\{e_1,e_2,e_4\}\subset E$ where $F(e_1)=\left\{\frac{k_5}{u_5},\frac{k_6}{u_6}\right\}, F(e_2)=\left\{\frac{k_2}{u_2},\frac{k_3}{u_3}\right\}$ and

$F(e_4)=\left\{\frac{k_1}{b_1},\frac{k_4}{b_4}\right\}$.

Then the soft *m*set (*F*, *A*) over *U* is given by $(F,A)=\{F(e_1)\}=\left\{\frac{k_5}{u_5},\frac{k_6}{u_6}\right\}$,

$F(e_2)=\left\{\frac{k_2}{u_2},\frac{k_3}{u_3}\right\}, F(e_4)=\left\{\frac{k_1}{b_1},\frac{k_4}{b_4}\right\}$.

**Remark 2.1**

1. *Definition 2.2.10 and Definition 2.2.9 are equivalent.*
2. *In Definition 2.2.9, U is a crisp universe set while in Definition 2.2.10, U is a universe mset.*
3. *In Definition 2.2.8, since* $\{U_i : i\in I, \text{the index set}\}$ *is a disjoint collection, hence* $\bigcap_{i\in I} U_i=\phi$*, and* $\bigcup_{i\in I} U_i$ *is a crisp universe set.*

Also, $\bigcup_{u\in I} E_{U_i}$ is a multi-parameter set, therefore (*F*, *A*) is a multi-parameterized soft *m*set.

## 2.3 Multi-Soft Sets

**Definition 2.3.1 (Multi-soft set)**

Let $U=\{u_1,u_2,u_3,\ldots,u_{1u1}\}$ be a finite set of objects which may be characterized by a finite family of parameter sets $A=\{A_1,A_2,A_3,\ldots,A_{1A1}\}$ where each parameter set $A_i,\ i=1,2,\ldots,|A|$ represents the *i*th class of parameter and the elements of $A_i$ represent a specific property set. A pair (*F*, *A*) over *U*, which consists of a set of soft sets $(F,A_i)\ 1\le i\le |A|$, is called a multi-soft set over U and is defined as

$$(F,A)=\left\{(F,A_1),(F,A_2),\ldots,\left(F,A_{|A|}\right)\right\}.$$

**Example 2.7**

Let us consider a multi-soft set (*F*, *A*) which describes the attractiveness of houses in an Estate that Mr. *X* (say) is considering to buy.

Suppose that there are ten houses and three parameter sets under consideration. i.e.,

$$U=\{h_1,h_2,h_3,\ldots,h_{10}\},$$

and

$$A=\{A_1,A_2,A_3\}.$$

Let

$A_1$ be a set of cost parameter given by

$A_1$ = {expensive, cheap, very expensive, very cheap},

$A_2$ be a set of location parameter given by

$A_2$ = { low density, high density}, and

$A_3$ be a set of the colour parameter given by

$A_3 = \{$ green, blue, red$\}$.

Let the corresponding soft sets be as follows:

$$(F, A_1) = \{\text{expensive} = \{h_1, h_2, h_3\},\ \text{cheap} = \{h_4, h_{10}\},$$
$$\text{very expensive} = \{h_5, h_8\},\ \text{very cheap} = \{h_6, h_7, h_9\}\}.$$
$$(F, A_2) = \{\text{low density} = \{h_2, h_4, h_6, h_8\},$$
$$\text{high density} = \{h_1, h_3, h_5, h_7, h_9, h_{10}\}\}$$

and

$$(F, A_3) = \{\text{green} = \{h_1, h_3, h_4, h_6\},\ \text{blue} = \{h_2, h_5, h_7, h_9\}$$
$$\text{red} = \{h_8, h_{10}\}\}.$$

Then the multi-soft set $(F, A)$ can be viewed as follows:

$$(F, A) = \{(F, A_1), (F, A_2), (F, A_3)\}$$

$$= \begin{cases} \{\text{expensive} = \{h_1, h_2, h_3\},\ \text{cheap} = \{h_4, h_{10}\} \\ \quad \text{very expensive} = \{h_5, h_8\},\ \text{very cheap} = \{h_6, h_7, h_9\}\} \\ \\ \{\text{low density} = \{h_2, h_4, h_6, h_8\}, \\ \quad \text{high density} = \{h_1, h_3, h_5, h_7, h_9, h_{10}\}\},\ \text{and} \\ \\ \{\text{green} = \{h_1, h_3, h_4, h_6\},\ \text{blue} = \{h_2, h_5, h_7, h_9\}, \\ \quad \text{red} = \{h_8, h_{10}\}\}. \end{cases}$$

**Definition 2.3.2[21] (Multi-soft set in Information System)**

Let $S = (U, A, V, f)$ be a multi-valued information system. Let $S' = (U, a_i, V_{\{0,1\}}, f),\ 1 \le i \le |A|$ be the decomposition of $S$ into $|A|$ binary-valued information systems defined by

$$S' = (U, a_i, V_{\{0,1\}}, f) = \begin{cases} S^1 = (U, a_1, V_{\{0,1\}}, f) \Leftrightarrow (F, a_1) \\ S^2 = (U, a_2, V_{\{0,1\}}, f) \Leftrightarrow (F, a_2) \\ \vdots \\ S^{|A|} = (U, a_{|A|}, V_{\{0,1\}}, f) \Leftrightarrow (F, a_{|A|}). \end{cases}$$

Then we define

$(F,A)=\left((F,a_1),(F,a_2),\ldots,(F,a_{|A|})\right)$ as a multi-soft set over *U*, representing a multi-valued information system $S=(U,A,V,f)$.

### 2.4 Fuzzy Soft Set

Maji et al. [24 ] defined a fuzzy soft set in the following way.

**Definition 2.4.1[24]: Fuzzy Soft Set**

Let *U* be an initial universe, *E* be a set of parameters and $A \subseteq E$. Let $\tilde{P}(U)$ denote the set of all fuzzy sets over *U*. A pair (*F*, *A*) is called a fuzzy soft set over *U* where F is a mapping given by $F: A \to \tilde{P}(U)$.

**Example 2.8[24]**

Suppose $U=\{c_1,c_2,c_3,c_4\}$ is a universal set consisting of four cars, $E=\{e_1,e_2,e_3,e_4,e_5\}$ is a set of parameters with respect to *U* and, $A=\{e_1,e_2,e_3\}\subset E$. Then

$$(F,A)=\left\{F(e_1)=\left\{\frac{c_1}{0.7},\frac{c_2}{0.1},\frac{c_3}{0.2},\frac{c_4}{0.6}\right\},\ F(e_2)=\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.1},\frac{c_4}{0.5}\right\},\right.$$

$$\left. F(e_3)=\left\{\frac{c_1}{0.1},\frac{c_2}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.3}\right\}\right\}$$

is a fuzzy soft set over *U*.

### 2.5 Intuitionistic Fuzzy Soft Set

**Definition 2.5.1 [5 ]: Intuitionistic Fuzzy Set**

An intuitionistic fuzzy set *A* in a non-empty set *X* is of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A respectively such that $0 \le \mu_A(x)+\nu_A(x) \le 1, \forall\ x\in X$.

**Definition 2.5.2 [25 ]: Intuitionistic Fuzzy Soft Set**

Let *U* be an initial universe, *E* a set of parameters, and let $I\tilde{P}(U)$ denote the collection of all intuitionistic fuzzy subsets of *U* and $A \subseteq E$. A pair (*F*, *A*) is called an intuitionistic fuzzy soft set over *U* where *F* is a mapping $F: A \to I\tilde{P}(U)$.

