A NEW PATH TO REACH THE SOLUTION OF FRACTIONAL MECHANICAL OSCILLATOR USING ADOMAIN DECOMPOSITION METHOD

By

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Abstract

The present paper describes the fractional mechanical oscillator of a simple system represented by fractional differential equation (FDE). The order of derivatives is $0 < \alpha \leq 1$. The system can be solved in terms of Mittag-Leffler function depending on the parameter α. In this paper, a new and developed approach Adomain Decomposition Method (ADM) has been worked out by using adomain polynomial as a proposed solution. In this regards, the concept of Jumarie derivative in terms of Mittag-Leffler function for FDE is utilized. With the help of MATHEMATICA software, the total displacement of the system has been studied for different values of α's $(0 < \alpha \leq 1)$.

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1. Introduction

In the last few decades, fractional based calculus spreads widely in various fields of science and engineering. The fractional calculus involves with fractional derivatives (of order $\alpha, 0 < \alpha \leq 1$) which has inbuilt integration (often called differeintegrals) and non-local in character. The order of the derivative classifies and quantifies the strong influence of past history if α is away from 1 and requires information about the past states. The closure to the value of 1 indicates that the influence of history is minimal. This type of memory effect can be represented by means of convolution between the function and a memorykernel (power of time).

Many authors have developed and defined the concepts of fractional integrals and derivatives in their own way. Of them, Riemann-Liouville, Weyl and Grunworld-Letnikov are worth mentioning. However, Caputo (1967) reformulated the more classic definition of the Riemann-Liouville fractional derivatives in order to solve fractional differential equation (FDE) with integer order initial conditions. The definitions of Riemann-Liouville and Caputo's are as follows [8, 16, 24, 30]:

Fractional Integration:

A repeated n-folded integration is defined as

$$D^{-n} f(t) = I^n f(t) = \frac{1}{(n-1)!}\int_0^t (t-u)^{n-1} f(u)du.$$

For any real number α>0, the repeated α- fold integration is defined as

$$D^{-\alpha} f(t) = I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-u)^{\alpha-1} f(u)du.$$

The following are some definitions of differeintegrals in the sense of several authors.

Riemann-Liouville (RL) Integration: For $\alpha > 0$

(i) *Forward Integration:*

$$ {}_aI_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-u)^{\alpha-1} f(u) du. $$

(ii) *Backward Integration:*

$$ {}_xI_b^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (u-x)^{\alpha-1} f(u) du. $$

Fractional derivatives

Riemann- Liouville left hand definition (RL LHD):

Let us select an integer $m > \alpha, \alpha$: fractional number such that

i) Integrate the function (m-α)- folds in the sense of forward RL

ii) Differentiate the above result by m

Then, *RL LHD* is defined as

$$ {}_0D_t^{\alpha} f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f(u)}{(x-u)^{\alpha+1-m}} du\right], \quad (m-1 \le \alpha < m). $$

Caputo right hand definition (Caputo RHD):

Let us select an integer $m > \alpha, \alpha$: fractional number such that

(i) Differentiate the function m times,

(ii) Integrate the above result $(m-\alpha)-$ *fold by RL LHD integration method.*

Thus, Caputo's RHD is defined as

$$ {}_0^CD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{\frac{d^m}{dt^m} f(u)}{(t-u)^{\alpha+1-m}} du, \ (m-1 \le \alpha < m). $$

It is to be noted that the RL LHD and Caputo RHD are equivalent.

But the Caputo derivative of a constant function is zero, whereas both left and right RL derivative of a constant (K) is non zero. In this regards, a modified left (right) RL derivative has been developed by Jumarie [14]. Thus the Jumarie derivative is defined as,

$$ {}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(-\alpha)} \int_0^t (t-\xi)^{-\alpha-1} f(\xi) d\xi, \ \alpha < 0, $$

$$ \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\xi)^{-\alpha} [f(\xi) - f(0)] d\xi, \ 0 < \alpha < 1, $$

$$ [f^{(\alpha-n)}(t)]^{(n)}, n \le \alpha < n+1, n \ge 1. $$

Using this definition, we get, ${}_0D_t^{\alpha}\{K\} = 0, \ 0 \le \alpha < 1.$

In 1903 Mittag-Leffler [8, 16, 24, 30] introduce a function defined by infinite series

$$ E_{\alpha}(at^{\alpha}) = 1 + \frac{at^{\alpha}}{\Gamma(1+\alpha)} + \frac{a^2t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{a^3t^{3\alpha}}{\Gamma(1+3\alpha)} + ... $$

is the one parameter Mittag-Leffler function. This function can be used as a trial solution of FDE like exponential function of whole order differential equation.

