COMPUTATIONAL MODELLING FOR THE FORMATION OF GEOMETRIC SERIES USING ANNAMALAI COMPUTING METHOD

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Abstract
This paper presents a mathematical model for the formation as well as computation of geometric series in a novel way. Using Annamalai computing method a simple mathematical model is established for analysis and manipulation of geometric series and summability. This new model could be used in the research fields of physics, engineering, biology, economics, computer science, queueing theory, and finance. In this paper, a novel computational model had also been developed such that $a\sum_{i=k}^{\infty} y^i = \frac{ay^k}{1-y}$ and $\sum_{i=0}^{\infty}\sum_{j=i}^{\infty} ay^j = \frac{a}{(1-y)^2}$, $(0 < y < 1)$. This could be very interesting and informative for current students and researchers.

Keywords: Annamalai computing method, Computational geometric series

1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in mathematics, science, management, and technology. The finite and infinite geometric sequence, series, and summability have important applications in engineering, economics, computer science, medicine, biology, physics, queueing theory, and finance [3, 4, 5, 8].
It is eventually understood that the summations of finite geometric series were

$$\sum_{i\backslash=0}^{n-1} ax^i = \frac{a(x^n - 1)}{(x-1)} \text{ and } \sum_{i=1}^{n-1} ax^i = \frac{a(x^n - x)}{x-1}, \quad x \neq 1$$

and the summations of infinite geometric series were

$$\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x} \text{ and } \sum_{i=1}^{\infty} ax^i = \frac{ax}{1-x} \ (0 < x < 1)$$

Geometric series can be used to convert the decimal to a fraction.
For examples,

(i) $0.9999999...... = \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + ... = \frac{ax}{1-x} = 1$ where $a = 9$ and $x = \frac{1}{10}$

(ii) $9.9999999...... = 9 + \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + ... = \frac{a}{1-x} = 10$ where $a = 9$ and $x = \frac{1}{10}$

(iii) $0.777777......... = \frac{7}{10} + \frac{7}{10^2} + \frac{7}{10^3} + ... = \frac{ax}{1-x} = \frac{7}{9}$ where $a = 7$ and $x = \frac{1}{10}$

2. Annamalai Computing Geometric Series

Annamalai computing method [1] provided a novel approach for computation of geometric series in a new way.

$$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Leftrightarrow ax^n = ax^n, \; \left(x \neq 1\right)$$

Proof

RHS $\Rightarrow ax^n = ax^n$

$\Rightarrow ax^n = a(x-1)x^{n-1} + ax^{n-1}$

$\Rightarrow ax^n = a(x-1)x^{n-1} + a(x-1)x^{n-2} + ... + a(x-1)x^i + ... + a(x-1)x^{-m} + ax^{-m}$

$ax^n = ax^n \Rightarrow \sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1}$ ---- (1)

LHS $\Rightarrow \sum_{i=-m}^{n-1} ar^i = \frac{a\left(x^n - x^{-m}\right)}{x-1}$

$\Rightarrow ax^n = a(x-1)x^{n-1} + a(x-1)x^{n-2} + ... + a(x-1)x^i + ... + a(x-1)x^{-m} + ax^{-m}$

$\Rightarrow ax^n = ax^n$

$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Rightarrow ax^n = ax^n$ ---- (2)

From (1) and (2) we get:

$$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Leftrightarrow ax^n = ax^n, \; \left(x \neq 1\right)$$

From the above result we can further find the following sumability :

(i) $a\sum_{i=k}^{n-1} x^i = \frac{a\left(x^n - 1 + 1 - x^k\right)}{x-1} = \frac{a\left(x^n - 1\right)}{x-1} - \frac{a\left(x^k - 1\right)}{x-1} = a\left(\sum_{i=0}^{n-1} x^i - \sum_{i=0}^{k-1} x^i\right)$

(ii) $\sum_{i=-m}^{n-1} ax^i = \sum_{i=-m}^{-1} ax^i + \sum_{i=0}^{n-1} ax^i = \sum_{i=1}^{m} \frac{a}{x^i} + \sum_{i=o}^{n-1} ax^i = a\left(\frac{\frac{1}{x} - \frac{1}{x^{m+1}}}{1 - \frac{1}{x}} + \frac{x^n - 1}{x-1}\right) = \frac{a\left(x^n - x^{-m}\right)}{x-1}$