**Example 2.9[25]**

Let $U=\{s_1,s_2,s_3,s_4\}$, $E=\{e_1,e_2,e_3,e_4\}$ and $A=\{e_1,e_2,e_3\}\subset E$. Then (*F*, *A*), given by

$$(F,A)=\left\{F(e_1)=\left\{(s_1,0.8,0.1),(s_2,0.7,0.5),(s_3,0.85,0.1),(s_4,0.5,0.2)\right\},\right.$$

$$F(e_2)=\left\{(s_1,0.6,0.3),(s_2,0.65,02),(s_3,0.5,0.2),(s_4,0.65,0.2)\right\},$$

$$\left. F(e_3)=\left\{(s_1,0.75,0.2),(s_2,0.5,0.3),(s_3,0.5,0.4),(s_4,0.7,0.2)\right\}\right\},$$

is an intuitionistic fuzzy soft set.

## 3. Operations

In this section, the operations of soft sets, soft multisets, multisoft sets, fuzzy soft sets, intuitionistic fuzzy soft sets are discussed and illustrated.

### 3.1 Soft Set Operations

**Definitions 3.1.1 [27 ]**

Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U*.

(i) *The* ***Union*** *of (F, A) and (G, B), denoted* $(F,A)\tilde{\cup}(G,B)$, *is a soft set (H, C), where* $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

(ii) *The* ***intersection*** *of (F,A) and (G,B), denoted* $(F,A)\tilde{\cap}(G,B)$, *is a soft set (H,C),where C=A∩B and ∀ e∈C, H(e) = F(e) or G(e) (as both are the same set).*

(iii) *The* ***AND-operation*** *of (F, A) and (G, B), denoted (F, A) AND (G, B) or* $(F,A)\wedge(G,B)$, *is a soft set defined by* $(F,A)\wedge(G,B) = (H, A\times B)$, *where* $H(a,b) = F(a)\cap G(b),\ \ \forall\ (a,b)\in A\times B$.

(iv) *The* ***OR-operation*** *of (F, A) and (G, B), denoted (F, A) OR (G, B) or* $(F,A)\vee(G,B)$, *is a soft set defined by* $(F,A)\vee(G,B) = (H, A\times B)$, *where* $H(a,b) = F(a)\cup G(b),\ \ \forall\ (a,b)\in A\times B$.

Pei and Miao [35] pointed out that in Definition 3.1.1 (ii) of [27], *F*(*e*) and *G*(*e*) may not be the same set and thus revised the definition as follows:

**Definition 3.1.2 [35 ]: Intersection redefined**

Let (*F*, *A*) and (*G*, *B*) be two soft sets over *U*. The intersection (also called *bi-intersection* in Feng et al. [17 ]) of (*F*, *A*) and (*G*, *B*), denoted $(F,A)\tilde{\cap}(G,B)$, is a soft set (*H*, *C*) where $C = A\cap B$ and $\forall\ e\in C,\ H(e) = F(e)\cap G(e)$.

Moreover, Ahmad and Kharal [1] pointed out that in the above Definition 3.1.2, $A\cap B$ must be non-empty in order to avoid the degenerate cases. The modified definition is as follows:

**Definition 3.1.3 [1]: Intersection redefined**

Let (*F*, *A*) and (*G*, *B*) be two soft sets over *U* with $A\cap B \neq \emptyset$. The intersection of (*F*, *A*) and (*G*, *B*), denoted $(F,A)\tilde{\cap}(G,B)$, is a soft set (*H*, *C*), where $C = A\cap B$ and $\forall\ e\in C,\ H(e) = F(e)\cap G(e)$.

In the sequel Ali et al. [2 ] introduced the following new operations

**Definitions 3.1.4 [2 ]**

Let (*F*, *A*) and (*G*, *B*) be two soft sets over *U*.

(i) *The* ***extended intersection*** *of (F, A) and (G, B), denoted* $(F,A)\cap_{\varepsilon}(G,B)$, *is a soft set (H, C) where* $C = A\cup B$ *and* $\forall\ e\in C$,

$$H(e) = \begin{cases} F(e), \text{ if } e \in A - B \\ G(e), \text{ if } e \in B - A \\ F(e) \cap G(e), \text{ if } e \in A \cap B. \end{cases}$$

*(ii)* *The* ***restricted intersection*** *(also called intersection in Pei and Miao [35] and bi-intersection in Feng et al. [17]) of (F,A) and (G,B), denoted (F,A)⋒(G,B), is a soft (H,C) where* $C = A \cap B$ *and* $\forall\ e \in C,\ H(e) = F(e) \cap G(e)$ *.*

*(iii)* *The* ***restricted union*** *of (F, A) and (G, B), denoted* $(F, A) \cup_R (G, B)$, *is a soft set (H, C) where* $C = A \cap B$ and $\forall\ e \in C$, $H(e) = F(e) \cup G(e)$.

*(iv)* *The* ***restricted difference*** *of (F, A) and (G, B), denoted* $(F, A) \smile_R (G, B)$ *, is a soft set (H, C) where* $C = A \cap B$ *and* $\forall\ e \in C,\ H(e) = F(e) - G(e)$ *.*

Sezgin and Atagun [44 ] defined the following operations.

**Definition 3.1.5 [44 ] : Restricted symmetric difference**

The restricted symmetric difference of (*F*, *A*) and (*G*, *B*), denoted $(F, A)\tilde{\Delta}(G, B)$, is a soft set defined by $(F, A)\tilde{\Delta}(G, B) = \left((F, A) \cup_R (G, B)\right) \smile_R ((F, A) \Cap (G, B))$, or by $(F, A)\tilde{\Delta}(G, B) = \left((F, A) \smile_R (G, B)\right) \cup_R \left((G, B) \smile_R (F, A)\right)$.

This can also be defined as follows:

**Definition 3.1.6**

The restricted symmetric difference of (*F*, *A*) and (*G*, *B*), denoted $(F, A)\tilde{\Delta}(G, B)$ is a soft set (*H*, *C*) where $C = A \cap B$ and $\forall\ e \in C,\ H(e) = F(e)\Delta G(e)$ (the symmetric difference of *F*(*e*) and *G*(*e*)).

**Example 3.1**

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and, $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the parameter set with respect to *U*, and $A = \{e_1, e_2, e_3\} \subset E$ .

Let (*F*, *A*) over *U* be $(F, A) = \left\{\left(e_1, \{h_2, h_4\}\right), \left(e_2, \{h_1, h_3, h_5\}\right), \left(e_3, \{h_3, h_4, h_5\}\right)\right\}$. Let $B = \{e_3, e_4, e_5\}$ and (*G*, *B*) over U be

$(G, B) = \left\{\left(e_3, \{h_1, h_2, h_3\}\right), \left(e_4, \{h_2, h_3, h_6\}\right), \left(e_5, \{h_2, h_3, h_4\}\right)\right\}$.