Using the definition of Jumarie derivative, several properties of Mittag-Leffler function has been deduced. Thus, one can deduce the following properties

1) $D^{\alpha}(E_{\alpha}(at^{\alpha})) = aE_{\alpha}(at^{\alpha})$, 2) $E_{\alpha}(at^{\alpha})E_{\alpha}(bt^{\alpha}) = E_{\alpha}((a+b)t^{\alpha})$.

Moreover, the analytical solution of linear fractional differential equation has been solved with the help of Jumarie derivative in terms of Mittag-Leffler function [8, 24, 30]. For example, the solution of

$(D^{\alpha} - a_1)(D^{\alpha} - a_2)\ldots(D^{\alpha} - a_n)y(t) = 0$, with all a_i's distinct is

$y = \Sigma A_i E_{\alpha}(a_i t^{\alpha})$, where A_i are arbitrary constants and $E_{\alpha}(a_i t^{\alpha})$ is a one-parameter family of Mittag-Leffler function. As a particular case, the solution of FDE

$D^{2\alpha}y - 2aD^{\alpha}y + a^2 y = 0$ is

$y = (At^{\alpha} + B)E_{\alpha}(at^{\alpha}), A = A_1/\Gamma(1+\alpha)$ where A and B are arbitrary constants.

Most of the applications and physical manifestations of fractional calculus have been found in 20[th] and 21[st] century [3, 5, 6, 7, 9-13, 15, 16, 18-21, 25-30]. The applications like Abel's fractional integral equation of Tatochrome, Modelling of speech signals, Modelling of cardiac tissue electrode interface, fractional differentiation of edge detection related to image processing, the solution of time dependent viscous diffusion fluid problems are worth mentioning. Some other fields which includes fractional calculus are polymer science, fractal phenomena, ultrasonic wave propagation in human cancellous bone.

The present paper developsthe solution of fractional oscillator equation [4] using Adomain decomposition method (ADM) [1, 2, 8] along with Jumarie formulation of FDE, which will be discussed later on.

The fractional differential equation corresponding to the mechanical system [24] (see figure 1) is given by

$$\frac{m}{\sigma^{2(1-\alpha)}}\frac{d^{2\alpha}x(t)}{dt^{2\alpha}} + \frac{\beta}{\sigma^{(1-\alpha)}}\frac{d^{\alpha}x(t)}{dt^{\alpha}} + kx(t) = F(t) \qquad (1)$$

where, m: mass measured in kg

β: damped coefficient measured in *N.s/m*

k: string constant measured in *N/m*

σ: auxiliary parameter (fractional time component of the system)

Equation (1) can be rewritten as

$$\frac{d^{2\alpha}x(t)}{dt^{2\alpha}} + \gamma\frac{d^{\alpha}x(t)}{dt^{\alpha}} + \omega^2 x(t) = \frac{F(t)}{m} \qquad (2)$$

$$x(0) = 0, \dot{x}(0) = 0$$

where,

$$\gamma = \frac{\beta\sigma^{1-\alpha}}{m},$$

$$\omega^2 = \omega_0^2\sigma^{2(1-\sigma)},$$

$\omega_0^2 = \dfrac{k}{m}$, a fundamental frequency when $\alpha = 1$.

ω: driving frequency.

The different form of forcing function *F(t)* is defined as

$$\frac{F(t)}{m} = \omega_0^2, t \geq 0$$

$$0, t < 0 \qquad (2a)$$

and

$$\frac{F(t)}{m} = F_0 \sin(\omega t) \tag{2b}$$

is a sinusoidal driving force, F_0 is the driven amplitude.