3. Computational Modelling

The equality $ax = ax$ [7] was used to design the computational modelling,

$ax = ax \Leftrightarrow ax = (x-1)a + a \Leftrightarrow ax = (x-1)\frac{a}{x^0} + (x-1)\frac{a}{x} + (x-1)\frac{a}{x^2} + ... + (x-1)\frac{a}{x^n} + \frac{a}{x^n}$

$ax = ax \Leftrightarrow \sum_{1=0}^{n} \frac{a}{x^i} = \frac{\left(ax - \frac{a}{x^n}\right)}{x-1} = \frac{ax\left(1 - \frac{1}{x^{n+1}}\right)}{x\left(1 - \frac{1}{x}\right)} = \frac{a\left(1 - y^{n+1}\right)}{1-y}$ where $y = \frac{1}{x}$

Now $\frac{a}{y} = \frac{a}{y} \Leftrightarrow \sum_{i=0}^{n-1} ay^i = \frac{a\left(y^n - 1\right)}{y-1}$ where it is understood that $\frac{a}{y} = \frac{a}{y} \Rightarrow ay = ay$

We know that if $0 < y < 1$, then $\sum_{i=0}^{n-1} ay^i = \frac{a\left(1 - y^n\right)}{1-y}$ and $\sum_{i=0}^{\alpha} ay^i = \frac{a}{1-y}$

Similarly, using Annamalai computing geometric series $a\sum_{i=k}^{n-1} y^i = \frac{a\left(y^n - y^k\right)}{y-1}$ $(y \neq 1)$, we can derive $a\sum_{i=k}^{n-1} y^i = \frac{a\left(y^k - y^n\right)}{1-y}$ and $a\sum_{i=k}^{\alpha} y^i = \frac{ay^k}{1-y}$ $(0 < y < 1)$ where $k > 0$ is an integer constant.

From the above result it can further found the following sum ability:

$$(i)\quad \sum_{i=0}^{\infty} ay^i - \sum_{i=k}^{\infty} ay^i = \frac{a}{1-y} - \frac{ay^k}{1-y} = \frac{a\left(1-y^k\right)}{1-y} = \sum_{i=0}^{k-1} ay^i$$

$$(ii)\quad \sum_{i=0}^{\infty}\sum_{j=i}^{\infty} ay^j = \sum_{j=0}^{\infty} ay^j + \sum_{j=1}^{\infty} ay^j + \sum_{j=2}^{\infty} ay^j + ... = \frac{a}{1-y} + \frac{ay}{1-y} + \frac{ay^2}{i-y} + ... = \frac{a}{(1-y)^2}$$

Conclusion

In the research study, a novel technique has been introduced to form the generalized geometric series and computing it. Also, a novel computational model was also developed such that $a\sum_{i=k}^{\infty} y^i = \frac{ay^k}{1-y}$ and $\sum_{i=0}^{\infty}\sum_{j=i}^{\infty} ay^j = \frac{a}{(1-y)^2}$, $(0 < y < 1)$. This new model can be used in the research fields of physics, engineering, biology, economics, computer science, queueing theory, and finance.

References

[1] C. Annamalai, Computational geometric series model with key applications in informatics, *Int. J. Comput. Intel. Res.*, **5** (4) (2009), 485-499.

[2] C. Annamalai, A novel computational technique for the geometric progression of powers of two, *J. Sci. Math. Res.*, **3** (2009), 16-17.

[3] C. Annamalai, Applications of exponential decay and geometric series in effective medicine dosage, *J. Adv. Biosci. Biotech.*, **1** (2010), 51-54.

[4] C. Annamalai, Computational model to study the dose concentration in bloodstream of patients, *Int. J. Med. and Pharm. Sci.*, **1** (5) (2011), 1-7.

[5] C. Annamalai, ACM cryptographic key exchange for secure communications, *Int. J. Cryp. Res.*, **3** (1) (2011), 27-33.

[6] C. Annamalai , A novel approach to ACM-geometric progression, *J. Bas. Appl. Res. Int.*, **2** (1) (2015), 39-40.

[7] C. Annamalai, Computing method for formation of geometric series using in science and technology, *Int. J. Sci. Adv. Res. Tech.*, **3** (8) (2017), 287-289.

[8] https://courses.lumenlearning.com/boundless-algebra/chapter/geometric-sequences-and-series/