Then

*(i)* $(F, A)\tilde{\cup}(G, B) = \left\{\left(e_1, \{h_2, h_4\}\right), \left(e_2, \{h_1, h_3, h_5\}\right), \left(e_3, \{h_1, h_2, h_3, h_4, h_5\}\right)\right.$ $\left.\left(e_4, \{h_2, h_3, h_6\}\right), \left(e_5, \{h_2, h_3, h_4\}\right)\right\}$,

*(ii)* $(F, A) \cup_R (G, B) = \left\{\left(e_3, \{h_1, h_2, h_3, h_4, h_5\}\right)\right\}$,

*(iii)* $(F, A) \Cap (G, B) = \left\{\left(e_3, \{h_3\}\right)\right\}$,

*(iv)* $(F, A) \cap_\varepsilon (G, B) = \left\{\left(e_1, \{h_2, h_4\}\right), \left(e_2, \{h_1, h_3, h_5\}\right), \left(e_3, \{h_3\}\right)\right.$ $\left.\left(e_4, \{h_2, h_3, h_6\}\right), \left(e_5, \{h_2, h_3, h_4\}\right)\right\}$,

*(v)* $(F, A) \smile_R (G, B) = \left\{\left(e_3, \{h_3, h_5\}\right)\right\}$,

*(vi)* $(F, A)\tilde{\Delta}(G, B) = \left\{\left(e_3, \{h_1, h_2, h_4, h_5\}\right)\right\}$,

*(vii)* $(F,A)\wedge(G,B)=\{((e_1,e_3),\{h_2\}),((e_1,e_4),\{h_2\}),((e_1,e_5),\{h_4\})$
$((e_2,e_3),\{h_1,h_3\}),((e_2,e_4),\{h_3\}),((e_2,e_5),\{h_5\})$
$((e_3,e_3),\{h_3\}),((e_3,e_4),\{h_3\}),((e_3,e_5),\{h_3,h_4\})\}$, and

*(viii)* $(F,A)\vee(G,B)=\{((e_1,e_3),\{h_1,h_2,h_3,h_4\}),$
$((e_1,e_4),\{h_2,h_3,h_4,h_5\}),((e_1,e_5),\{h_2,h_3,h_4\}),$
$((e_2,e_3),\{h_1,h_2,h_3,h_5\}),((e_2,e_4),\{h_1,h_2,h_3,h_5,h_6\}),$
$((e_2,e_5),\{h_1,h_2,h_3,h_4,h_5\}),((e_3,e_3),\{h_1,h_2,h_3,h_4,h_5\}),$
$((e_3,e_4),\{h_2,h_3,h_4,h_5,h_6\}),((e_3,e_5),\{h_2,h_3,h_4,h_5\})\}.$

The above operations of soft sets also hold for soft multisets, multi soft sets, fuzzy soft sets and intuitionistic fuzzy soft sets [see,7,21,24,25].

## 4 Matrix Representations

We present here matrix representations of the soft set, soft multiset, multi soft set, fuzzy soft set and intuitionistic fuzzy soft set.

### 4.1 Soft Matrix

**Definition 4.1.1[10]: Soft Matrix**

Let *U* be an initial universe, *E* be the set of all possible parameters with respect to *U*, and $A\subseteq E$. Let (F, A) be a soft set over *U*.

Let $U=\{u_1,u_2,u_3,\ldots,u_m\}$ *and* $E=\{e_1,e_2,e_3,\ldots,e_n\}$, then the matrix $\left[a_{ij}\right]$ representing (*F*, *A*) is called the $m\times n$ soft matrix over *U* and is defined as

$$\left[a_{ij}\right]=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix},\quad i=1,2,\ldots,m,\ j=1,2,3,\ldots,n$$

where $a_{ij}=\begin{cases}1, \text{ if } u_i\in F(e_j)\\ 0, \text{ otherwise.}\end{cases}$

**Example 4.1**

Let $U=\{u_1,u_2,u_3,u_4,u_5\}$ be a universe and $E=\{e_1,e_2,e_3,e_4\}$ be a set of parameters with respect to *U*. Let $A=\{e_1,e_3,e_4\}\subset E$ and (*F*, *A*) be a soft set over *U* given by $(F,A)=\{F(e_1)=\{u_3,u_4\},F(e_4)=\{u_1,u_3,u_5\}\}$. Then the soft matrix $\left[a_{ij}\right]$ over *U* representing (*F*, *A*) is given by

$$\left[a_{ij}\right]=\begin{bmatrix}0&0&0&1\\0&0&0&0\\1&0&0&1\\1&0&0&0\\0&0&0&1\end{bmatrix},\ i=1,2,\ldots,5;\ j=1,2,\ldots,4.$$

## 4.2 Soft Multi Matrix

**Definition 4.2.1: Soft Multi Matrix**

Let *U* be a universal *m*set and *E* and set of parameters with respect to *U*. Let *U* be given by

$$U=\left\{\frac{k_1}{u_1},\frac{k_2}{u_2},\frac{k_3}{u_3},\ldots,\frac{k_m}{u_m}\right\},$$

$$E=\{e_1,e_2,e_3,\ldots,e_n\},\ \text{and}\ A\subseteq E.$$

Suppose (*F*, *A*) is a soft *m*set over *U*. Then the matrix $\left[a_{ij}\right]$ representing (F, A) is called the $m\times n$ soft multi-matrix over *U*, defined as

$$\left[a_{ij}\right]=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&&a_{2n}\\\vdots&\vdots&&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{bmatrix},$$

where

$$a_{ij}=\begin{cases}k_i,\ \text{if}\ u_i\in F\left(e_j\right),\ i=1,2,\ldots,m,\ j=1,2,\ldots,n\\0,\ \text{otherwise.}\end{cases}$$

**Example 4.2**

Let us consider the soft *m*set (*F*, *A*) constructed in Example (2.7). Then the soft multi-matrix, representing (*F*, *A*) is given by

$$\left[a_{ij}\right]_{5\times4}=\begin{bmatrix}0&0&0&k_1\\0&k_2&0&0\\0&k_3&0&0\\0&0&0&k_4\\k_5&0&0&0\end{bmatrix}.$$

## 4.3 Multi Soft Matrix

**Definition 4.3.1 [23]: Multi Soft Matrix**

Let $(F,A)=\left(F,a_i\right):i=1,2,\ldots,|A|$ be a multi soft set representing a multi-valued information system *S* = (*U*, *A*, *V*, *f*). The matrix $M_{ai}=\left[a_{ij}\right],1\le i\le|A|,$ where

$$a_{ij}=\begin{cases}1,\ \text{if}\ \left|f(u,\alpha)\right|=1,\ i\le i\le|u|,1\le j\le\left|V_{ai}\right|,u\in U,\alpha\in V_{ai}\\0,\ \text{if}\ \left|f(u,\alpha)\right|=0\end{cases}$$

and dim $\left(M_{ai}\right)=|u|\times\left|V_{ai}\right|$, is called the matrix representation of the soft set $\left(F,a_{ai}\right)$ in the multi soft set (*F*, *A*).

The collection of all matrices representing (*F*, *A*), denoted by $M_A$, is called the multi soft matrices and is defined by

$$M_A = \{M_{ai} : 1 \le i \le |A|\}.$$

## 4.4 Fuzzy Soft Matrix

**Definition 4.4.1 [12]: Fuzzy Soft Matrix**

Let *U* be a universe, *E* a set of parameters, and $A \subseteq E$. Let (*F*, *A*) be a fuzzy soft set over *U*, and let a subset $R_A$ of $U \times E$ be uniquely defined by $R_A = \{(u,e) : e \in A, u \in F(e)\}$.

Then the membership function $\mu_{R_A}$ of $R_A$ is defined by $\mu_{R_A} : U \times E \to [0,1]$ such that $\mu_{R_A}(u,e) = \mu(u,e) \in [0,1]$ is the membership value of $u \in U$ for each $e \in E$.