$\frac{d^{2\alpha}}{dt^{2\alpha}}, \frac{d^{\alpha}}{dt^{\alpha}}$ $(0 < \alpha < 1)$ is a fractional order derivative in the sense of Riemann Liouville. The parameter σ is introduced in order to make the time dimensionality to be consistent. This means that the fractional time derivatives operator $\frac{d^{\alpha}}{dt^{\alpha}}$ has $s^{-\alpha}$ $(0 < \alpha \leq 1)$ but if the new parameter is introduced in the form $\frac{1}{\sigma^{1-\alpha}}\frac{d^{\alpha}}{dt^{\alpha}}$ $(0 < \alpha \leq 1)$, then $[\frac{1}{\sigma^{1-\alpha}}\frac{d^{\alpha}}{dt^{\alpha}}] = 1/s$ (3)

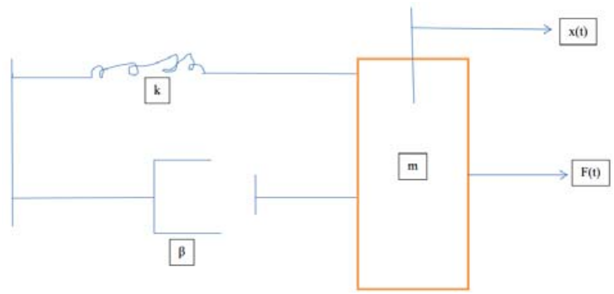

Figure 1. Damped Harmonic Oscillator

In fact (3) is dimensionally consistent if and only if $[\sigma] = s$. The present paper discusses the solution of (2) with forcing function as (2a).

In order to solve the system (2), a Mittag-Leffler function has been used, which depends on the order of the derivative α [7, 10, 24, 30]. However, a more developed and realistic approach viz.,Adomain Decomposition Method (ADM) is carried out to solve the system (2).The concept of Jumarie derivative in terms of Mittag-Leffler function is also utilized in the initial step. The major advantages of ADM is that, it does not involve any discretizations (and hence free from rounding off errors) and not huge amount of computer memory is required. Moreover, with Riemann-Liouville derivatives, the initial condition may not require fractional order state rather it would be an integer order state. However, ADM is close to physical reality and visualize the reaction system by decomposing the total gross reactions from zeroth mode to infinity mode. The sum of all these modes is the solution of (2).

The whole matter of the paper is organized as follows:

Section 1 is the introductory one. In section 2, a generalized dynamic system and the idea of solution of FDE using ADM (which involves adomain polynomial) is discussed. Section 3 gives the solution of the system (2) using the ADM. Finally in section 4, based on ADM a numerical simulation is done on fractional mechanical oscillator using MATHEMATICA software.

2. Generalized Dynamic system and its solution using Adomain Decomposition Method (ADM) [7, 8]

General physics law states that a system will react to external stimulus and will have opposition to changes and the process is describe by the extraordinary differential equation,

$$Fu = G, \tag{4}$$

where, F: general linear or non-linear differential equation which may be fractional in the sense of Riemann-Liouville. This operator can be decomposed as,

$$Fu = L_0 + Ru + Nu = G, \tag{5}$$

L_0: highest order derivative (may be positive integer or fractional order), which is invertible i.e.

$L_0 u = \frac{d^m u(x)}{dx^m} = D_x^m u(x)$, which is linear operator.

R: linear differential (remainder) operator of order less than that of L_0; this can also be fractional linear operator i.e.

$Ru \equiv a_1 D_x^{m-1} + a_2 D_x^{m-2} + a_3 D_x^{m-3} + ... + a_{m-1} D_x^1$, where a_0, a_1, a_2,... a_{m-1} are constants.

N: non- linear part which will be decomposed into infinite sum of adomain polynomial (this term may be linear or constant) i.e.

$Nu \equiv a_m u(x) + b_k [u(x)]^k + b_{k-1}[u(x)]^{k-1} + b_{k-2}[u(x)]^{k-2} + ... + b_0 u(x)$

G(x): sum of all external stimulus source/ sink.

The R and N are internal stimuli and when it get balanced with the external stimulus G then the process of the parameter remains static without any growth or decay. Otherwise, the process parameter will have a solution as infinite (or finite) decomposed modes; generated by the system itself to oppose stimulus generated internally by previous mode.

The idea of solution to (4) is a new approach called Adomain Decomposition Method (ADM). ADM method is an analytical method and has a certain advantage over standard numerical techniques. The decomposed parts of ADM method are related to system reactions of various modes from zeroth mode to infinity mode. The sum of all these modes is the solution of (4). In (4), if L_0 is the fractional differential operator in the sense of RL, then the initial conditions should be in fractional order say, $u^m(0)$, $u^{m-1}(0)$ for m to be fractional. These states are hard to visualize. But with this ADM, the RL formulation does not need these fractional initial states instead requires $u(0)$, $u^{(1)}(0)$, $u^{(2)}(0)$ etc.; the integer order states gives the solution and thus physically understandable.