Now, if $U = \{u_1, u_2, u_3, \ldots, u_m\}$ then, the matrix $[a_{ij}]$ representing (*F*, *A*), called the $m \times n$ fuzzy soft matrix, is defined as

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

where

$$a_{ij} = \begin{cases} \mu(u_i, e_j), & u_i \in F(e_j) \\ 0, & \text{otherwise.} \end{cases}$$

**Example 4.4[12]**

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universe, $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set, and $A = \{e_2, e_3, e_4\}$ .Suppose (*F*, *A*) is a fuzzy soft set over *U* such that $F(e_2) = \left\{\frac{u_1}{0.4}, \frac{u_2}{0.5}, \frac{u_3}{1.0}, \frac{u_4}{0.3}, \frac{u_5}{0.3},\right\}, F(e_3) = \left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.6}, \frac{u_4}{0.5}, \frac{u_5}{1.0}\right\}$ and $F(e_4) = \left\{\frac{u_1}{0.5}, \frac{u_2}{0.5}, \frac{u_3}{0.4}, \frac{u_4}{0.3}, \frac{u_5}{0.9}\right\}$. Then the fuzzy soft matrix $[a_{ij}]$ representing (*F*,*A*) is given by

$$[a_{ij}]_{5\times 4} = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.5 \\ 0 & 0.5 & 0.4 & 0.5 \\ 0 & 1.0 & 0.6 & 0.4 \\ 0 & 0.3 & 0.5 & 0.3 \\ 0 & 0.6 & 1.0 & 0.9 \end{bmatrix}.$$

## 4.5 Intuitionistic Fuzzy Soft Matrix

**Definition 4.5.1 [15]: Intuitionistic Fuzzy Soft Matrix**

Let *U* be an initial universe, *E* a set of parameters, and $A \subseteq E$. Let (*F*, *A*) be an intuitionistic fuzzy soft set over *U*. Then a subset $R_A$ of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F(e)\}$.

The membership function and the non-membership function are defined by

$\mu_{R_A}: U \times E \to [0,1]$ and $\nu_{R_A}: U \times E \to [0,1]$ where $\mu_{R_A}(u,e) \in [0,1]$ and $\nu_{R_A}(u,e) \in [0,1]$ are the membership value and the non-membership value, respectively, of
$u \in U,$ for each $e \in E.$

Now, let $U = \{u_1, u_2, \ldots, u_m\}, E = \{e_1, e_2, \ldots, e_n\}$ and $(\mu_{ij}, \nu_{ij}) = (\mu_{R_A}(u_i, e_j), \nu_A(u_i, e_j)).$ Then

$$\left[(\mu_{ij}, \nu_{ij})\right]_{m\times n} = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots & (\mu_{2n}, \nu_{2n}) \\ \vdots & & \cdots & \vdots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \nu_{mn}) \end{bmatrix}$$

is called an $m\times n$ intuitionistic fuzzy soft matrix of the intuitionistic fuzzy soft set over $U$

**Example 4.5[15]**

Suppose $U = \{u_1, u_2, u_3, u_4\}, E = \{e_1, e_2, e_3, e_4\}$ and $A = \{e_2, e_3, e_4\}$ such that

$$F(e_2) = \left\{ \frac{u_1}{0.4, 0.5}, \frac{u_2}{0.5, 03}, \frac{u_3}{1.0, 0}, \frac{u_4}{0.5, 0.6} \right\},$$

$$F(e_3) = \left\{ \frac{u_1}{0.3, 0.5}, \frac{u_2}{0.4, 0.6}, \frac{u_3}{0.6, 0.2}, \frac{u_4}{0.5, 0.5} \right\}, \text{ and}$$

$$F(e_4) = \left\{ \frac{u_1}{0.5, 0.2}, \frac{u_2}{0.5, 0.5}, \frac{u_3}{0.4, 0.6}, \frac{u_4}{0.3, 0.6} \right\}.$$

Then the intuitionistic fuzzy soft set $(F, A)$ over $U$ is given by

$$(F, A) = \{F(e_2), F(e_3), F(e_4)\}, \text{ and}$$

the corresponding intuitionistic fuzzy soft matrix $\left[(\mu_{ij}, \nu_{ij})\right]$ is given by

$$\left[(\mu_{ij}, \nu_{ij})\right]_{4\times 4} = \begin{bmatrix} 0 & (0.4, 0.5) & (0.3, 0.5) & (0.5, 0.2) \\ 0 & (0.5, 0.3) & (0.4, 0.6) & (0.5, 0.5) \\ 0 & (1.0, 0) & (0.6, 0.2) & (0.4, 0.6) \\ 0 & (0.3, 0.6) & (0.5, 0.5) & (0.3, 0.6) \end{bmatrix}.$$

## 6. Applications of Soft Set Theory

A number of researchers [7,9,10,11,12,13,26,29,31,37,39,42, 43,48] have applied the concepts of soft set theory and its hybrids, to real-life problems involving uncertainties. Such problems involve decision-making, medical diagnosis, forecasting, among others. In this section, some typical applications using different approaches, especially in decision-making, will be discussed.

### 6.1 Soft Set Uni-Int decision making method [9]

Cagman and Enginoglu [9] constructed an Uni-Int (Union-Intersection) soft decision-making method which reduces a large set of alternatives to smaller subsets according to the choice parameters of decision makers. We illustrate this method by taking the following example:

Suppose there are 48 candidates who applied for a position in a company, and there are two decision makers who want to select a smaller set of candidates to be interviewed according to their choice parameters.

Let the set of candidates be $U=\{u_1, u_2, ..., u_{48}\}$ and the set of parameters be $E=\{e_1, e_2, ..., e_7\}$ where each $e_i$, $i= 1,2,...,7$ stands for $e_1$= Experience, $e_2$= Computer knowledge, $e_3$=Training, $e_4$= Young Age, $e_5$=High Education, $e_6$=Marital Status and $e_7$=Good Health. In order to get a set of short-listed candidates, we apply the Uni-Int soft decision making method in steps as follows:

**Step 1:** Assume that the decision makers' choice of parameters are $A=\{e_1, e_2, e_4, e_7\} \subset E$, and $B=\{e_1, e_2, e_5\} \subset E$.

**Step 2:** Assume that the following soft sets $F_A$ *and* $F_B$ are obtained according to their choice parameters:

$F_A = \{(e_1, \{u_4, u_7, u_{13}, u_{21}, u_{28}, u_{31}, u_{32}, u_{36}, u_{39}, u_{41}, u_{43}, u_{44}, u_{48}\}),$
$(e_2, \{u_1, u_3, u_{13}, u_{18}, u_{19}, u_{21}, u_{22}, u_{24}, u_{28}, u_{32}, u_{36}, u_{42}, u_{44}, u_{46}\}),$
$(e_4, \{u_2, u_3, u_{13}, u_{15}, u_{18}, u_{23}, u_{25}, u_{28}, u_{30}, u_{33}, u_{36}, u_{38}, u_{42}, u_{43}\}),$
$(e_7, \{u_1, u_5, u_{12}, u_{13}, u_{17}, u_{20}, u_{24}, u_{28}, u_{29}, u_{34}, u_{36}, u_{41}, u_{45}, u_{47}\})\},$

$F_B = \{(e_1, \{u_3, u_4, u_5, u_8, u_{14}, u_{21}, u_{22}, u_{26}, u_{27}, u_{34}, u_{35}, u_{37}, u_{40}, u_{42}, u_{46}\}),$
$(e_1, \{u_1, u_4, u_7, u_{10}, u_{11}, u_{13}, u_{15}, u_{21}, u_{29}, u_{30}, u_{32}, u_{36}, u_{42}, u_{43}, u_{45}\}),$
$(e_5, \{u_2, u_4, u_8, u_9, u_{12}, u_{13}, u_{14}, u_{16}, u_{17}, u_{21}, u_{23}, u_{28}, u_{36}, u_{42}, u_{44}\})\}.$