Equation (5) can be rewritten as

$L_0 u = G - Ru - Nu.$

Applying invertible operator both sides, we get

$$u = \Phi + L_0^{-1} G - L_0^{-1}[R(u)] - L_0^{-1}[N(u)], \tag{6}$$

Φ: solution of homogeneous equation $L_0 u=0$, so that $L_0 \Phi=0$ and this comes from initial and boundary condition.

Then with ADM, we have,

$u(\lambda)=u_0+\lambda u_1+\lambda^2 u_2+\lambda^3 u_3+\dots$

This is a Maclaurin's series with respect to λ with coefficient u_n $(n = 1,2, ...)$ and $u_n = u^{(n)}(0)/n!$ around $\lambda = 0$. Then, $N(u)$ is a Maclaurin's series with respect to λ and obtain

$$N(u) = \sum_{n=0}^{\infty} \lambda^n A_n \text{, where } A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\right].$$

For $N(u)$ linear, $A_0 = u_0$ and $A_n = u_n$.

For $N(u)$ to be non- linear,

$A_0=N(u_0)$, $A_1 = u_1 N'(u_0)$, $A_2 = u_2 N'(u_0) + \frac{1}{2} u_1^2 N''(u_0)$,

$A_3 = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{u_1^3}{3!} N'''(u_0)$ etc.

The parameter λ is just an identifier for collecting terms in suitable way, such that u_n depends on $u_0, u_1, u_2, \dots, u_{n-1}$ and later on we will set λ=1. Parameterizing the equation (5), we get,

$$u = \Phi + L_0^{-1} G - \lambda L_0^{-1}[R(u)] - \lambda L_0^{-1}[N(u)] \tag{7}$$

Expanding the decomposition, (7) gives

$$u = \Phi + L_0^{-1}G - \lambda L_0^{-1}R\left(\sum_{n=0}^{\infty}\lambda^n u_n\right) - \lambda L_0^{-1}N\left(\sum_{n=0}^{\infty}\lambda^n A_n\right) \quad . \tag{8}$$

Comparing the like powers of λ in the expression for $n = 0$, we have,

$u_0 = \Phi + L_0^{-1}G$. Similarly, for n =1,2,…, we have

$$u_n = -L_0^{-1}R(u_{n-1}) - L_0^{-1}(A_{n-1}) \text{ for } n \geq 1. \tag{9}$$

3. Solution of fractional mechanical oscillator (2) using Adomain Decomposition Method

We rewrite the fractional mechanical oscillator (2) as

$$ {}_0D_t^{2\alpha}x + \gamma\, {}_0D_t^{\alpha}x + \lambda x = F_0, \tag{10}$$

$$x(0) = 0, \dot{x}(0) = 0,$$

where, $\gamma = \dfrac{\beta\sigma^{1-\alpha}}{m}, \lambda = \omega^2, F_0 = \omega_0^2.$

Equation (10) is again rewritten as $Fx=G$, where

$Fx=L_0x+Rx+Nx$ and $G=F_0$ (constant) and

$L_0 = {}_0D_t^{2\alpha}, R = \gamma\, {}_0D_t^{\alpha},$

$N(x)=\lambda x$ (constant).

The decomposed equation can be written as,

$L_0x = G\text{-}R(x)\text{-}N(x)$.

Applying invert operator on both sides, we get

$$x = \phi + L_0^{-1}G - L_0^{-1}[R(x)] - L_0^{-1}[N(x)],$$

where φ is the solution of the homogeneous equation $L_0(x) = 0$, so that $L_0(\varphi)=0$.

Consider the homogeneous equation

$${}_0D_t^{2\alpha}x = 0,$$

Let $x = AE_\alpha(mt^\alpha)$ ($\neq 0$) be a nontrivial trial solution of the homogeneous equation.

By using Jumarie derivative, we have $D^{2\alpha}x = Am^2E_\alpha(mt^\alpha)$.

Therefore, $x = A+Bt^\alpha$ and hence $x = 0$, by using initial conditions

$x(0) = 0, \dot{x}(0) = 0.$

By using the definition of Jumarie derivative, the ADM method thus generates the mode as

$$x_0 = \phi + L_0^{-1}G = {}_0D_t^{-2\alpha}F_0 = F_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

$A_0 = N(x_0) = \lambda x_0.$

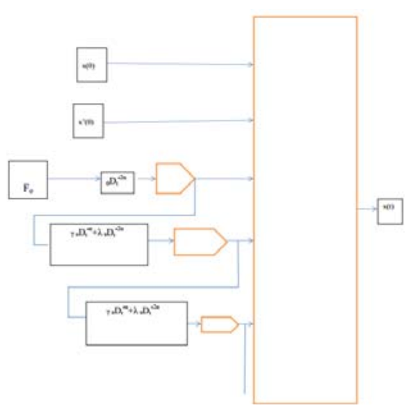

Figure 2. Block Diagram of fractional mechanical oscillator

Again, by using Jumarie derivative,

$$x_1 = -L_0^{-1}R(x_0) - L_0^{-1}A_0 = \left(-{}_0D_t^{-2\alpha}\gamma\,{}_0D_t^{\alpha} - {}_0D_t^{-2\alpha}\lambda\right)x_0$$

$$= \left(-\gamma\,{}_0D_t^{-\alpha} - \lambda\,{}_0D_t^{-2\alpha}\right)\left(F_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right) = -\gamma F_0\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \lambda F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)},$$

$A_1 = x_1$,

$$x_2 = \left(-\gamma\,{}_0D_t^{-\alpha} - \lambda\,{}_0D_t^{-2\alpha}\right)x_1$$

$$= \left(-\gamma\,{}_0D_t^{-\alpha} - \lambda\,{}_0D_t^{-2\alpha}\right)\left(-\gamma F_0\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \lambda F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}\right)$$

$$= \gamma^2 F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + 2\gamma\lambda F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \lambda^2 F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)},$$

$A_2 = x_2$

$$x_3 = \left(-\gamma\,{}_0D_t^{-\alpha} - \lambda\,{}_0D_t^{-2\alpha}\right)x_2$$

$$= \left(-\gamma\,{}_0D_t^{-\alpha} - \lambda\,{}_0D_t^{-2\alpha}\right)\left(\gamma^2 F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + 2\gamma\lambda F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \lambda^2 F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)}\right)$$

$$= -\gamma^3 F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} - 3\gamma^2\lambda F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)} - 3\gamma\lambda^2 F_0\frac{t^{7\alpha}}{\Gamma(7\alpha+1)} - \lambda^3\frac{t^{8\alpha}}{\Gamma(8\alpha+1)} \text{ etc.}$$

The following table indicates the modal force and displacement for the fractional mechanical oscillator with fractional order derivative α ($0 < \alpha \le 1$):

Table 1:

Mode	**Force**	**Displacement**
0	F_0	$F_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$
1	$-F_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$	$-\gamma F_0\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \lambda F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}$
2	$\gamma F_0\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \lambda F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)}$	$\gamma^2 F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + 2\gamma\lambda F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \lambda^2 F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)}$
3	$-\gamma^2 F_0\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - 2\gamma\lambda F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} - \lambda^2 F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)}$	$-\gamma^3 F_0\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} - 3\gamma^2\lambda F_0\frac{t^{6\alpha}}{\Gamma(6\alpha+1)} - 3\gamma\lambda^2 F_0\frac{t^{7\alpha}}{\Gamma(7\alpha+1)} - \lambda^3\frac{t^{8\alpha}}{\Gamma(8\alpha+1)}$
...	...	

Therefore the solution of (10) is given by

$x = x_0 + x_1 + x_2 + \ldots$

$$= F_0 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \gamma F_0 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \left(\gamma^2 - \lambda\right) F_0 \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + \left(2\gamma\lambda - \gamma^3\right) F_0 \frac{t^{5\alpha}}{\Gamma(5\alpha+1)}$$

$$+ \left(\lambda^2 - 3\gamma^2\lambda\right) F_0 \frac{t^{6\alpha}}{\Gamma(6\alpha+1)} + (\gamma^4 - 3\gamma\lambda^2) F_0 \frac{t^{7\alpha}}{\Gamma(7\alpha+1)} + (4\gamma^3\lambda - \lambda^3) F_0 \frac{t^{8\alpha}}{\Gamma(8\alpha+1)} + \ldots.$$

$$F_0 = \frac{k}{m}, \gamma = \frac{\beta\sigma^{1-\alpha}}{m}, \lambda = \frac{k}{m}\sigma^{2(1-\alpha)}.$$