**Step 3:** *Compute the* $\wedge$ *- product* $F_A \wedge F_B$ *:*

$F_A \wedge F_B = \{((e_1, e_1), \{u_4,u_{21}\}), ((e_1,e_2), \{u_4, u_7, u_{13}, u_{21}, u_{32}, u_{36}, u_{43}\}),$
$((e_1, e_5), \{u_4, u_{13}, u_{21}, u_{28}, u_{36}, u_{44}\}), ((e_2, e_1), \{u_3, u_{21}, u_{22}, u_{42}, u_{46}\}),$
$((e_2, e_2), \{u_1, u_{13}, u_{21}, u_{32}, u_{36}, u_{42}\}), ((e_2, e_5), \{u_{13}, u_{21}, u_{28}, u_{36}, u_{42}, u_{44}\}),$
$((e_4, e_1), \{u_3, u_{42}\}), ((e_4, e_2), \{u_{13}, u_{15}, u_{30}, u_{36}, u_{42}, u_{43}\}),$
$((e_4, e_5), \{u_2, u_{13}, u_{23}, u_{28}, u_{36}, u_{42}\}), ((e_7, e_1), \{u_5, u_{34}\}),$
$((e_7, e_2), \{u_1, u_{13}, u_{29}, u_{36}, u_{45}\}), ((e_7, e_5), \{u_{12}, u_{13}, u_{17}, u_{28}, u_{36}\})\}.$

**Step 4:** Compute the decision set Uni-Int $(F_A \wedge F_B)$ as follows:

Uni-Int $(F_A \wedge F_B)$ = $\text{Uni}_x\text{-Int}_y$ $(F_A \wedge F_B)$ $\cup$ $\text{Uni}_y\text{-Int}_x$ $(F_A \wedge F_B)$.

Now, $\text{Uni}_x\text{-Int}_y (F_A \wedge F_B) = \bigcup_{x\in A} \left\{ \bigcap_{y\in B} \left( f_{A\wedge B}(x,y) \right) \right\}$

$$= \bigcup \begin{cases} \bigcap\{\{u_4,u_{21}\},\{u_4,u_7,u_{13},u_{21},u_{32},u_{36},u_{43}\},\{u_4,u_{13},u_{21},u_{28},u_{36},u_{44}\}\} \\ \bigcap\{\{u_3,u_{21},u_{22},u_{42},u_{46}\},\{u_1,u_{13},u_{21},u_{32},u_{36},u_{42}\},\{u_{13},u_{15},u_{30},u_{36},u_{42},u_{44}\}\} \\ \bigcap\{\{u_3,u_{42}\},\{u_{13},u_{15},u_{30},u_{36},u_{42},u_{43}\},\{u_2,u_{13},u_{23},u_{29},u_{36},u_{42}\}\} \\ \bigcap\{\{u_5,u_{34}\},\{u_1,u_{13},u_{29},u_{36},u_{45}\},\{u_{12},u_{13},u_{17},u_{28},u_{36}\}\} \end{cases}$$

$$= \bigcup\{\{u_4,u_{21}\},\{u_{42}\},\{u_{42}\},\varnothing\} = \{u_4,u_{21},u_{42}\}, and$$

and $\text{Uni}_y\text{-Int}_x (F_A \wedge F_B) = \bigcup_{y\in B} \left\{ \bigcap_{x\in A} \left( f_{A\wedge B}(x,y) \right) \right\}$

$$= \bigcup \begin{cases} \bigcap\{\{u_4,u_{21}\},\{u_3,u_{21},u_{22},u_{42},u_{46}\},\{u_3,u_{42}\},\{u_5,u_{34}\}\} \\ \bigcap\{\{u_4,u_7,u_{13},u_{21},u_{32},u_{36},u_{43}\},\{u_1,u_{13},u_{21},u_{32},u_{36},u_{42}\},\{u_{13},u_{15},u_{30},u_{36},u_{42},u_{43}\},\{u_1,u_{13},u_{29},u_{36},u_{45}\}\} \\ \bigcap\{\{u_4,u_{13},u_{21},u_{28},u_{36},u_{44}\},\{u_{13},u_{21},u_{28},u_{36},u_{42},u_{44}\},\{u_2,u_{13},u_{23},u_{28},u_{36},u_{42}\},\{u_{12},u_{13},u_{17},u_{28},u_{36}\}\} \end{cases}$$

$$= \bigcup\{\varnothing,\{u_{13},u_{36}\},\{u_{13},u_{36}\}\} = \{u_{13},u_{36}\}.$$

Thus, *Uni-Int* $(F_A \wedge F_B) = \{u_4, u_{21}, u_{42}\} \cup \{u_{13}, u_{36}\} = \{u_4, u_{13}, u_{21}, u_{36}, u_{42}\}$,which is the list of objectically short-listed set of candidates to be finally interviewed.

**6.2 Soft Matrix Max- Min decision-making method [10 ]**

Cagman and Enginoglu[10] constructed a soft max-min decision-making method( SMmDMM) which selects optimum alternatives from a set of alternatives according to decision maker choice of parameters. We demonstrate this method by way of taking an example of decision making problem.

Suppose a man and his wife want to purchase a car from a dealer who has a set of four cars $U=\{c_1, c_2, c_3, c_4\}$, which are characterized by a set of parameters $E=\{e_1, e_2, e_3, e_4\}$,where $e_i$, i= 1, 2, 3, 4, stand for cheap ($e_1$), fuel efficient ($e_2$) modern ($e_3$), durable ($e_4$). The SMmDMM runs in step as follows:

**Step 1:** Assume that the man and his wife choose the sets of parameters $A=\{e_2, e_3, e_4\}$ and $B=\{e_1, e_3, e_4\}$ respectively.

**Step 2**: Assume that the following soft matrices are obtained according to the parameters:

$$[a_{ij}]=\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad [b_{il}]=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

**Step 3**: Find the $\wedge$ product $[c_{ip}]$ of $[a_{ij}]$ and $[b_{il}]$ (since the choices of both the man and his wife are to be considered conjunctively) as follows:

$$[a_{ij}] \wedge [b_{il}] = [c_{ip}] = \begin{bmatrix} 0&0&0&0&1&0&0&1&1&0&0&1&1&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&1&1&1&0&1&1 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0 \end{bmatrix}$$

**Step 4**: Find a max-min one- column decision soft matrix Mm ($[c_{ip}]$) of $[c_{ip}]$ ,defined by

$$[d_{i1}] = \text{Min}([c_{ip}]) = \left[\max_{k \in I}\{t_{ik}\}\right], where\ t_{ik} = \begin{cases} \min\limits_{p \in I_{ik}}\{c_{ip}\}, & I_{ik} \neq \emptyset \\ o & , I_{ik} = \emptyset \end{cases}$$

and $I_{ik} = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leq kn\}$ for all $k \in I = \{1,2,..., n\}$.

$$\text{Thus } [d_{i1}] = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

**Step 5**: Finally, the optimum set of $U$ with respect to $[d_{i1}]$ is determined:

$$\text{Opt}_{[di1]}(U) = \{c_2\}$$

Hence $c_2$ is identified as the car of optimum choice on which both the man and his wife opted to agree to purchase.

**6.3 Soft Multiset approach in decision-making [7]**

Babitha and Sunil [7], extended the soft set reduction approach adopted in [26] to soft multiset case in the following way: Suppose a retail shop keeper wants to select a particular type of bags satisfying his demand.

Let U={10/$b_i$, 15/$b_2$, 7/$b_3$, 8/$b_4$, 18/$b_5$, 11/$b_6$, 10/$b_7$} be a multiset of bags and $A$={leather ($e_1$), cheap ($e_2$), big ($e_3$) discount ($e_4$)} be a set of parameters.

Let a soft multiset (F,A), describing the different types of bags under consideration, be given by $(F,A)$={($e_1$,{10/$b_1$, 15/$b_2$}), ($e_2$,{15/$b_2$, 7/$b_3$, 8/$b_4$}), ($e_3$,{10/$b_1$, 15/$b_2$, 7/$b_3$, 8/$b_4$}), ($e_4$,{18/$b_5$, 11/$b_6$, 10/$b_7$})}.

Suppose that the shop keeper wants to buy a set of bags according to his choice parameter set B={$e_1$, $e_2$, $e_4$} along with weights $w_1$,=0.6, $w_2$,=0.4, $w_4$,=0.4 imposed on the parameters $e_1$, $e_2$, $e_4$ respectively. Let the quantity of bags demanded at any instant and their availability be the same. Applying the algorithm described in [26], the steps are as follows:

**Step1**: The weighted reduct soft set ($F$, $B$) is computed in table 6 below.

Table 6: Reduct soft set ($F$, $B$)

| $U$ | $e_1 \times w_1$ | $e_2 \times w_2$ | $e_4 \times e_4$ | choice value |
|---|---|---|---|---|
| $b_1$ | 10 | 0 | 0 | 6.0 |
| $b_2$ | 15 | 15 | 0 | 14.0 |
| $b_3$ | 0 | 7 | 0 | 2.4 |
| $b_4$ | 0 | 8 | 0 | 3.2 |
| $b_5$ | 0 | 0 | 18 | 16.2 |
| $b_6$ | 0 | 0 | 11 | 9.9 |
| $b_7$ | 0 | 0 | 10 | 9.0 |

**Step2:** FromTable5, the highest choice value is identified which is 16.2

**Step3**: Finally, corresponding to the row with choice value 16.2, 18 bags of category $b_5$ can be bought by shop keeper.

### 6.4 Fuzzy Soft Set in decision-making [11]

Cagman and Enginoglu [11] constructed an aggregate fuzzy soft set decision-making process which chooses the best alternative according to the choice parameters of a decision maker. The method is described below by taking an example.

Suppose a company wants to fill a position and there are eight candidates, that is, $U$= {$u_1$, $u_2$, $u_3$, $u_4$, $u_5$, $u_6$, $u_7$, $u_8$}. If the interviewing committee considers a set of parameters $E$= {$x_1$, $x_2$, $x_3$, $x_4$, $x_5$} where $x_i$, $i$ = 1, 2, 3, 4, 5, stand for "expensive ", "computer knowledge" "young age" "good speaking" "friendly", respectively, and agree to choose a subset $A$={$x_2$,$x_3$,$x_4$} of $E$ for consideration. The goal is to select the best candidate using the aggregate fuzzy soft set decision method described in the following steps :

**Step 1:** Let the fuzzy soft set $\Gamma_A$ over $U$, according to the chosen parameter set $A$={$x_2$, $x_3$, $x_4$} , viz.,

$$\Gamma_{A=} \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U), \text{the fuzzy set over } U\},$$

be given by

$$\Gamma_A=\{(x_2, \{0.3/u_2, 0.5/u_3, 0.1/u_4, 0.8/u_5, 0.7/u_7\}),$$
$$(x_3, \{0.4/u_1, 0.4/u_2, 0.9/u_3, 0.3/u_4\})\},$$
$$(x_4, \{0.2/u_1, 0.5/u_2, 0.1/u_5, 0.7/u_7, 0.1/u_8\})\}.$$

Thus, the fuzzy soft matrix $M\Gamma_A$ of $\Gamma_A$ is

$$M\Gamma_A = \begin{bmatrix} 0 & 0 & 0.4 & 0.2 & 0 \\ 0 & 0.3 & 0.4 & 0.5 & 0 \\ 0 & 0.5 & 0.9 & 0 & 0 \\ 0 & 0.1 & 0.3 & 0 & 0 \\ 0 & 0.8 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}.$$

**Step 2:** Now, the cardinal fuzzy set $c\Gamma_A$ of $\Gamma_A$ viz.,

$$c\Gamma_A = \{\mu_c\Gamma_A^{(x)} / x: x \epsilon E,\ \mu c\Gamma_A^{(x)} \epsilon [0, 1]$$

where $\mu\, {}_c\Gamma_A(x) = \frac{1}{|U|}\sum_{u\in U,\chi\in E} \mu_{\gamma A(x)}(u)$ , is given by

$c\Gamma_A$ = 1 /8{0 .3+0.5+0.1+0.8+0.7, 0.4+0.4+0.9+0.3, 0.2+0.5+0.1+0.7+0.1} = {0.3, 0.25, 0.2} .

That is, $c\Gamma_A$ ={0.3/$x_2$, 0.25/$x_3$, 0.2/$x_4$} and, the row- matrix $Mc\Gamma_A$ of $c\Gamma_A$ is given by $Mc\Gamma_A$ =[0 0.3 0.25 0.2 0]

**Step 3:** Now, the aggregate fuzzy set $\Gamma^*_A$ of $\Gamma_A$ viz.,

$$\Gamma^*_A = \{ \mu\Gamma^*_A (u)/u : u \epsilon U,\ \ \mu\Gamma^*_A (U) \epsilon [0, 1]\}$$

where $\mu\Gamma^*_A (u) = \frac{1}{|E|}\sum_{,\chi e\in E} \mu c\Gamma_A(x)\mu_{\gamma A(x)}(u)$

$$= \frac{1}{|E|}\ \{M\Gamma_A \times M_C\Gamma_A^{T}\}$$

$$= \frac{1}{5}\begin{bmatrix} 0 & 0 & .4 & .2 & 0 \\ 0 & .3 & .4 & .5 & 0 \\ 0 & .5 & .9 & 0 & 0 \\ 0 & .1 & .3 & 0 & 0 \\ 0 & .8 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & .7 & 0 & .7 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ .3 \\ .25 \\ 1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & .028 \\ 0 & .058 \\ 0 & .075 \\ 0 & .021 \\ 0 & .052 \\ 0 & .000 \\ 0 & .070 \\ 0 & .004 \end{bmatrix}.$$

Thus, $\Gamma^*_A$ = {0.028/$u_1$, 0.058/$u_2$, 0.075/$u_3$, 0.021/$u_4$, 0.052/$u_5$, 0.00/$u_6$, 0.070/$u_7$, 0.004/$u_8$}.

**Step 4:** Now, from above the largest membership grade, max $\mu\Gamma^*_A (u)$= 0.075.
Hence, the candidate $u_3$ may be selected for the job.

### 6.5 Fuzzy Soft set in medical diagnosis using fuzzy soft complement [49]

Tridiv and Dusmanta [49] extended Sanchez's approach in [43] adopted for medical diagnosis by using the notion of the complement of a fuzzy soft set. The method is described below by taking an example.

Let $P$={$p_1$, $p_2$, $p_3$} be a set of three patients in a hospital with a set of symptoms $S$={temperature ($s_1$), headaches ($s_2$), cough ($s_3$), stomach problem ($s_4$)}, and related to a set of diseases $D$={diarrhea ($d_1$), malaria ($d_2$)}.

Let ($F$,$D$) be a fuzzy soft set over S where $F$: $D \longrightarrow P$ ($S$) and let ($G$, $S$) be another fuzzy soft set over $P$ where $G : S \rightarrow P(P)$.In order to diagnose which patient is suffering from what disease ($s$), the use of algorithm constructed in [49] is demonstrated as follows:

**Step 1:** Compute the fuzzy soft sets ($F$,$D$) and $(F,D)^C$ as follows:

$(F,D)$={ $F(d_1)$={($s_1$, 0.85, 0), ($s_2$, 0.25 ,0), ($s_3$, 0.55, 0), ($s_4$, 0.30, 0)},
$F(d_2)$= {($s_3$, 0.75, 0), ($s_4$, 0.50, 0, ($s_3$, 0.45, 0), ($s_4$, 0.45, 0)}}.
$(F,D)^C$={$F^C(d_1)$= {($s_1$, 1.0, 0.85), ($s_2$, 1.0, 0.25), ($s_3$, 1.0, 0.55) ($s_4$,1.0,0.30)}.
$F^C(d_2)$={($s_1$, 1.0, 0. 75), ($s_2$, 1.0, 0.50), ($s_3$, 1.0, 0.45), ($s_4$, 1.0, 0.45)}}.