4. Numerical Simulation

Based on ADM, a numerical simulation is done by using MATHEMATICA software for fractional mechanical oscillator given by (10). The step length for t-axis has been taken to be 0.5. We take the parameter value for mechanical oscillator as m = 250 kg, F = 2450 N, x = 0.5 m, so that $k = F/x$ = 4900 N/m. It is to be noted that the parameter α, which represents the fractional order time derivative can be related to the auxiliary parameter σ, which characterizes the existence in the system of fractional structure. For $\beta = 0$, the system (10) gives the relation

$$\alpha = \frac{\sigma}{\sqrt{m/k}} = \sigma\omega_0, \quad 0 < \sigma \leq \sqrt{m/k}.$$

Thus for $\beta = 0$; $\gamma = 0$, we have the sets of values of the parameters give as follows:

Table 2.

α	σ	F_0	λ
1	0.22	19.6	19.6
0.75	0.17	19.6	8.036
0.5	0.11	19.6	2.156
0.25	0.06	19.6	0.196

The corresponding graph for fractional mechanical oscillator with αth order element where, α=1, 0.75, 0.5, 0.25 is shown in Figure 3 with step length in t-axis as 0.5.

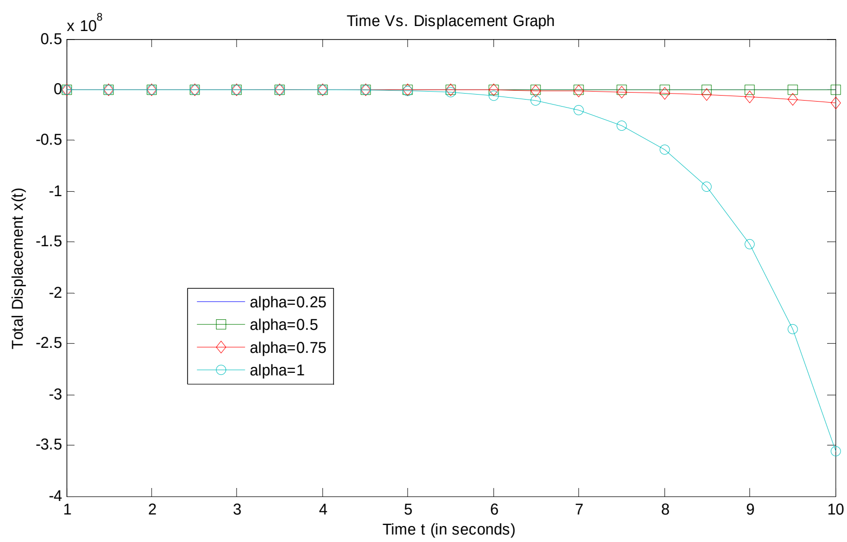

Figure 3. Time Vs. Total Displacement of mechanical oscillator system

Conclusion

In this study, a solution of fractional mechanical oscillator is represented by fractional differential equation (FDE) with αth order derivative in the sense of Riemann Liouville type is developed. It is generally given by driven harmonic oscillators and the equation is a second order differential equation of integer order. In order to tackle the solution to the system, a typical analytical method can be used by using Mittag-Leffler function as well as a series solution can be decomposed as Maclaurin's series at the origin with all its derivative components. But, in the present problem, a very new approach, Adomain Decomposition Method (ADM) is used to obtain the total displacement. It is illustrated in the block diagram (Figure 2) that the total displacement can be decomposed from zeroth mode to infinity mode, where the process parameter will have a solution generated by the system itself to oppose stimulus generated internally by the previous mode with initial conditions $x(0) = 0 = x'(0)$. However, it is demonstrated that using the capabilities of MATHEMATICA software, the total displacement of the oscillator by taking several values of parameters given in Table 2. It is clear from Figure 3, a sharp bent occurs for second order mechanical oscillator system i.e. at α =1. But in reality, it is slightly deviated when past states i.e. the fractional order element say α = 0.75, 0.5, 0.25 is taken into consideration.

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