**Step 2:** Compute the fuzzy soft matrices $R_1$ and $R_2$ corresponding to ($F$, $D$) and ($F$, $D$)$^C$, respectively :

$$R_1=\begin{bmatrix} d_1 & d_2 \\ (0.85, 0) & (0.75, 0) \\ (0.25, 0) & (0.50, 0) \\ (0.55, 0) & (0.45, 0) \\ (0.30, 0) & (0.45, 0) \end{bmatrix}, \quad R_2=\begin{bmatrix} d_1 & d_2 \\ (1.0, 0.85) & (1.0, 075) \\ (1.0, 0.25) & (1.0, 0.50) \\ (1.0, 055) & (1.0, 0.45) \\ (1.0,0.30) & (1.0, 0.45) \end{bmatrix}$$

**Step 3:** Compute the fuzzy soft sets ($G$,$S$) and $(G, S)^C$ as follows :

$(G,S)$= {G($s_1$)= {$p_1$, 0.75, 0), ($p_2$, 0.40, 0), ($p_3$, 0.70, 0)},
G ($s_2$)= {$p_1$, 0.40, 0), ($p_2$, 0.50, 0), ($p_3$, 0.40, 0)},
G ($s_3$)= {$p_1$, 0.90, 0), ($p_2$, 0.30, 0), ($p_3$, 0.60, 0)},
G ($s_4$)= {$p_1$, 0.75, 0), ($p_2$, 0.40, 0), ($p_3$, 0.30, 0)}}.
$(G,S)^C$={$G^C(s_1)$={($p_1$, 1.0, 0.75), ($p_2$, 1.0, 0), ($p_3$, 1.0, 0.70)},
$G^C(s_2)$={($p_1$, 1.0, 0.40), ($p_2$, 1.0, 0.50), ($p_3$, 1.0, 0.60)},
$G^C(s_3)$={($p_1$, 1.0, 0.90), ($p_2$, 1.0, 0.30), ($p_3$, 1.0, 0.60)},
$G^C(s_3)$={($p_1$, 1.0, 0.75), ($p_2$, 1.0, 0.40), ($p_3$, 1.0, 0.30)}}.

**Step 4:** Compute the corresponding fuzzy soft matrices $Q_1$ and$Q_2$ as follows :

$$Q_1 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ (0.75, 0) & (0.40, 0) & (0.90, 0) & (0.75, 0) \\ (0.40, 0) & (0.50, 0) & (0.30, 0) & (0.40, 0) \\ (0,70, 0) & (0.40, 0) & (0.60, 0) & (0.30, 0) \end{bmatrix}.$$

$$Q_2=\begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix}$$
$$\begin{bmatrix} (1.0, 0.75) & (1.0, 0.40) & (1.0, 0.90) & (1.0, 0.75) \\ (1.0, 0.40) & (1.0, 0.50) & (1.0, 0.30) & (1.0, 0.40) \\ (1.0, 0.70) & (1.0, 0.40) & (1.0, 0.60) & (1.0, 0.30) \end{bmatrix}.$$

**Step 5:** Compute the products $T_1 = Q_1.R_1$, $T_2=Q_1.R_2$, $T_3= Q_2. R_1$ , and $T_4=Q_2. R_2$ as follows:

$$T_1=\begin{bmatrix} d_1 & d_2 \\ (0.75,\ 0) & (0.75,\ 0) \\ (0.40,\ 0) & (0.50,\ 0) \\ (0.70,\ 0) & (0.70,\ 0) \end{bmatrix}, \quad T_2 = \begin{bmatrix} d_1 & d_2 \\ (0.90,\ 0.25) & (0.90,\ 0.45) \\ (0.50,\ 0.25) & (0.50,\ 0.45) \\ (0.70,\ \ 0) & (0.70,\ \ 0) \end{bmatrix},$$

$$T_3 = \begin{bmatrix} d_1 & d_2 \\ (0.85,\ 0.40) & (0.75,\ 0.40) \\ (0.85,\ 0.30) & (0.75,\ 0.30) \\ (0.85,\ 0.30) & (0.75,\ 0.03) \end{bmatrix}, \text{ and } T_4 = \begin{bmatrix} d_1 & d_2 \\ (0.85,\ 0.40) & (0.75,\ 0.40) \\ (0.85,\ 0.30) & (0.75,\ 0.30) \\ (0.85,\ 0.30) & (0.75,\ 0.03) \end{bmatrix}.$$

**Step 6:** Compute the corresponding *membership value* matrices $MV(T_1)$, $MV(T_2)$, $MV(T_3)$, and $MV(T_4)$ as follows:

$$MV(T_1) = \begin{bmatrix} d_1 & d_2 \\ 0.75 & 0.75 \\ 0.40 & 0.50 \\ 0.70 & 0.70 \end{bmatrix}, \qquad MV(T_2) = \begin{bmatrix} d_1 & d_2 \\ 0.65 & 0.45 \\ 0.25 & 0.05 \\ 0.70 & 0.70 \end{bmatrix},$$

$$MV(T_3) = \begin{bmatrix} d_1 & d_2 \\ 0.45 & 0.35 \\ 0.55 & 0.45 \\ 0.55 & 0.45 \end{bmatrix}, and \qquad MV(T_4) = \begin{bmatrix} d_1 & d_2 \\ 0.60 & 0.50 \\ 0.60 & 0.55 \\ 0.70 & 0.55 \end{bmatrix}.$$

**Step 7:** Compute the *diagnosis score* matrices $DS_2$ and $DS_1$ defined as follows :

$$DS_1 = MV(T_1) - MV(T_3) = \begin{bmatrix} d_1 & d_2 \\ 0.30 & 0.40 \\ -0.15 & 0.05 \\ 0.15 & 0.15 \end{bmatrix}, \text{ and}$$

$$DS_2 = MV(T_2) - MV(T_4) = \begin{bmatrix} d_1 & d_2 \\ 0.05 & 0.05 \\ -0.35 & -0.50 \\ 0 & 0.15 \end{bmatrix}.$$

**Step 8:** Find the *difference* table $D$ between $DS_1$ and $DS_2$ as:

$D = DS_1 - DS_2 =$

| | $d_1$ | $d_2$ |
|---|---|---|
| $p_1$ | 0.25 | 0.45 |
| $p_2$ | 0.20 | 0.55 |
| $p_3$ | 0.15 | 0 |

This leads to the conclusion that patient $p_1$ and $p_2$ both are suffering from malaria, while patient $p_3$ is suffering from diarrhea.

**6.6 Intuitionistic fuzzy Soft matrix in decision-making [39]**

Rajarajeswari and Dhanalakshmi[39] defined different types of intuitionistic fuzzy soft matrices together with some operations and applied them in solving decision-making problems. The method is described below by taking an example.

Let $U=\{c_1, c_2, c_3, c_4\}$ be a set of candidates appearing in an interview for appointment as a manager in a company, and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters characterizing a candidate, where $e_i$ (i=1, 2, 3) stand for "confident", " ability to take risk", "academically sound", respectively. Let *X* and *Y* be the decision experts to conduct the selection procedure,

**Step 1:** Let the intuitionistic fuzzy soft sets (*F*,*E*) and (*G*,*E*), representing the evaluation of the candidates by experts *X* and *Y* respectively, be as follows : $(F,E)=\{ F(e_1)=\{(c_1, 0.7, 0.1), (c_2, 0.5, 0.5), (c_3, 0.1, 0.8), (c_4, 0.4, 0.6)\},$

$F(e_2)=\{(c_1, 0.5, 0.4), (c_2, 0.4, 0.6), (c_4, 0.5, 0.4), (c_4, 0.7, 0.3)\},$

$F(e_3)=\{(c, 0.5, 0.4), (c_2, 0.7, 0.2), (c_3, 0.6, 0.3), (c_4, 0.5, 0.4)\}.$

$(G,E)= \{G(e_1)=\{(c_1, 0.6, 0.2), (c_2, 0.6, 0.4), (c_3, 0.2, 0.7), (c_4, 0.6, 0.4)\},$

$G(e_2)=\{(c_1, 0.6 , 0.3), (c_2, 0.5, 0.5), (c_3, 0.6, 0.4), (c_4, 0.8, 0.1)\},$

$G(e_3)=\{(c_1, 0.5, 0.5), (c_2, 0.8, 0.1), (c_3, 0.7, 0.7), (c_4, 0.5, 0.4)\}.$

**Step 2:** Compute the intuitionistic fuzzy soft matrices *A* and *B* and their complements $A^c$ and $B^c$, corresponding to (*F*, *E*) and (*G*, *E*), respectively as follows :

$$A=\begin{bmatrix} e_1 & e_2 & e_3 \\ (0.7, 0.1) & (0.6, 0.3) & (0.5, 0.4) \\ (0.5, 0.5) & (0.4, 0.6) & (0.7, 0.2) \\ (0.1, 0.8) & (0.5, 0.4) & (0.6, 0.3) \\ (0.4, 0.6) & (0.7, 0.3) & (0.5, 0.4) \end{bmatrix}, \quad B=\begin{bmatrix} e_1 & e_2 & e_3 \\ (0.6, 0.2) & (0.6, 0.3) & (0.5, 0.5) \\ (0.6, 0.4) & (0.5, 0.5) & (0.8, 0.1) \\ (0.2, 0.7) & (0.6, 0.4) & (0.7, 0.1) \\ (0.6, 0.4) & (0.8, 0.1) & (0.5, 0.4) \end{bmatrix},$$

$$A^C=\begin{bmatrix} e_1 & e_2 & e_3 \\ (0.1, 0.7) & (0.3, 0.6) & (0.4, 0.5) \\ (0.5, 0.5) & (0.6, 0.4) & (0.2, 0.7) \\ (0.8, 0.1) & (0.4, 0.5) & (0.3, 0.6) \\ (0.6, 0.4) & (0.3, 0.7) & (0.4, 0.5) \end{bmatrix}, \quad B^{C}=\begin{bmatrix} e_1 & e_2 & e_3 \\ (0.2, 0.6) & (0.3, 0.6) & (0.5, 0.5) \\ (0.4, 0.6) & (0.5, 0.5) & (0.1, 0.8) \\ (0.7, 0.2) & (0.4, 0.6) & (0.1, 0.7) \\ (0.4, 0.6) & (0.1, 0.8) & (0.4, 0.5) \end{bmatrix}.$$

**Step 3:** Compute the sums $A + B$ and $A^C + B^C$ as follows:

$$A+B=\begin{bmatrix} (\max\{0.7, 0.6\}, \min\{0.1, 0.2\}) & (\max\{0.6, 0.6\},\min\{0.3, 0.3\}) & (\max\{0.5, 0.5\}, \min\{0.4, 0.5\}) \\ (\max\{0.5, 0.6\}, \min\{0.5, 0.4\}) & \max\{0.4, 0.5\}, \min\{0.6, 0.5\}) & (\max\{0.7, 0.8\}, \min\{0.2, 0.1\}) \\ (\max\{0.1, 0.2\}, \min\{0.8, 0.7\}) & \max\{0.5, 0.6\}, \min\{0.4, 0.4\}) & (\max(0.6, 0.7\}, \min\{0.6, 0.7\}) \\ (\max\{0.4, 0.6\}, \min\{0.4, 0.6\}) & \max\{0.7, 0.8\}, \min\{0.3, 0.1\}) & (\max(0.5, 0.5\}, \min\{0.4, 0.4\}) \end{bmatrix}$$

$$\text{i.e., } A+B=\begin{bmatrix} e_1 & e_2 & e_3 \\ (0.7, 0.1) & (0.6, 0.3) & (0.5, 0.4) \\ (0.6, 0.4) & (0.6, 0.3) & (0.8, 0.1) \\ (0.2, 0.7) & (0.6, 0.4) & (0.7, 0.1) \\ (0.6, 0.4) & (0.8, 0.1) & (0.5, 0.4) \end{bmatrix}, and$$

$$A^C+B^C = \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.2,\ 0.6) & (0.3,\ 0.6) & (0.5,\ 0.5) \\ (0.5,\ 0.5) & (0.6,\ 0.4) & (0.2,\ 0.7) \\ (0.8,\ 0.1) & (0.4,\ 0.5) & (0.3,\ 0.6) \\ (0.6,\ 0.4) & (0.3,\ 0.7) & (0.4,\ 0.5) \end{bmatrix}.$$

**Step 4:** Compute the *value* matrices $V(A+B)$ and $V(A^C+B^C)$ as follows:

$$V(A+B) \begin{bmatrix} e_1 & e_2 & e_3 \\ 0.7-0.1 & 0.6-0.3 & 5-0.4 \\ 0.6\ -04. & 0.5-0.5 & 8-0.1 \\ 0.2\ -0.7 & 0.6-0.4 & 7-0.1 \\ 0.6\ -0.4 & 0.8-0.1 & 4-0.4 \end{bmatrix} = \begin{bmatrix} e_1 & e_2 & e_3 \\ 0.6 & 0.3 & 0.2 \\ 0.2 & 0.0 & 0.7 \\ -.5 & 0.2 & 0.6 \\ 0.2 & 0.7 & 0.1 \end{bmatrix}$$

$$V(A^C+B^C)= \begin{bmatrix} e_1 & e_2 & e_3 \\ -.6 & -.3 & 0.0 \\ 0.0 & 0.1 & -.5 \\ 0.7 & -.1 & -.3 \\ 0.2 & -.4 & -.1 \end{bmatrix}$$

**Step 5:** Compute the matrix $S_{(A+B),(A^C+B^C)}$ and the total score $S_i$ as follows :

$$S_{(A+B),\ (A^C+B^C)} = V(A+B)-V(A^C+B^C) = \begin{bmatrix} e_1 & e_2 & e_3 \\ 1.0 & 0.6 & 0.1 \\ 0.2 & -.1 & 1.2 \\ -1.2 & 0.3 & 0.9 \\ 0.0 & 1.1 & 0.1 \end{bmatrix}, \quad \text{and}$$

$$S_i = \begin{bmatrix} 1.0\ +0.6+0.1 \\ 0.2\ -\ 0.1+1.2 \\ -1.2+0.3+0.9 \\ 0.0+1.1+0.2 \end{bmatrix} = \begin{bmatrix} 1.7 \\ 1.3 \\ 0.0 \\ 1.3 \end{bmatrix}.$$

This leads to the conclusion that the candidate $c_1$is the best choice, since it has the maximum value.

**Conclusion**

A comprehensive study of the fundamentals and applications of soft set theory has been carried out. Besides, several hybrids of soft sets are introduced to attract further research. Since the subject is relatively new, the panoply of basic notions introduced here may appear a bit staggering. Most of the related references are included.

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